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LOCAL KODAMA-TYPE TRANSFORMATIONS AND INTEGRABILITY LIMITS OF THE EXTENDED FIFTH-ORDER KORTEWEG-DE VRIES EQUATION

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Abstract. The extended fifth-order Korteweg-de Vries (KdV) equation $u_t + 6uu_x + u_{xxx} + \varepsilon(\alpha_1 u^2 u_x + \gamma_1 uu_{xxx} + \gamma_2 u_x u_{xx} + \beta_1 u_{5x}) = 0$ models long internal waves in a two-layer stratified fluid with next-order dispersive and nonlinear corrections. Whether these four correction terms preserve integrability under a purely local change of the dependent variable, without a non-local term, had not been settled as an exact statement.

We apply the local near-identity transformation $u = v + \varepsilon(av^2 + bv_{xx})$ and compute its effect on the four correction coefficients to $O(\varepsilon)$. We evaluate the time derivative of the third KdV conserved density under the same transformation, examine the compatibility of the 2×2 AKNS Lax scheme with the quadratic term $6uu_x$, and carry out a linear dispersion analysis of the corrected equation.

The local transformation removes the coefficients α_1 and γ_2 completely, for any values, but cannot remove γ_1 or β_1 for any choice of the transformation parameters. The equation reduces exactly to KdV if and only if $\gamma_1 = \beta_1 = 0$. The third KdV invariant is conserved along solutions under the same condition. The symmetric AKNS reduction $q = u$, $r = -u$ is structurally incompatible with the $6uu_x$ term, the reduction $q = u$, $r \equiv -1$ is the correct one. The correction to the group-phase velocity gap from fifth-order dispersion is obtained in closed form, $\Delta V(k, \varepsilon) - \Delta V(k, 0) = 4\varepsilon\beta_1 k^4$.

The results give an exact, checkable criterion for which correction terms in the fifth-order internal-wave equation are removable by local means and which reflect genuine higher-order physics, relevant to estimating wave-packet asymmetry in stratified oceanic flows.

Keywords: extended Korteweg-de Vries equation, Kodama transformation, conservation law, Lax hierarchy, Lax pair, group and phase velocity

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КЕНЕТЕЙЛГЕН БЕСІНШІ РЕТТІ КОРТЕВЕГ-ДЕ ФРИЗ ТЕНДЕУІНІҢ ЛОКАЛДЫ КОДАМ ТИПТІ ТҮРЛЕНДІРУІ МЕН ИНТЕГРАЛДАНУ ШЕКТЕРІ

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Аннотация. Бесінші ретті кеңейтілген Кортевег-де Фриз (КдФ) теңдеуі $u_t + 6uu_x + u_{xxx} + \varepsilon(\alpha_1 u^2 u_x + \gamma_1 uu_{xxx} + \gamma_2 u_x u_{xx} + \beta_1 u_{5x}) = 0$ е к і қабатты стратификацияланған сұйықтықтағы ұзын ішкі толқындарды келесі ретті дисперсиялық және бейсызықты түзетулермен сипаттайды. Осы төрт түзету мүшесінің тәуелді айнымалыны тек локальді түрлендіру арқылы, локальді емес мүшелерсіз өзгерткеде интегралдануын сақтайтындығы осы уақытқа дейін нақты тұжырым түрінде шешілмеген.

Осы жұмыста $u = v + \varepsilon(av^2 + bv_{xx})$ түріндегі локальді жуықтап бірлік түрлендіруі қарастырылып, оның төрт түзету коэффициентіне әсерін $O(\varepsilon)$ дәлдігімен есептелген. Сонымен қатар, осы түрлендіру аясында КдФ теңдеудің үшінші сақталу тығыздығының уақыт бойынша туындысы табылып, 2×2

өлшемді АКНС Лакс схемасының $6uu_x$ квадраттық мүшесімен үйлесімділігі зерттелген және түзетілген теңдеудің сызықтық дисперсиялық талдауы жасалынған.

Локальді түрлендіру α_1 және γ_2 коэффициенттерін олардың кез келген мәндерінде толығымен жояды. Алайда түрлендіру параметрлерін қалай тандалса да, γ_1 немесе β_1 коэффициенттерін жойылуы мүмкін емес. Теңдеу дәл КдФ теңдеуіне тек $\gamma_1 = \beta_1 = 0$ болғанда ғана келтіріледі. КдФ теңдеуі үшінші инварианты да дәл осы шарт орындалғанда ғана шешімдер бойымен сақталады. Симметриялы $q = u$, $r = -u$ АКНС редукциясы $6uu_x$ мүшесімен құрылымдық тұрғыдан үйлеспейді, дұрыс редукция $q = u$, $r \equiv -1$ болып табылады. Бесінші ретті дисперсиядан туындайтын топтық және фазалық жылдамдықтар айырымына түзету тұйық түрде алынған: $\Delta V(k, \varepsilon) - \Delta V(k, 0) = 4\varepsilon\beta_1 k^4$.

Алынған нәтижелер бесінші ретті ішкі толқындар теңдеуіндегі қандай түзету мүшелерінің локальді тәсілдермен жоюға болатынын және қайсылары жоғары ретті нақты физикалық әсерлерді сипаттайтынын анықтауға мүмкіндік беретін дәл әрі тексерілетін критерий береді. Бұл критерий стратификацияланған мұхит ағындардағы толқындық пакеттердің асимметриясын бағалау үшін маңызды.

Түйін сөздер: кеңейтілген Кортевег-де Фриз теңдеуі, Кодам түрлендіруі, сақталу заңы, Лакс иерархиялары, Лакс жұбы, топтық және фазалық жылдамдықтар

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ЛОКАЛЬНЫЕ ПРЕОБРАЗОВАНИЕ ТИПА КОДАМЫ И ПРЕДЕЛЫ ИНТЕГРИРУЕМОСТИ РАСШИРЕННОГО УРАВНЕНИЯ КОРТЕВЕГА-ДЕ ФРИЗА ПЯТОГО ПОРЯДКА

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Аннотация. Расширенное уравнение Кортевега-де Фриза (КдФ) пятого порядка $u_t + 6uu_x + u_{xxx} + \varepsilon(\alpha_1 u^2 u_x + \gamma_1 uu_{xxx} + \gamma_2 u_x u_{xx} + \beta_1 u_{5x}) = 0$ моделирует длинные внутренние волны в двухслойной стратифицированной жидкости с учетом дисперсионных и нелинейных поправок следующего порядка.

Вопрос о том, сохраняют ли эти четыре поправочных члена интегрируемость при чисто локальной замене зависимой переменной, без введения нелокального члена, до сих пор не получил точного ответа.

Мы применяем локальное преобразование, близкое к тождественному $u = v + \varepsilon(av^2 + bv_{xx})$ и вычисляем его влияние на четыре коэффициента поправок с точностью до $O(\varepsilon)$. Мы вычисляем производную по времени от третьей сохраняющейся плотности КдФ при том же преобразовании, исследуем совместность матричной схемы Лакса АКНС размера 2×2 с квадратичным членом $6uu_x$ и проводим анализ линейной уравнения с поправкой.

Локальное преобразование полностью устраняет коэффициенты α_1 и γ_2 при любых их значениях, но не может исключить γ_1 или β_1 ни при каком выборе параметров преобразования. Уравнение сводится точно к КдФ тогда и только тогда, когда $\gamma_1 = \beta_1 = 0$. При этом же условии вдоль решений сохраняется третий инвариант КдФ. Поправка к разности между групповой и фазовой скоростями, обусловленная дисперсией пятого порядка, получена в замкнутом виде: $\Delta V(k, \varepsilon) - \Delta V(k, 0) = 4\varepsilon\beta_1 k^4$.

Полученные результаты дают точный, легко проверяемый критерий того, какие именно поправочные члены в уравнении внутренних волн пятого порядка могут быть устранены локальными средствами, а какие отражают подлинную физику высших порядков, что важно для оценки асимметрии волновых пакетов в стратифицированных океанических течениях.

Ключевые слова: расширенное уравнение Кортевеги-де Фриза, преобразование Кодама, закон сохранения, иерархия Лакса, пара Лакса, групповая и фазовая скорость

Introduction. Long internal waves in the ocean rarely behave like the textbook KdV soliton once you look closely. Real stratified fluids add dispersive and nonlinear corrections at the next order in wave amplitude, and the equation that captures this, the extended fifth-order KdV equation

$$u_t + 6uu_x + u_{xxx} + \varepsilon(\alpha_1 u^2 u_x + \gamma_1 uu_{xxx} + \gamma_2 u_x u_{xx} + \beta_1 u_{xxxxx}) = 0 \quad (1.1)$$

has been used for decades to model exactly this situation in a two-layer stratified ocean (Holloway et al., 1999; Marchant et al., 1990; Horikis et al., 2022). Strip away the ε terms and you get back the plain KdV equation, $u_t + 6uu_x + u_{xxx} = 0$, which is integrable by the inverse scattering transform. This has been known since Gardner, Greene, Kruskal, and Miura, and was put on a Lax pair footing shortly after (Gardner et al., 1967; Lax, 1968). The natural question, and the one this paper is really about, is easy to state but not so easy to answer: do the four correction terms destroy this integrability, or can it survive in some form?

Kodama already gave a general answer in 1985. If the transformation back to KdV is allowed to include a non-local piece, then (1.1) is asymptotically integrable to $O(\varepsilon)$

regardless of the values of the four coefficients (Kodama, 1985). Marchant and Smyth later checked this numerically. Soliton collisions under (1.1) turn out to be elastic to $O(\varepsilon)$, with any inelastic radiation appearing only at higher order (Marchant, Smyth, 1996). So in the broad sense, the integrability question was settled long ago.

What has not been settled, and what we address here, is a narrower and more concrete version of the question. What happens if the non-local term is not allowed? If the only permitted transformation is a local, purely polynomial change of variable, how much of the equation's structure can actually be simplified away? The answer turns out to be exact and clean. Of the four coefficients $\alpha_1, \gamma_1, \gamma_2, \beta_1$, exactly two, γ_1 and β_1 , cannot be removed by any local transformation, no matter how the transformation parameters are chosen. The other two, α_1 and γ_2 can always be removed completely. This clean split of the parameter space into a removable part and a stubborn part is the main result of the paper.

There is also a more physical question worth asking. What does fifth-order dispersion actually do to the shape of a localized pulse, the kind of thing associated with a rogue wave? Section 4.5 gives a short, elementary linear estimate of this effect, just enough to quantify the group phase velocity mismatch it produces.

The rest of the paper follows a simple structure. Section 2 reviews the relevant prior literature. Section 3 describes the methods used. Section 4 contains the actual analysis: the local transformation, the third conservation law, the connection to the Lax hierarchy, the AKNS Lax pair structure, and the dispersion analysis. Section 5 discusses what these results do and do not tell us, and Section 6 concludes.

Literature Review. The inverse scattering transform was introduced for the KdV equation by Gardner, Greene, Kruskal, and Miura, and it showed that KdV carries an infinite hierarchy of conserved densities (Gardner et al., 1967). Around the same time Lax gave the equation a Lax pair formulation, $L_t = [A, L]$, with L a Schrödinger operator (Lax, 1968). Run the same Lax construction with higher-order operators A and you get the whole polynomial KdV hierarchy: each member is its own separate, independently integrable equation, all sharing the KdV soliton profile but evolving in time differently. The fifth-order member that comes directly out of this construction (Lax, 1968) is the equation we work with in Section 4. There are two other classical fifth-order integrable equations worth mentioning, the Sawada-Kotera equation and the Kaup-Kuperschmidt equation, but their nonlinear structure and coefficient ratios are different from the Lax equation, so we leave them aside here.

The AKNS scheme (Ablowitz et al., 1974) gives a more general 2×2 matrix Lax pair formalism, and depending on how you reduce the off-diagonal potentials, it produces either the KdV hierarchy or the modified KdV (mKdV) hierarchy. The symmetric mKdV-type reduction, tied to Wadati's exact solution (Wadati, 1972), is not the same as the KdV reduction, and this distinction turns out to matter a lot in Section 4. For a standard treatment of inverse scattering and soliton theory more broadly, Drazin and Johnson's book is a good reference (Drazin & Johnson, 1989).

Kodama put the integrability question for perturbed KdV-type equations on solid footing through near-identity normal-form transformations (Kodama, 1985). His

approach shows that coefficients of a fairly broad class of correction terms can be removed to a given order in the perturbation parameter, as long as the transformation is allowed to be non-local. This is exactly what explains the elastic soliton collisions found numerically for the extended KdV equation (Marchant & Smyth, 1990, 1996), and it underlies a lot of oceanographic work on internal tide transformation and internal solitary waves (Holloway et al., 1999).

The extended KdV framework has kept growing since then. Horikis, Frantzeskakis, and Smyth derived a broader family of extended shallow-water models, including the extended KdV equation itself, directly from the Euler equations by pushing the asymptotic expansion one order beyond the usual weakly nonlinear long-wave approximation (Horikis et al., 2022). Karczewska and Rozmej derived a related family of (2+1)-dimensional extended KdV and Gardner equations from the same ideal-fluid setup (Karczewska & Rozmej, 2023, 2024). Khusnutdinova, Stepanyants, and Tranter looked at solitary-wave solutions of fifth-order KdV equations and showed how the higher-order nonlinear-dispersive terms change the existence, polarity, profile, and stability of embedded and regular solitons (Khusnutdinova et al., 2018). Kumar and Mohan found multiple soliton solutions for a generalized nonlinear fifth-order KdV-type equation using Painleve analysis and the Hirota bilinear method (Kumar & Mohan, 2022). Baqer, Horikis, and Frantzeskakis reviewed the resulting non-convex dispersive hydrodynamics of the extended KdV equation, including its near-identity reduction to Hamiltonian form and the resonant radiation this non-convexity produces ahead of dispersive shock waves, which connects directly to the fifth-order dispersion we treat in Section 4.5 (Baqer et al., 2025). This same non-convexity of the linear dispersion relation was worked out at the fully nonlinear level by Hoefer, Smyth, and Sprenger, who found a modulation-theory solution for the resulting nonclassical traveling dispersive shock waves of the fifth-order KdV equation (Hoefer, Smyth & Sprenger, 2019). The linear estimate in Section 4.5 gives the leading-order picture behind that nonlinear structure. Kwak established global well-posedness and energy estimates for the fifth-order modified KdV equation, a related integral member of the same hierarchy (Kwak, 2024). Most relevant to what we do here, Sidorovas, Tseluiko, Choi, and Khusnutdinova applied a Kodama-Fokas-Liu near-identity transformation to map the extended KdV equation for a two-layer fluid onto an improved Gardner equation, which is a practical application of exactly the class of local transformations we examine structurally in Section 4.1 (Sidorovas et al., 2025). Rozmej and Karczewska separately worked out soliton, periodic, and superposition solutions for the nonlocal (2+1)-dimensional extended KdV equation within the same ideal-fluid framework (Rozmej & Karczewska, 2023).

Wave-packet asymmetry in strongly localized events, the kind tied to oceanic rogue waves, has been studied for the nonlinear Schrodinger equation through exact rational solutions. The linear-dispersion mechanism we look at in Section 4 gives an analogous, KdV-specific estimate. Explicit breather construction for KdV-type equations, which would be a natural next step from the structural results here, has recently been done on a non-zero periodic background by Hoefer, Mucalica, and Pelinovsky (Hoefer, Mucalica & Pelinovsky, 2023).

What none of this work provides is an exact statement, for the four-coefficient extended equation (1.1), of which corrections survive a local transformation and which do not. Kodama's theorem answers the question for the more permissive class of non-local transformations. Marchant and Smyth's results are numerical rather than exact algebraic statements about individual coefficients. And the recent Kodama-Fokas-Liu mapping by Sidorovas et al. is applied to a specific physical parameter regime rather than derived as a general coefficient-space criterion (Sidorovas et al., 2025). That gap is what Section 4 closes.

Materials and Methods. The analysis relies on plain algebraic computation, backed up at every step by independent symbolic and numerical checks.

Local near-identity transformation. We plug the candidate local transformation $u = v + \varepsilon(av^2 + bv_{xx})$ into (1.1) and expand it out to $O(\varepsilon)$. The coefficients that come out in front of v^2v_x , vv_{xxx} , v_xv_{xx} , and v_{xxxxx} are collected term by term from each of the three pieces of the original equation, $(u_t, 6uu_x, u_{xxx})$, both by hand and with symbolic computation, just to be sure nothing gets dropped.

Conservation-law test. We calculate the time derivative of the third KdV conserved density separately from the transformation result, using repeated integration by parts on the perturbed equation. For the unperturbed KdV case we also ran a direct numerical check, integrating the equation with a pseudo-spectral, integrating-factor Runge-Kutta solver on a non-soliton initial profile made of two Gaussians. A single soliton profile would not have been a good test here, since it is a degenerate case for any translation-invariant functional.

AKNS reduction analysis. We look at whether the 2×2 AKNS matrix scheme is compatible with the quadratic nonlinearity $6uu_x$ by comparing the symmetry properties of two candidate potential reductions against the symmetry actually present in the target equation.

Linear dispersion analysis. The equation is linearized around $u = 0$, and the resulting dispersion relation, phase velocity, and group velocity are obtained in closed form. The correction to the gap between them coming from fifth-order dispersion is then checked against the independently derived nonlinear soliton speed, using analytical continuation of the wavenumber.

Results. *Local Kodama-type transformation.*

Let $\varphi = av^2 + bv_{xx}$, so that $u = v + \varepsilon\varphi$. Direct differentiation gives

$$u_x = v_x + \varepsilon\varphi_x, \varphi_x = 2avv_x + bv_{xxx}, \quad (4.1)$$

$$u_{xxx} = v_{xxx} + \varepsilon\varphi_{xxx}, \varphi_{xxx} = 6av_xv_{xx} + 2avv_{xxx} + bv_{xxxxx}. \quad (4.2)$$

For the nonlinear term uu_x , keeping only terms up to $O(\varepsilon^2)$, one finds

$$uu_x = vv_x + \varepsilon(3av^2v_x + bv_{xx}v_{xxx} + bv_xv_{xx}). \quad (4.3)$$

For the time derivative u_t , the leading-order relation $v_t \approx -(6vv_x + v_{xxx})$, is enough inside terms that are already of order ε . This gives

$$2avv_t \rightarrow -12av^2v_x - 2avv_{xxx}, \quad (4.4)$$

$$bv_{xxt} = b\partial_x^2 v_t \rightarrow -18bv_xv_{xx} - 6bv_{xxx} - bv_{xxxxx}. \quad (4.5)$$

Now collect the $O(\varepsilon)$ in $u_t + 6uu_x + u_{xxx} = 0$. The key point is that the coefficients of vv_{xxx} and v_{xxxxx} cancel identically for any choice of a and b . So a local transformation of the form used here can never generate or remove those two terms. By contrast, the coefficients of v^2v_x and v_xv_{xx} do change, so they can be adjusted.

The coefficients of v^2v_x , vv_{xxx} , v_xv_{xx} and v_{xxxxx} are collected term by term from each of the three pieces of the original equation, u_t , $6uu_x$ and u_{xxx} both by hand and with symbolic computation, just to be sure nothing gets dropped. Table 1 shows the contribution of individual terms of the original equation to the $O(\varepsilon)$ correction coefficients.

Table 1. Contribution of individual terms of the original equation to the $O(\varepsilon)$ correction coefficients.

Source	v^2v_x	vv_{xxx}	v_xv_{xx}	v_{xxxxx}
From u_t	$-12a$	$-2a - 6b$	$-18b$	$-b$
From $6uu_x$	$18a$	$6b$	$6b$	0
From u_{xxx}	0	$2a$	$6a$	b
Total	$6a$	0	$6a - 12b$	0

The transformed equation becomes

$$v_t + 6vv_x + v_{xxx} + \varepsilon[(\alpha_1 + 6a)v^2v_x + \gamma_1 vv_{xxx} + (\gamma_2 + 6a - 12b)v_xv_{xx} + \beta_1 v_{xxxxx}] = 0. \quad (4.6)$$

Choosing

$$a = -\frac{\alpha_1}{6}, b = \frac{\gamma_2 - \alpha_1}{12} \quad (4.7)$$

removes the v^2v_x and v_xv_{xx} terms completely, for arbitrary values of α_1 and γ_2 . The equation then reduces to

$$v_t + 6vv_x + v_{xxx} + \varepsilon(\gamma_1 vv_{xxx} + \beta_1 v_{xxxxx}) = 0. \quad (4.8)$$

This is the main structural point of the transformation calculation. The parameters α_1 and γ_2 are removable by a local near-identity change of variable, while γ_1 and

β_1 are not. In particular, the extended equation is locally equivalent to the pure KdV equation if and only if $\gamma_1 = \beta_1 = 0$, regardless of the values of α_1 and γ_2 .

The geometry of this non-removable coefficient space is shown in Figure 1.

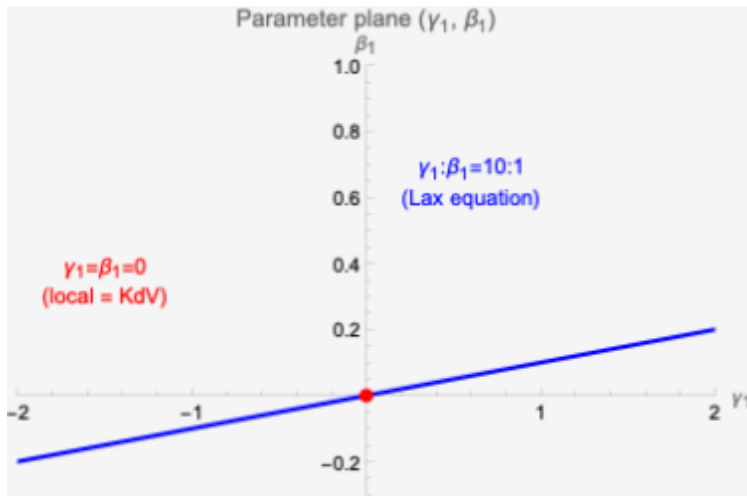


Figure 1. Coefficient plane (γ_1, β_1) , the two non-removable parameters of Eq. (1.1).

The figure illustrates the plane of non-removable coefficients after the local transformation (4.6). The origin $\gamma_1 = \beta_1 = 0$ is the unique point of exact local equivalence to KdV. The shallow line $\beta_1 = \gamma_1/10$ (ratio 10:1) marks where the transformed coefficients coincide with those of the classical fifth-order Lax equation (4.19); this is a coefficient-matching condition, not itself evidence of integrability away from this line.

Third conservation law. The next step is to check whether the same condition also appears from the conservation-law side. For the KdV equation the third invariant can be written as

$$I_3 = \frac{1}{6} \int_{-\infty}^{\infty} \left(v^3 - \frac{1}{2} v_x^2 \right) dx, \quad (4.9)$$

which is proportional to the standard density $2v^3 - v_x^2$ in the KdV hierarchy (Gardner et al., 1967; Lax, 1968; Drazin and Johnson, 1989). Taking the time derivative and integrating by parts, with $v \rightarrow 0$ as $|x| \rightarrow \infty$ gives

$$\frac{dI_3}{dt} = \int_{-\infty}^{\infty} \left(\frac{v^2}{2} + \frac{1}{6} v_{xx} \right) v_t dx. \quad (4.10)$$

Now substitute $v_t = -(6vv_x + v_{xxx}) + \varepsilon(\gamma_1 vv_{xxx} + \beta_1 v_{xxxxx})$ from (4.8). The $O(1)$ part becomes

$$\int_{-\infty}^{\infty} \left(\frac{v^2}{2} + \frac{1}{6} v_{xx} \right) [-(6vv_x + v_{xxx})] dx =$$

$$-3 \int v^3 v_x dx - \frac{1}{2} \int v^2 v_{xxx} dx - \int vv_x v_{xx} dx - \frac{1}{6} \int v_{xx} v_{xxx} dx. \quad (4.11)$$

The first and the last terms are exact derivatives, so they vanish. After integrating by parts, the two middle terms cancel exactly:

$$-\frac{1}{2} \int v^2 v_{xxx} dx - \int vv_x v_{xx} dx = 0. \quad (4.12)$$

This cancellation depends on the coefficient $\frac{1}{2}$ in front of v_x^2 in (4.9). If that coefficient were replaced by 1, the same calculation would leave a nonzero residual term proportional to $\int v_x^3 dx$. So the density has to be normalized in the standard KdV way. Leaving only the $O(\varepsilon)$ part, we get

$$\frac{dI_2}{dt} = \varepsilon \int_{-\infty}^{\infty} \left(\frac{v^2}{2} + \frac{1}{6} v_{xx} \right) (\gamma_1 v v_{xxx} + \beta_1 v_{xxxxx}) dx. \quad (4.13)$$

Now integrate by parts repeatedly. Using the identities

$$\int v^3 v_{xxx} dx = 3 \int vv_x^3 dx, \quad (4.14)$$

$$\int vv_{xx} v_{xxx} dx = -\frac{1}{2} \int v_x v_{xx}^2 dx, \quad (4.15)$$

$$\int v^2 v_{xxxxx} dx = -5 \int v_x v_{xx}^2 dx, \quad (4.16)$$

and

$$\int v_{xx} v_{xxxxx} dx = 0, \quad (4.17)$$

the derivative of the invariant reduces to

$$\frac{dI_2}{dt} = \varepsilon \left[\frac{3\gamma_1}{2} \int vv_x^3 dx - \left(\frac{\gamma_1}{12} + \frac{5\beta_1}{2} \right) \int v_x v_{xx}^2 dx \right]. \quad (4.18)$$

The two functions that appear here are independent. They do not have the same derivative structure and they do not have the same degree in v , so one cannot cancel the other for a general localized profile. Because of that, the right-hand side vanishes identically for arbitrary v only if both coefficients are zero. This gives $\gamma_1 = \beta_1 = 0$.

So the conservation-law test gives exactly the same condition as the local transformation in Section 4.1. The equation is locally equivalent to pure KdV only

when the two non-removable coefficients vanish. The values of α_1 and γ_2 do not affect this criterion.

Proposition 1. Equation (1.1) is locally Kodama-equivalent to KdV, and the third invariant (4.9) is conserved identically, if and only if $\gamma_1 = \beta_1 = \mathfrak{C}$, independent of α_1 and γ_2 .

Connection to the Lax hierarchy and its limits. The classical fifth-order Lax equation is an independently integrable member of the polynomial KdV hierarchy (Lax, 1968):

$$u_t = 30u^2u_x + 20u_xu_{xx} + 10uu_{xxx} + u_{xxxxx}. \quad (4.19)$$

The ratio of coefficients at uu_{xxx} and u_{xxxxx} in (4.19) is **10:1**. Two other classical fifth-order integrable equations are the Sawada-Kotera equation and the Kaup-Kupershmidt equation; their nonlinear structures and coefficient ratios differ from the Lax equation and are not treated here.

Section 4.1 showed that α_1 and γ_2 are gauge and that the non-removable part of the equation is determined only by the ratio $\gamma_1:\beta_1$. This is a direct consequence of that section: since a, b are two free parameters, they can map any pair (α_1, γ_2) to any other prescribed pair without changing γ_1, β_1 . Fixing $\gamma_1:\beta_1 = 10:1$ therefore automatically makes (1.1), for any α_1, γ_2 , locally Kodama-equivalent to the equation with $\alpha_1 = 30\kappa, \gamma_1 = 10\kappa, \gamma_2 = 20\kappa, \beta_1 = \kappa$, that is, to the fifth-order Lax flow in (4.19), integrable for an independent reason: the existence of its own Lax pair, not through the behavior of I_3 (Lax, 1968).

Proposition 2. At the ratio $\gamma_1:\beta_1 = 10:1$, the local transformation carries the non-removable coefficients of (1.1) into coincidence with those of the classical fifth-order Lax equation. This is a statement about coefficient matching under the transformation, not a proof that the deformed equation (1.1) is integrable in general; integrability at this ratio is a separate, already established fact, resting on the Lax pair of (4.19) itself.

Some care is needed in stating this precisely. Section 4.2 showed that at $\gamma_1:\beta_1 = 10:1$ (with $\gamma_1, \beta_1 \neq 0$), the third invariant I_3 is *not* conserved: (4.18) is nonzero for a general profile at this ratio as at any other nonzero γ_1, β_1 . This is not a contradiction: the Lax equation (4.19), as a separate flow of the hierarchy, carries its own infinite series of conserved densities, in general distinct from the undeformed $I_3[v]$. The general theory of commuting flows states that all members of a hierarchy share the same conserved integrals only when those integrals are consistently deformed together with the transformation (4.6), in the spirit of Kodama's full, Hamiltonian-consistent construction, which includes a non-local term. The naive substitution $v \rightarrow u$ into the unchanged functional $I_3[v]$, used in Section 4.2, performs no such deformation and therefore cannot serve as an integrability criterion for equation (4.19) itself; it is informative only as a criterion for local equivalence to *pure* KdV, exactly the claim made in Section 4.2.

Integrability of (1.1) at $\gamma_1:\beta_1 = 10:1$ is therefore established not by computing

dI_3/dt , but by the independent fact that (4.19) has its own Lax pair (Lax, 1968). At ratios $\gamma_1:\beta_1$ other than $10:1$, the residual equation does not coincide with the fifth-order Lax flow; whether a more general, possibly non-local normal-form transformation can still relate it to an integrable hierarchy is outside the scope of this work.

Structural constraint on the AKNS Lax pair. Constructing a matrix Lax pair for (4.19) within the AKNS scheme (Ablowitz et al., 1974) depends materially on the choice of reduction of the 2×2 system. The symmetric reduction

$$U = \begin{pmatrix} -i\lambda & u \\ -u & i\lambda \end{pmatrix}, \quad q(u) = u, \quad r(u) = -u, \quad (4.20)$$

is invariant under the simultaneous change $u \rightarrow -u$, so the zero-curvature condition with a polynomial matrix V in λ reproduces only equations sharing this symmetry, the mKdV family ($u^2 u_x$ nonlinearity; (Wadati, 1972)), not equations containing $6uu_x$, which does not possess this symmetry. Reduction (4.20) is structurally unsuitable as a Lax pair for (4.19) or (1.1).

The correct AKNS reduction, generating the KdV hierarchy (Drazin & Johnson, 1989), is

$$U = \begin{pmatrix} -i\lambda & u \\ -1 & i\lambda \end{pmatrix}, \quad q(u) = u, \quad r \equiv -1. \quad (4.21)$$

On the zero background $u = 0$, matrix (4.21) is not diagonal (the lower-left entry equals -1), and the linear system $\Phi_x = U\Phi$ is solved not by elementary exponentials $e^{\pm i\lambda x}$, as for the symmetric reduction, but by a triangular system with a resonant solution, a known property of the Schrödinger spectral problem ($\psi_{xx} = (u - \lambda)\psi$ in first-order form), to which (4.21) reduces. This is why smooth, localized oscillating (breather) solutions for KdV-type equations are typically constructed not on the zero background but on a non-zero periodic (cnoidal) background, as demonstrated directly by Hoefer, Mucalica, and Pelinovsky for pure KdV (Hoefer, Mucalica & Pelinovsky, 2023).

Constructing the fifth-order matrix V for reduction (4.21) and the resulting breather solutions of (4.19) on a non-zero background is a separate task requiring independent symbolic verification; explicit formulas are not given here.

The one result available without the matrix V is the speed of the single-soliton solution of (4.19): since all flows of one polynomial KdV hierarchy share a common τ -function and differ only in their time-evolution law, the profile $u = 2a^2 \operatorname{sech}^2[a(x - ct)]$ remains an exact solution of (1.1) at $\gamma_1:\beta_1 = 10:1$, with speed

$$c(a) = 4a^2 + 16\epsilon\beta_1 a^4, \quad (4.22)$$

verified by direct substitution. It also follows from the general hierarchy result that the n -th flow of the KdV hierarchy propagates its common soliton at speed $(2a)^{2n}$: the

$n = 1$ (KdV) term gives $4a^2$ and the $n = 2$ (Lax) term, entering with coefficient $\varepsilon\beta_1$, gives $16a^4$, and, equivalently, from the linear phase velocity of Section 4.5 by analytic continuation $k \rightarrow 2ia$ (not $k \rightarrow ia$): substituting $k = 2ia$ into (4.25), $k^2 = -4a^2$, $k^4 = 16a^4$, gives $V_p(2ia) = 4a^2 + 16\varepsilon\beta_1 a^4 = c(a)$ (Figures 2-5). The factor of 2 in $k = 2ia$ reflects a known property of sech^2 -solitons of KdV: the soliton pole in the spectral plane sits at $k = 2ia$, not at $k = ia$.

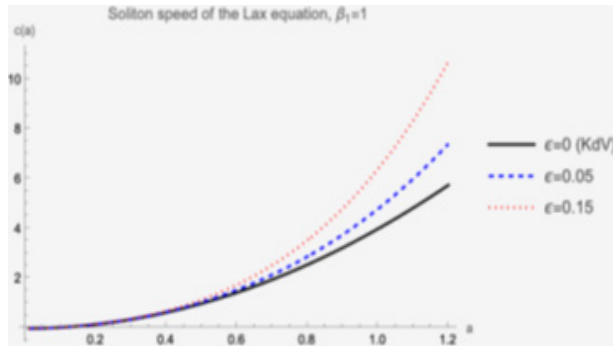


Figure 2. Analytic soliton speed $c(a)$ as a function of the amplitude parameter a .

The figure shows the analytical soliton speed plotted against the amplitude parameter a fixed $\beta_1 = 1$. The curves represent the closed-form expression rather than a numerical fit, contrasting the unperturbed KdV limit for pure KdV ($\varepsilon = 0$) with the fifth-order Lax equation regimes at $\varepsilon = 0.05$ and $\varepsilon = 0.15$. An increase in the perturbation parameter $\varepsilon\beta_1$ leads to a systematic, non-linear increase in the soliton propagation speed at a given amplitude.

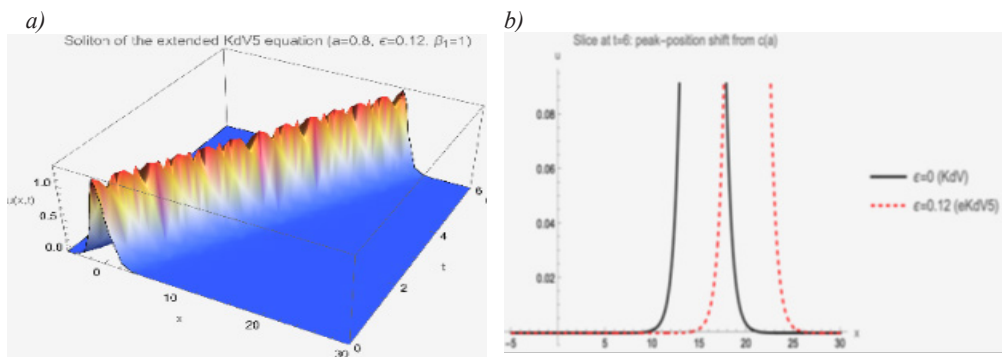


Figure 3. Evolution of the fifth-order Lax-equation soliton: the closed-form speed correction (4.22) accumulates linearly in t as a shift of the peak position. a) Space-time evolution of $u(x,t)$ at $a = 0.8$, $\varepsilon = 0.12$, $\beta_1 = 1$; an illustrative parameter choice, not a fitted regime; b) Slice at $t = 6$: peak position of the Lax-equation soliton relative to a pure-KdV soliton of the same amplitude a , both started from the same initial peak location.

Linear dispersion analysis. Linearizing (1.1) about $\mathbf{u} = \mathbf{0}$ (terms of degree higher than one in $\mathbf{u}$ vanish at linear order):

$$u_t + u_{xxx} + \varepsilon\beta_1 u_{xxxxx} = 0. \quad (4.23)$$

For $u = A \exp[i(kx - \omega t)]$, the dispersion relation (Figure 6) is

$$\omega(k) = -k^3 + \varepsilon\beta_1 k^5. \quad (4.24)$$

Phase and group velocities (Figure 7) are

$$V_p(k) = \frac{\omega}{k} = -k^2 + \varepsilon\beta_1 k^4, \quad (4.25)$$

$$V_g(k) = \frac{d\omega}{dk} = -3k^2 + 5\varepsilon\beta_1 k^4. \quad (4.26)$$

The difference

$$\Delta V(k, \varepsilon) = V_g - V_p = -2k^2 + 4\varepsilon\beta_1 k^4 \quad (4.27)$$

is nonzero already at $\varepsilon = 0$: a general property of any cubic-in- k dispersion relation, including pure KdV, unrelated to the fifth order. This non-convex dispersion structure is also what allows resonant radiation ahead of dispersive shock waves in the extended KdV equation (Baquer et al., 2025), and underlies the nonclassical traveling dispersive shock waves obtained at the fully nonlinear level by Hoefer, Smyth, and Sprenger (Hoefer, Smyth & Sprenger, 2019). What is meaningful for the present analysis is not ΔV itself but its change due to fifth-order dispersion (Figure 9):

$$\Delta V(k, \varepsilon) - \Delta V(k, 0) = 4\varepsilon\beta_1 k^4. \quad (4.28)$$

This correction grows as k^4 : high-frequency (narrow) components of the wave packet acquire a disproportionately larger shift of the group path relative to the phase path. For a localized pulse represented as a superposition of components spread over k , this gives a quantitative, elementary estimate of the growth of envelope–carrier mismatch with increasing ε , without recourse to the still-unfinished nonlinear construction of Section 4.4.

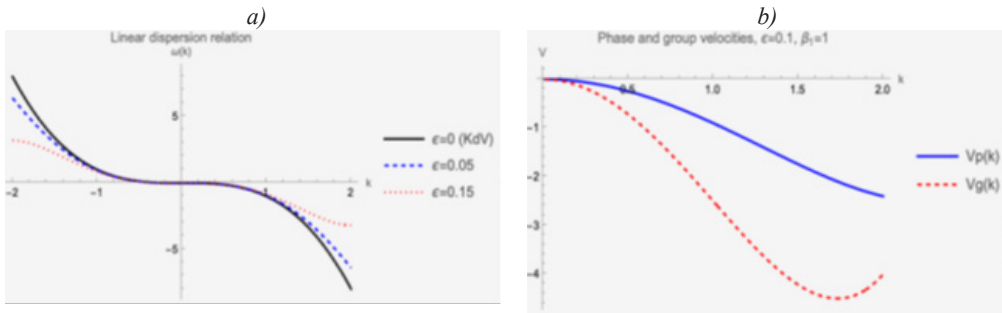


Figure 4. Linear dispersion of equation (1.1). a) Linear dispersion relation $\omega(k) = -k^3 + \epsilon\beta_1 k^5$ (4.24) for pure KdV versus the $O(\epsilon)$ -corrected equation; b) Phase velocity $V_p(k)$ and group velocity $V_g(k)$ at $\epsilon = 0.1$, $\beta_1 = 1$; the gap $\Delta V = V_g - V_p$ is nonzero already at $\epsilon = 0$ (a generic feature of cubic-in- k dispersion) and is not itself the fifth-order effect.

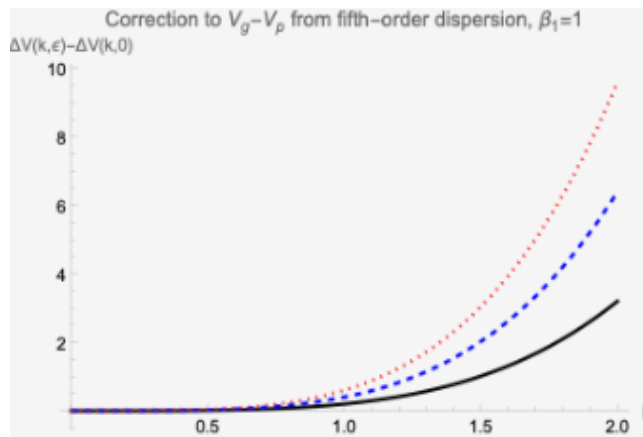


Figure 5. Pure fifth-order correction to the group-phase velocity gap. $\Delta V(k, \epsilon) - \Delta V(k, 0) = 4\epsilon\beta_1 k^4$ plotted against k for several values of $\epsilon\beta_1 = 1$. The k^4 growth shows that short-wavelength (high- k) components of a wave packet are disproportionately affected. This is the physical mechanism underlying the envelope-carrier mismatch and wave-packet asymmetry discussed in detail in Section 5.

Discussion. Splitting the four correction coefficients into a removable pair (α_1, γ_2) and a non-removable pair (γ_1, β_1) shows that, to the order considered, the only physics beyond KdV that survives locally comes through the mixed nonlinear-dispersive term $\gamma_1 u u_{xxx}$ and the pure fifth-order dispersion $\beta_1 u_{xxxxx}$. The cubic-nonlinear term $\alpha_1 u^2 u_x$ and the mixed term $\gamma_2 u_x u_{xx}$ can always be absorbed into a redefinition of the dependent variable, so they carry no independent signature at this order. For modelers working with equations of the form (1.1), this points to which coefficients actually need independent measurement or fitting and which are simply equivalent choices of variable, matching the practical use of local near-identity transformations already applied to the extended KdV equation in (Sidorovas et al., 2025).

Non-conservation of I_3 at the Lax ratio $\gamma_1:\beta_1 = 10:1$ (Section 4.3) does not undermine the third-invariant test; it simply marks where the test stops applying. A

fixed, undeformed functional such as $I_3[v]$ can only detect equivalence to plain KdV, not to other members of the hierarchy, since testing those would require deforming the functional together with the transformation, as in Kodama's full construction (Kodama, 1985).

The AKNS argument in Section 4.4 clarifies, independently of the fifth-order construction itself, why the symmetric reduction used for mKdV-type equations cannot simply be reused for equations containing $6uu_x$, and why the correct reduction places KdV-type breathers on a non-zero periodic background rather than on the zero background. This matches the explicit construction of KdV breathers on a cnoidal background by Hoefer, Mucalica, and Pelinovsky (Hoefer, Mucalica & Pelinovsky, 2023), and suggests that building a fifth-order breather for (4.19) is simply an unfinished extension of the present structural argument, not a question of choosing the right reduction.

The closed-form correction $\Delta V(k, \varepsilon) - \Delta V(k, 0) = 4\varepsilon\beta_1 k^4$ to the group-phase velocity gap gives a way to estimate wave-packet asymmetry without waiting on the still-unfinished nonlinear breather construction. Since the correction scales as k^4 , narrow, high-wavenumber components of a localized pulse shift far more than broad ones, consistent with the non-convex-dispersion mechanism shown to drive resonant radiation ahead of dispersive shocks in the same equation (Baquer et al., 2025). Comparing this linear estimate against a fully nonlinear multiple-scale envelope calculation remains future work, and would show how far the estimate holds once nonlinear effects are included.

This k^4 growth suggests that shorter-wavelength components of a localized packet are shifted more strongly, which can enhance envelope asymmetry and contribute to a steeper leading edge in rogue-wave-like events, although in real ocean waves this effect acts together with nonlinear bound-wave and focusing mechanisms.

One limitation is that the analysis solves exactly only the ratio $\gamma_1:\beta_1 = 10:1$, the case that coincides with the Lax flow; at any other ratio, whether even a non-local Kodama-type transformation restores integrability remains an open question we do not attempt here. Much of the cited literature dates from the 1960s through the 1990s simply because that is when inverse scattering, the AKNS scheme, and the polynomial KdV hierarchy were worked out; more recent papers are cited specifically where they bear on the physical model, its dispersive-shock and breather behavior, or point to unresolved questions.

Conclusion. The local Kodama-type transformation (4.6) removes α_1 and γ_2 from the extended fifth-order KdV equation (1.1) completely, for any values of these coefficients; γ_1 and β_1 cannot be removed by such a transformation, no matter how its parameters are chosen. The third KdV invariant is conserved along solutions of (1.1) exactly when $\gamma_1 = \beta_1 = 0$ is the same criterion that comes out of the transformation itself. At the ratio $\gamma_1:\beta_1 = 10:1$, the transformation maps the equation's coefficients onto those of the classical fifth-order Lax equation; this is a coincidence in coefficient space, and the integrability it reflects rests on the Lax equation's own Lax pair, not on any property of I_3 , which in fact fails to be conserved at this ratio. The symmetric AKNS

reduction $q = u, r = -u$ cannot serve as a Lax pair for equations containing $6uu_x$; the reduction $q = u, r \equiv -1$ does. Finally, the correction to the group-phase velocity gap from fifth-order dispersion comes out exactly as $\Delta V(k, \varepsilon) - \Delta V(k, 0) = 4\varepsilon\beta_1 k^4$.

Several questions remain open: constructing an explicit fifth-order breather on a non-zero periodic background using reduction (4.21); checking whether a non-local Kodama transformation can remove the residual $\gamma_1 v v_{xxx} + \beta_1 v_{xxxxx}$, even approximately, away from the 10:1 ratio; and confirming the wave-packet asymmetry result at the fully nonlinear level, for instance through a multiple-scale envelope expansion built on (4.24), or by comparing directly with the resonant-radiation regimes recently classified for this equation (Baquer et al., 2025).

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