ISSN 2518-1726 (Online),
ISSN 1991-346X (Print)

ҚАЗАҚСТАН РЕСПУБЛИКАСЫ
ҰЛТТЫҚ ҒЫЛЫМ АКАДЕМИЯСЫНЫҢ

ӘЛЬ-ФАРАБИ АТЫНДАҒЫ
ҚАЗАҚ ҰЛТТЫҚ УНИВЕРСИТЕТІНІҢ

Х А Б А Р Л А Р Ы

ИЗВЕСТИЯ

НАЦИОНАЛЬНОЙ АКАДЕМИИ НАУК
РЕСПУБЛИКИ КАЗАХСТАН

КАЗАХСКИЙ НАЦИОНАЛЬНЫЙ
УНИВЕРСИТЕТ ИМЕНИ АЛЬ-ФАРАБИ

N E W S

OF THE NATIONAL ACADEMY OF SCIENCES
OF THE REPUBLIC OF KAZAKHSTAN

AL-FARABI KAZAKH
NATIONAL UNIVERSITY

ФИЗИКА-МАТЕМАТИКА СЕРИЯСЫ
♦
СЕРИЯ ФИЗИКО-МАТЕМАТИЧЕСКАЯ
♦
PHYSICO-MATHEMATICAL SERIES

6 (322)

ҚАРАША – ЖЕЛТОҚСАН 2018 ж.
НОЯБРЬ – ДЕКАБРЬ 2018 г.
NOVEMBER – DECEMBER 2018

1963 ЖЫЛДЫҢ ҚАҢТАР АЙЫНАН ШЫҒА БАСТАҒАН
ИЗДАЕТСЯ С ЯНВАРЯ 1963 ГОДА
PUBLISHED SINCE JANUARY 1963

ЖЫЛЫНА 6 РЕТ ШЫҒАДЫ
ВЫХОДИТ 6 РАЗ В ГОД
PUBLISHED 6 TIMES A YEAR

АЛМАТЫ, ҚР ҰҒА АЛМАТЫ, НАН РК ALMATY, NAS RK

«ҚР ҰҒА Хабарлары. Физика-математикалық сериясы».

ISSN 2518-1726 (Online), ISSN 1991-346X (Print)

Меншіктенуші: «Қазақстан Республикасының Ұлттық ғылым академиясы» РҚБ (Алматы қ.)
Қазақстан республикасының Мәдениет пен ақпарат министрлігінің Ақпарат және мұрағат комитетінде 01.06.2006 ж. берілген **№5543-Ж** мерзімдік басылым тіркеуіне қойылу туралы куәлік

Мерзімділігі: жылына 6 рет.
Тиражы: 300 дана.

Редакцияның мекенжайы: 050010, Алматы қ., Шевченко көш., 28, 219 бөл., 220, тел.: 272-13-19, 272-13-18,
www:nauka-nanrk.kz / physics-mathematics.kz

Типографияның мекенжайы: «Аруна» ЖК, Алматы қ., Муратбаева көш., 75.

«Известия НАН РК. Серия физико-математическая».

ISSN 2518-1726 (Online), ISSN 1991-346X (Print)

Собственник: РОО «Национальная академия наук Республики Казахстан» (г. Алматы)
Свидетельство о постановке на учет периодического печатного издания в Комитете информации и архивов Министерства культуры и информации Республики Казахстан **№5543-Ж**, выданное 01.06.2006 г.

Периодичность: 6 раз в год.
Тираж: 300 экземпляров.

Адрес редакции: 050010, г. Алматы, ул. Шевченко, 28, ком. 219, 220, тел.: 272-13-19, 272-13-18,
www:nauka-nanrk.kz / physics-mathematics.kz

Адрес типографии: ИП «Аруна», г. Алматы, ул. Муратбаева, 75.

News of the National Academy of Sciences of the Republic of Kazakhstan. Physical-mathematical series.

ISSN 2518-1726 (Online), ISSN 1991-346X (Print)

Owner: RPA "National Academy of Sciences of the Republic of Kazakhstan" (Almaty)
The certificate of registration of a periodic printed publication in the Committee of information and archives of the Ministry of culture and information of the Republic of Kazakhstan N 5543-Ж, issued 01.06.2006

Periodicity: 6 times a year
Circulation: 300 copies

Editorial address: 28, Shevchenko str., of. 219, 220, Almaty, 050010, tel. 272-13-19, 272-13-18,
www:nauka-nanrk.kz / physics-mathematics.kz

Address of printing house: ST "Aruna", 75, Muratbayev str, Almaty

N E W S
OF THE NATIONAL ACADEMY OF SCIENCES OF THE REPUBLIC OF KAZAKHSTAN
PHYSICO-MATHEMATICAL SERIES
ISSN 1991-346X **https://doi.org/10.32014/2018.2518-1726.11**
Volume 6, Number 322 (2018), 5 – 14

UDC 536.46:532.517.4
IRSTI 29.03.77; 29.03.85

A.S. Askarova[1], S.A. Bolegenova[2], P. Safarik[3], S.A. Bolegenova[2], V.Yu. Maximov[1], M.T. Beketayeva[1], A.O. Nugymanova[2]

[1] Scientific Research Institute of Experimental and Theoretical Physics of al-Farabi Kazakh National University, SRI ETP of KazNU, Almaty, Kazakhstan;
[2] Physical and Technical Physics Faculty of al-Farabi Kazakh National University, Al-Farabi KazNU, Almaty, Kazakhstan
[3] Fluid Dynamics and Thermodynamics Department of Czech Technical University in Prague, CTU in Prague, Prague, Czech Republic
E-mail: Aliya.Askarova@kaznu.kz, Saltanat.Bolegenova@kaznu.kz, pavel.safarik@fs.cvut.cz, bolegenova.symbat@kaznu.kz, valeriy.maximov@kaznu.kz, Beketayeva.m@gmail.com, aijoka01@gmail.com

MODERN COMPUTING EXPERIMENTS ON PULVERIZED COAL COMBUSTION PROCESSES IN BOILER FURNACES

Abstract. The aim of the work is to create new computer technologies for 3D modeling of heat and mass transfer processes in high-temperature physico-chemical-reactive environments that will allow to determine the aerodynamics of the flow, heat and mass transfer characteristics of technological processes occurring in the combustion chambers in the operating coal TPP RK. The novelty of the research lies in the use of the latest information technologies of 3D modeling, which will allow project participants to obtain new data on the complex processes of heat and mass transfer during the burning of pulverized coal in real combustion chambers operating in the CHP of RK. Numerical simulation, including thermodynamic, kinetic and three-dimensional computer simulation of heat and mass transfer processes when burning low-grade fuel, will allow finding optimal conditions for setting adequate physical, mathematical and chemical models of the technological process of combustion, as well as conduct a comprehensive study and thereby develop ways to optimize the process of ignition, gasification and burning high ash coals. The proposed methods of computer simulation are new and technically feasible when burning all types of coal used in pulverized coal-fired power plants around the world. The developed technologies will allow replacing or eliminating the conduct of expensive and labor-consuming natural experiments on coal-fired power plants.

Key words. Combustion, boundary conditions, computer simulation, low-grade coal, pulverized coal, reacting mixture, combustion chamber, numerical experiment.

Introduction

Kazakhstan is currently a developed country rich in natural resources. The fuel and energy complex is the basis for life support and economic development. Kazakhstan coal has high ash content (~ 40%) so they rated as low-grade, despite of it this organic fuel covers more than 40% of the demand for primary energy resources. The use of such quality coal leads to economic and ecological problems, related with ineffective incomplete combustion of fuel, which causes a high level of carbon and nitrogen compounds in the atmosphere.

In this regard, the President of the Republic Nazarbayev N.A. identified the global energy-environmental strategy for sustainable development of Kazakhstan, where he expressed ideas about sustainable energy. According to the adopted “Sustainable Energy Strategy for the Future of Kazakhstan until 2050” [1], the factors of energy independence and development principles include the requirements of ensuring the interests of the new generation and preserving the environment, which are determined by

the following parameters: ensuring the world level of economic and technical efficiency throughout the country's energy sector; control the level of environmental impact of energy; the existence of an internal policy aimed at ensuring the availability of all types of energy; possession of the optimal institutional structure of the energy-complex; ensuring participation in international energy markets.

In the main, the country is currently dependent upon fossil fuels for power generation. As shown in Fig. 1 13% of Kazakhstan's power is generated by hydroelectric power plants, and whilst 90% is from thermal-powered plants (75% coal-fired stations).

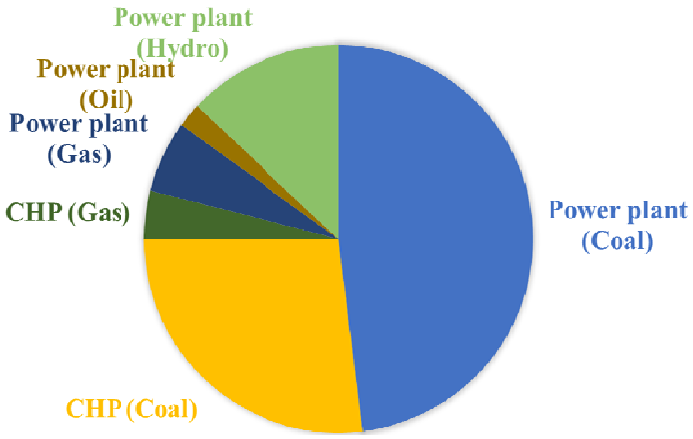


Fig 1 - Kazakhstan's electricity generating capacity (%)

To optimize the combustion of solid fuels, to develop and implement “clean” technologies and to protect the environment and ensure the efficiency of power plants, a deeper study of the issues of burning solid fuels in the combustion chambers of boilers and conducting research of technological processes taking place at TPPs is needed. It is possible with a combination of physical, scientific and applied, technological and engineering research in the field of optimization of solid fuel combustion processes [2-7]. In this regard, it becomes relevant to conduct computational experiments on the study of ignition, heat transfer, and mechanisms for burning out a coal-dust torch in the combustion chambers of boilers of energy facilities.

Methods of pulverized coal combustion research

At present, the intensive development of computer technologies and numerical simulation methods ensures a sufficiently high accuracy, the convergence of numerical results and their agreement with the results of field experiments. The use of computational fluid dynamics CFD allows one to obtain data without field experiments, which can then be used to substantiate the parameters and modes of thermal and hydrodynamic processes in the preparation of subsequent experimental studies on real energy facilities.

To study the complex physicochemical processes occurring while the pulverized coal combustion in furnace of boilers, it is necessary to have certain conditions required for carrying out computational experiments, including a multiprocessor computing system, an adequate physical, mathematical and chemical model and an exact method for solving a system of differential equations that describe the real technological process of burning pulverized coal in the existing power plant.

Numerical simulation uses numerical methods for solving the fundamental equations of heat and mass transfer processes using powerful computers. The theoretical analysis of vortex flows is based on the Navier-Stokes and Reynolds equations [8]. However, due to the nonlinearity and interconnectedness of these equations, their solution in the general case can be found only numerically [9]. The predominant method in the numerical simulation of subsonic currents and heat and mass transfer is the well-proven algorithm of SIMPLE Patankar-Spalding [10].

The description of the numerical model is based on a number of physical laws of conservation of mass, momentum, energy [11]. The mathematical model consists of a system of differential equations, algebraic closing relations and boundary (initial and boundary) conditions.

Since most practical flows are turbulent, the conservation equations must be considered in averaged and filtered by time or spatial forms, which must be closed using additional turbulent models [12]. For the formulation of a mathematical model, we consider the basic equations.

Since there are no sources of mass, only the transformation of the constituent components takes place. In this case, the equation of conservation of mass or the continuity equation takes the form (where the first term of the equation describes the flow nonstationarity, the second term is convective transport):

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_i}(\rho u_i) = 0 \quad (1)$$

$$\frac{\partial (\rho u_i)}{\partial t} + \frac{\partial}{\partial x_j}(\rho u_i u_j) = -\frac{\partial p}{\partial x_i} + \frac{\partial \tau_{ij}}{\partial x_j} + \rho g_i + F_i \quad (2)$$

The first term of the equation describes the nonstationarity of the flow, the second - convective transport, the third and fourth terms - surface forces (pressure gradient and molecular diffusion), the fifth - mass forces (gravity), the sixth - external mass forces.

The energy conservation equation takes into account energy transfer due to conductivity, diffusion, and viscous dissipation:

$$\frac{\partial (\rho h)}{\partial t} + \frac{\partial}{\partial x_i}(\rho h u_i) = \frac{\partial p}{\partial t} + u_i \frac{\partial p}{\partial x_i} - \frac{\partial}{\partial x_i}(k_{eff} \frac{\partial T}{\partial x_i}) - \frac{\partial}{\partial x_{ij'}} h_{j'} J_{j'} + (\tau_{ij'})_{eff} \frac{\partial u_j}{\partial x_j} + S_h \text{ , (3)}$$

where $h = \sum_{j'} m_{j'} h_{j'}$ – enthalpy for ideal gases, $h = \sum_{j'} m_{j'} h_{j'} + {}^{p}\!/_{\rho}$ – enthalpy for incompressible flow of gas, $h_{j'} = \int_{T_{ref}}^{T} c_{p,j'} dT$ – enthalpy for flow $J_{j'}$ diffusion substance, $k_{eff} = k_l + k_t$ – effective thermal conductivity (the sum of laminar and turbulent thermal conductivity), $(\tau_{ij'})_{eff}$ – effective stress tensor, S_h – source term that takes heat into account due to chemical reactions and other volumetric energy sources (heat due to radiation, convective exchange between particles and the gas phase, and heat of combustion).

To study the turbulent burning flow of an industrial flame, the averaged conservation equations are used, supplemented by a two-parametric k-ε model of turbulence [13].

Simulation of the combustion process in the gas phase is a complex process involving numerous chemical reactions of fuel and oxidizer through the formation of intermediates and final products of combustion. The task is further complicated because of the interaction between turbulence and the kinetics of the combustion process, in view of the fact that turbulent reactive flows are characterized by sharp fluctuations in temperature and density, under the strong influence of exothermic reactions of the combustion process. To simulate the combustion of the gas phase, a simple chemical reaction system developed by Spalding is used. The model describes the global nature of the combustion process, where the complex mechanism of chemical kinetics is replaced by infinitely fast chemical reactions between fuel and oxidant [14].

So for mathematical modeling of processes occurring in combustion devices during coal combustion, the FLOREAN computer program [15-16] based on numerical solution of three-dimensional equations of energy and substance transfer taking into account chemical reactions is used. All mathematical models represent a complex system of nonlinear three-dimensional partial differential equations. They consist of the equations of continuity of the medium, the state of an ideal gas and the motion of a two-phase medium, heat transfer equations, chemical kinetics, and diffusion for the components of the reacting mixture, taking into account the radiative and turbulent transport described by the k-ε model of turbulence. For numerical calculation were used the primary and boundary conditions, also control volume method for solving the differential equations [17].

Setting of the computing experiments in boiler of RK

In the present work, for carrying out computational experiments on pulverized coal combustion used software package FLOREAN. Creating a database for modeling is carried out using the PREPROZ

software package [18], where the generated files contain the geometric data of the process under study, the initial and boundary conditions for modeling the process of heat and mass transfer in the reacting flows. General view of the boiler BKZ is shown in Fig. 2.

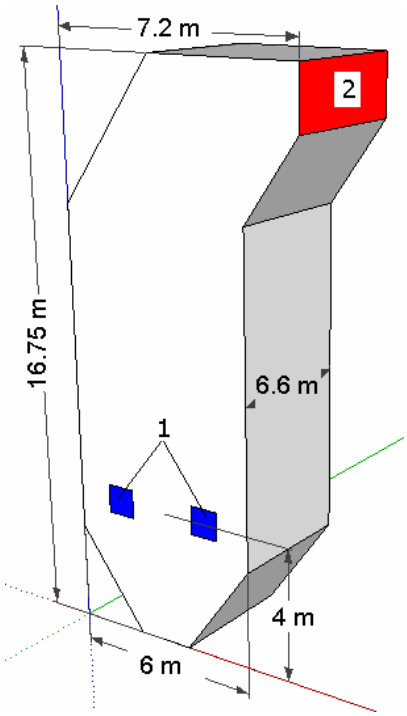


Fig. 2 - General view of the boiler BKZ-75 of Shakhtinskaya CHPP (RK)

Pipes front, rear screens and the lower part form in the furnace space the area of the cold funnel. In tables 1 and 2 there showed basic geometric parameters also technical parameters of the combustion chamber of the BKZ-75 boiler. The characteristics of the coal are presented in Table 3.

Table 1 - Basic geometric parameters of the combustion chamber of the BKZ-75 boiler

Name	Symbol	Unit	Value
Height of the combustion chamber	*(Z)* $H_т$	m	16.75
Width of the combustion chamber	*(X)* $b_т$	m	6
Depth of the combustion chamber	*(Y)* $Г_т$	m	6.6
Frontal and posterior wall area	F_{fr}, F_p	m^2	90.675
Area of the right side wall	F_{s1}	m^2	92.4
Area of the left side wall	F_{s2}	m^2	110.55
Ceiling wall Area	F_s	m^2	27.72
Area of hearth wall	F_h	m^2	7.26
Cross-sectional area of the air-blast channel in the burner	F_a	m^2	0.12
The cross-sectional area of the secondary air duct in the burner	F_{sa}	m^2	0.25

The furnace chamber of the BKZ-75 boiler is equipped with four axial-blade vortex pulverized-coal burners, which are located in one stage of two burners on the side walls of the chamber and direct dust injection from individual dust preparation systems is used.

Table 2 - Technical parameters of the combustion chamber of the boiler BKZ 75-39FB Shakhtinskaya CHP

Name	Value
Number of burners on the boiler, $N_{b,}$ pc.	4
The performance of a single burner for fuel, $B_{b,}$ t/h	3.2
The primary air flow to the boiler, $V_{pa.}$, Nm^3/h	31797
Secondary air consumption per boiler, V_{sa}, Nm^3/h	46459
The temperature of hot air, t_{ha}, °C	290
The excess air factor in the furnace, α	1.2
Value of the suction cup, $\Delta\alpha$ Firebox and festoon Superheater Economizer Air Heater	 0.1 0.03 0.02 0.03
Estimated fuel consumption per boiler, B_c, t/h	12.49
Cold air temperature, t_{ca}, °C	30
Pressure at the inlet, P, mbar	$1.013 \cdot 10^3$
Hydrodynamic resistance of the burner air mixture channel, *ΔP,* mm of water column	67.1
The temperature of the air mixture, t_{am}, °C	140
The wall temperature, t_w, °C	430.15

Table 3 - Characteristics of Karaganda coal grade KR-200

Name	**Symbol**	**Unit**	**Value**
Type of coal	KR-200	-	-
Milling dispersity	R_{90}	%	20
Coal density	ρ	kg/m^3	1350
Heat of combustion	Q_y	kJ/kg	$3.4162 \cdot 10^4$
Ash	A^c	%	35.10
Volatiles	V^r	%	22.00
Humidity	W^p	%	10.60
Carbon	C	%	43.21
Hydrogen	H_2	%	3.6
Oxygen	O_2	%	5.24
Sulfur	S_2	%	1.04
Nitrogen	N_2	%	1.21
Chemical composition of ash (macrocomponents)			
	SiO_2	%	60.2
	Al_2O_3	%	25.5
	Fe_2O_3	%	5.85
	CaO	%	3.65
	MgO	%	1.05
	TiO_2	%	0.95
	SO_3	%	0.8
	K_2O	%	1.65
	Na_2O	%	1.06

Considered coals are difficult to enrich. Their inner component is almost indestructible (the organic part consists of plant matter, brought from mineral impurities deposited with plant residues, and the infiltration part of the mineral salts contained in the water circulating through the cracks). Thus, their enrichment does not justify the economic costs associated with the enrichment process.

Results of computing experiments on pulverized coal combustion processes in boiler furnace of RK

The aerodynamics of two-phase turbulent flows during the combustion of pulverized coal which is vortex transfer [19] causes the nature of the leaking of the entire combustion process. The main role of the aerodynamic structure of the vortex flow is the perfect mixture of fuel mixture with oxidant. Fig. 3a shows that the flow of the air mixture with the combustion products has a vortical character in the burners' zone (Z ~ 4 m). It can be seen that the total velocity vector has its maximum values V ~ 16 m/s there. This is because the counter flow currents, blown from the burner devices, are directed at maximum speed to the center of the furnace space, collide. And here, dissecting into several vortices, form a return flow up and down over the furnace space. This vorticity character arises from turbulence (Fig. 3b) due to the interaction of the air mixture with the oxidant [20].

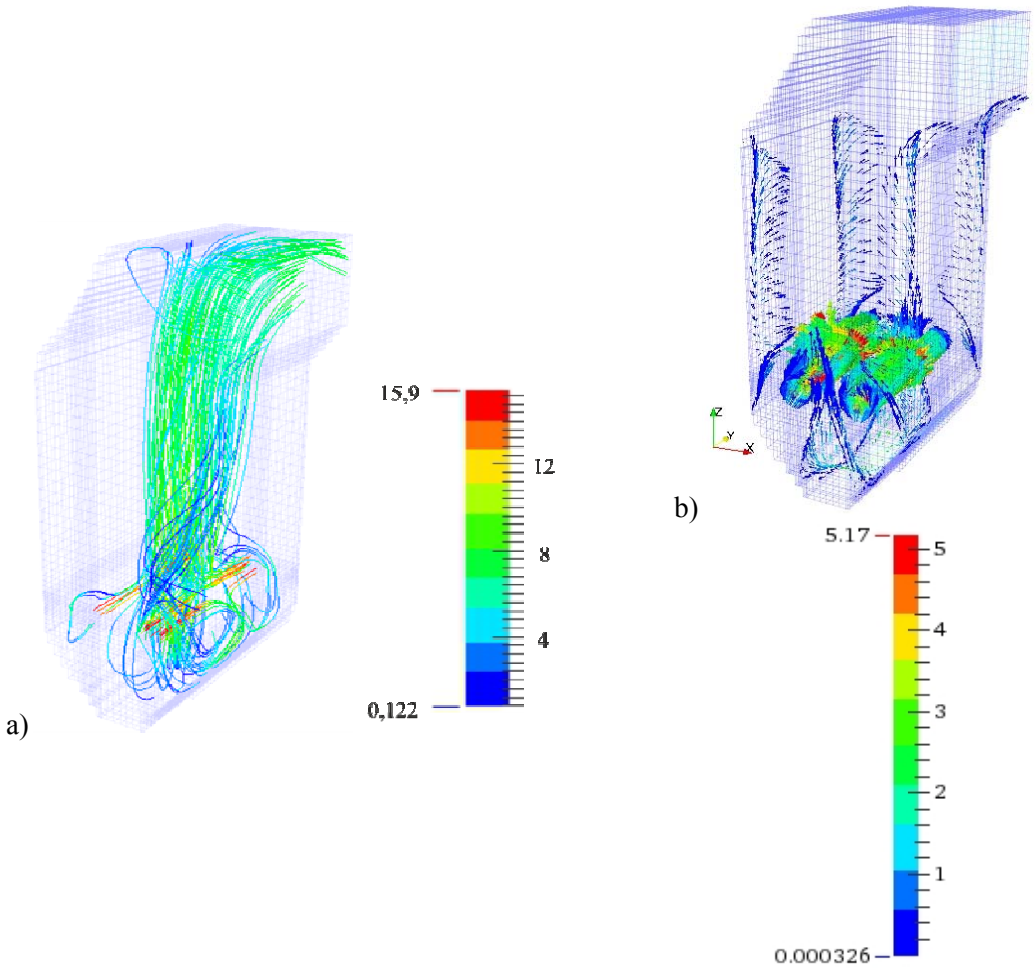


Fig. 3 - Three-dimensional distribution of the full-velocity vector (a) and turbulence (b) in the volume of the combustion chamber of the boiler BKZ-75-39FB Shakhtinskaya CHPP

The fuel mixture and oxidizer (air) coming from the opposite burners, so the temperature values reach their maximum in the core region of the torch at an altitude of about three meters, it is the lower part of burners. Here, due to the vortex nature of the flow maximum convective transfer and an increase in the

residence time of coal particles are observed. As a result temperature rises to its value 1100°C (see Fig. 4). On the height of the combustion chamber, it can be seen a gradual decrease in temperature to the exit from the furnace. At the exit the temperature fields are equalized: in the rotary region of the furnace, the average temperature is T = 941°C, and at the outlet from the combustion space T = 879°C.

Analyzing the verification of obtained computational data with the theoretical calculation at the outlet from the boiler [21] and with natural data from TPP [22]. Analyzing the results it is seen a good agreement.

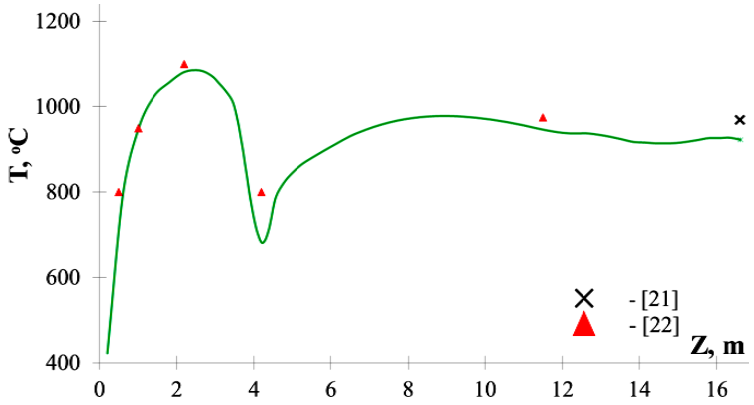


Fig. 4 - Distribution of the temperature by the height
of the combustion chamber and its verification with the known data [21-22]

The distribution of CO_2 concentration in the central part (Fig. 5) is less than at the outlet. The final stages of complete combustion of energy fuel, with the greatest amount of formation of combustion products of CO_2 take place to the output area. The verification of obtained results with known data [22] from real TPP shows the good compliance.

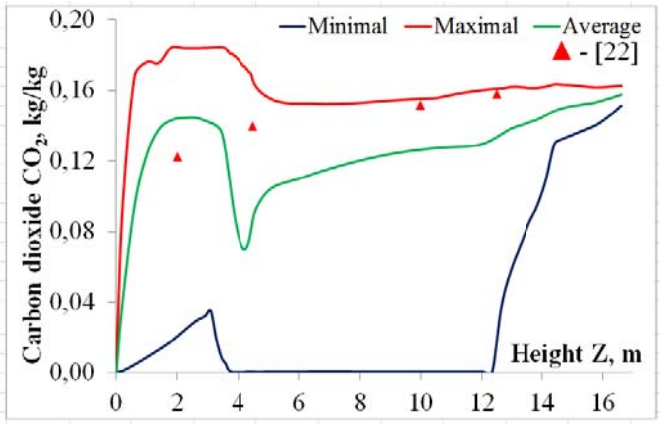


Fig. 5. Two-dimensional distribution of the CO_2 concentrations by height
of the boiler and its verification with the known data

Concentration of nitrious oxides NOx as shown in figure 6 has a maximal value at the burners zone (Z=4 m) in a region where the injected flows are met (central part). At the outlet from the chamber the average value of the NOx is about ~ 700 mg/Nm^3. From the comparing with experimental data [22-24] and MPC norms [25], as it shown in Fig. 8 it can be said that obtained results correspond quite well to the picture of real formation of NOx emissions with acceptable values for ecologically clean operation of energy fuel in this boiler.

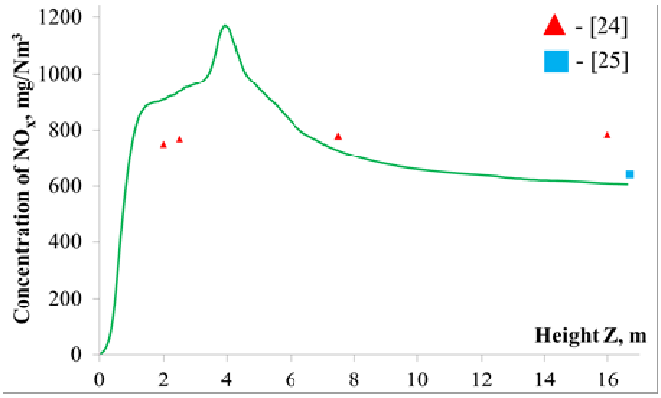


Fig.6 - Distribution of the NOx concentrations by height and its verification with the known data

3D computer simulation of the combustion of pulverized coal fuel allows for better understanding of the problems of computational fluid dynamics (CFD), mathematical and numerical modeling of solid fuel combustion processes, and the mechanism of chemical interaction between combustion products. The results of the conducted research contribute to the solution of the actual problems of thermal physics, technical physics, thermal power engineering and environmental safety, since they make it possible to give recommendations on optimization of burning processes of low-grade energy fuels in order to increase energy efficiency and improve the ecological situation and create “clean” energy production.

Conclusion

In order to reduce emissions and meet the growing demand for electricity, it is urgently necessary to develop and implement new cost-effective and environmentally friendly (safe) technologies, as well as to modernize existing energy supply facilities.

Concluding results of conducting research we can propose the new physical-mathematical and chemical models of simulation low-grade Kazakhstan coal combustion in the real chambers of the energy objects. Used method give an adequate character of the processes of heat and mass transfer and formation of emissions of harmful substances during burning of low-quality Karaganda coal of grade KR-200 with high ash content (more than 35%) in the combustion chamber of the existing power boiler BKZ-75 of Shakhtynskaya CHPP.

By comparisons of numerical experiment results held in this work with natural data from TPP we can propose the observed method of research of combustion processes is reliable. The results carried out in this work and used method of computational study can be useful in the design and development of new, as well as in the improvement of existing combustion chambers of power boilers of TPP.

Acknowledgment

Grant of the Ministry of Education and Science of the Republic of Kazakhstan № AP05133590 “Creation of new computer technologies for 3D modeling of heat and mass transfer processes in high-temperature physicochemically reactive media” (2018-2020).

REFERENCES

[1] Nazarbaev NA (**2011**) Strategii ustojchivoj jenergetiki budushhego Kazahstana do 2050 goda. Jekonomika, Rossija-Kazahstan. Online: http://www.akorda.kz/

[2] Vockrodt S, Leithner R, etc (**1999**) Firing technique measures for increased efficiency and minimization of toxic emissions in Kasakh coal firing, Combustion and incineration, 1492:93-97. (in Eng).

[3] Bolegenova SA, Maximov VYu, Beketayeva MT (**2018**) Modeling of heat mass transfer in high temperature reacting flows with combustion, High Temp, 5:1-6. (in press).

[4] Ergaliyeva A., Gabitova Z., etc (**2016**) Three-dimensional modelling of heat and mass transfer during combustion of low-grade Karaganda coal. Proceedings of 19th Conference on Process Integration, Modelling and Optimisation for Energy Saving and Pollution Reduction (PRES 2016). Prague, Czech Republic. P.0734.

[5] Bolegenova SA, Beketayeva MT, etc (**2015**) Computational Method for Investigation of Solid Fuel Combustion in Combustion Chambers of a Heat Power Plant, High Temp, 53:751-757. DOI: 10.1134/S0018151X15040021 (in Eng).

[6] Askarova AS, Bolegenova SA, etc (**2016**) Influence of boundary conditions to heat and mass transfer processes, International journal of mechanics, 10:320-325. (in Eng).

[7] Bolegenova SA, Bolegenova SA, etc (**2017**) A computational experiment for studying the combustion of thermochemically-gasified coal in the combustion chamber of the boiler BKZ-160, News of the National Academy of Sciences of the Republic of Kazakhstan-Series physico-mathematical, 2:75-80. DOI: https://doi.org/10.32014/2018.2518-1726.

[8] Reynolds WC (**1987**) Fundamentals of turbulence for turbulence modeling and simulation. Lecture Notes for Von Karman Institute Agard.

[9] Leithner R., Ospanova S., etc. (**2016**) Computational modeling of heat and mass transfer processes in combustion chamber at power plant of Kazakhstan. MATEC Web of Conferences 76, 06001. Corfu Island, Greece. P.5.

[10] Launder B, Spalding D (**1974**) The numerical computation of turbulent flows, Comp Meths Appl Mech Eng, 3:269-289. (in Eng)

[11] Askarova AS, Bolegenova SA, etc (**2017**) Numerical modeling of burning pulverized coal in the combustion chamber of the boiler PK 39, News of the National Academy of Sciences of the Republic of Kazakhstan-Series physico-mathematical, 2:58-63. DOI: https://doi.org/10.32014/2018.2518-1726.

[12] Görner K (**1991**) Technische Verbrennungssysteme – Grundlagen, Modellbildung, Simulation. Springer Verlag, Berlin Heidelberg.

[13] Leithner R., Müller H. (**2003**) CFD studies for boilers. Proceedings of Second M.I.T. Conference on Computational Fluid and Solid Mechanics. Cambridge, United Kingdom. P.172.

[14] Bekmuhamet A., Beketayeva M.T. etc. (**2014**) Analysis of formation harmful substances formed as a result of burning the low-grade coal in the combustion chamber of the industrial boiler of Kazakhstan using CFD-code FLOREAN. Proceedings of 17th Conference on Process Integration, Modelling and Optimisation for Energy Saving and Pollution Reduction (PRES 2014). Prague, Czech Republic. P.7.160.

[15] Leithner R (**2006**) Numerical Simulation. Computational Fluid Dynamics CFD: Course of Lecture. IWBT, Braunschweig.

[16] Heierle EI, Manatbayev R, etc (**2016**) CFD study of harmful substances production in coal-fired power plant of Kazakhstan, Bulgarian Chemical Communications, 48:260-265. (in Eng)

[17] Müller H (**1997**) Numerische Simulation von Feuerungen. CFD-Vorlesung. IWBT, Braunschweig.

[18] Pauker W (**1997**) Creating data sets for Florean using the tool PREPROZ. IWBT, Braunschweig.

[19] Askarova AS, Bolegenova SA, etc (**2017**) Investigation of aerodynamics and heat and mass transfer in the combustion chambers of the boilers PK-39 and BKZ-160, News of the National Academy of Sciences of the Republic of Kazakhstan-Series physico-mathematical, 2:27-38. DOI: https://doi.org/10.32014/2018.2518-1726.

[20] Nugymanova A., Shortanbayeva Zh. Etc. (**2017**) Simulation of the Aerodynamics and Combustion of a Turbulent Pulverized-Coal Flame. Proceedings of 4th International Conference on Mathematics and Computers in Sciences and in Industry (MCSI 2017). Corfu Island, Greece. P.92-97.

[21] Kuznecov NV i dr. (**1998**) Teplovoj raschet kotlov: Normativnyj metod. AOOT «NPO CKTI», Rossija.

[22] Alijarov BK, Alijarova MB (**2011**) Szhiganie kazahstanskih uglej na TJeS i na krupnyh kotel'nyh: opyt i perspektivy. RGP Gylym ordasy, Kazakhstan.

[23] Ustimenko BP, Alijarov BK, Abubakirov EK (**1982**) Ognevoe modelirovanie pyleugol'nyh topok. Nauka, Kazakhstan.

[24] Bolegenova SA (**2015**) Otchet o nauchno-issledovatel"skoj rabote "issledovanie ajerodinamicheskih i teplofizicheskih harakteristik pyleugol"nogo fakela – 1589/GF3". № 0113РК00498.

[25] RND 34.02.303-91 (**2005**) Otraslevaja instrukcija po normirovaniju vrednyh vybrosov v atmosferu dlja teplovyh jelektrostancij i kotel'nyh. Astana, Kazakhstan. (in Russ)

А.С. Асқарова[1], С.Ә.Бөлегенова[2], П. Шафаржик[3],
С.Ә. Бөлегенова[2], В.Ю. Максимов[1], М.Т. Бекетаева[1], А.О. Нұғманова[2]

[1] Әл-Фараби атындағы Қазақ ұлттық университетіне қарасты эксперименталдық және теориялық физиканың ғылыми-зерттеу институты, әл-Фараби атындағы ҚазҰУ ЭТФ ҒЗИ, Алматы, Қазақстан;
[2] Әл-Фараби атындағы Қазақ ұлттық университетін физика-техникалық факультеті, әл-Фараби атындағы ҚазҰУ, Алматы, Қазақстан;
[3] Прага қ. Чех Техникалық университетінің сұйық динамикасы мен термодинамика факультеті, Прага қ. Чех Техникалық университеті, Прага, Чех Республикасы

ҚАЗАНДЫҚТАРДЫҢ ЖАНУ КАМЕРАЛАРЫНДА ШАҢТЕКТЕС КӨМІРДІҢ ЖАНУ ПРОЦЕСТЕРІНІҢ ЗАМАНАУИ КОМПЬЮТЕРЛІК ТӘЖІРИБЕЛЕРІ

Аннотация. Жобаның мақсаты жоғарытемпературалы физика-химиялық әрекеттесуші орталардағы жылу масса тасымалдану процестерін 3D моделдеудің жаңа компьютерлік технологияларын құру болып табылады. Олар ҚР қызмет ететін көмірлі ЖЭС-ң жану камераларында болатын технологиялық процестердің ағыстар аэродинамикасын, жылу масса алмасу сипаттамаларын анықтауға мүмкіндік туғызады. Зерттеулердің жаңашылдығы ең жаңа 3D моделдеудің ақпараттық технологияларын қолдану болып табылады. Ол жоба қатысушыларына ҚР іске қосылған ЖЭС-ң реалды жану камераларындағы шаң-көмірлі

отынның жануы кезінде жылу масса тасымалданудың күрделі процестерінің жаңа мәліметтерін алуға жәрдемдеседі. Төменгісұрыпты отынды жағу кезінде термодинамикалық, кинетикалық және жылу және масса тасымалдау процестерін үш өлшемді компьютерлік модельдеуді ескергендегі сандық моделдеу жану процесінің технологиялық процесінің адекватты физикалық, математикалық және химиялық үлгілерін орналастыру үшін оңтайлы жағдайларды табуға мүмкіндік береді, сондай-ақ жан-жақты зерттеулер жүргізуге көмек береді, осылайша жану процесінің тұтану, газдану және жоғары күлді көмірлерді жағуда жаңа жолдарды ұсынуға мүмкіндік береді. Жобаның дамуына ұсынылған компьютерлік модельдеу әдістері - бүкіл әлем бойынша көмірмен жұмыс істейтін жылу электр станцияларында қолданылатын көмірдің барлық түрлерін жағуда жаңа және техникалық мүмкіншілігі зор болып табылады. Ұсынылып отырған технологиялар көмірмен жұмыс істейтін электр станцияларында орындалатын қымбат және еңбек шығындары қажет етілетін табиғи эксперименттерді ауыстыруға немесе жоюға мүмкіндік береді.

Түйін сөздер. Жану, шекаралық шарттар, компьютерлік моделдеу, төменгі сұрыпты көмір, шаңкөмірлі отын, әсерлесетін қоспа, жану камерасы, сандық тәжірибелер

А.С.Аскарова[1], С.А.Болегенова[2], П.Шафаржик[3],
С.А.Болегенова[2], В.Ю.Максимов[1], М.Т.Бекетаева[1], А.О.Нугманова[2]

[1] Научно-исследовательский институт экспериментальной и теоретической физики при Казахском национальном университете имени аль-Фараби, НИИ ЭТФ при КазНУ имени аль-Фараби, Алматы, Казахстан;
[2] Физико-технический факультет Казахского национального университета имени аль-Фараби, КазНУ имени аль-Фараби, Алматы, Казахстан;
[3] Департамент динамики жидкости и термодинамики Чешского Технического Университета г.Прага, ЧТУ г. Прага, Прага, Чешская Республика

СОВРЕМЕННЫЕ КОМПЬЮТЕРНЫЕ ЭКСПЕРИМЕНТЫ ПРОЦЕССОВ СЖИГАНИЯ УГОЛЬНОЙ ПЫЛИ В ТОПОЧНЫХ КАМЕРАХ КОТЛОВ

Аннотация. Целью работы является создание новых компьютерных технологий 3D моделирования процессов тепломассопереноса в высокотемпературных физико-химически реагирующих средах, которые позволят определять аэродинамику течения, тепломассообменные характеристики технологических процессов, происходящих в топочных камерах в действующих угольных тепловых электрических станций Республики Казахстан. Новизна исследований заключается в использовании новейших информационных технологий 3D моделирования, которые позволят участникам проекта получить новые данные о сложных процессах тепломассопереноса при горении пылеугольного топлива в реальных топочных камерах, действующих ТЭЦ РК. Численное моделирование, включающее термодинамическое, кинетическое и трехмерное компьютерное моделирование процессов тепломассопереноса при сжигании низкосортного топлива позволит найти оптимальные условия для постановки адекватной физико-математической и химической модели технологического процесса горения, а также провести комплексное исследование и тем самым разрабатывать пути оптимизирования процесса воспламенения, газификации и сжигания высокозольных углей. Предлагаемые к разработке методы компьютерного моделирования являются новыми и технически реализуемы при сжигании всех типов углей, используемых на пылеугольных ТЭС по всему миру. Разрабатываемые технологии позволят заменить или исключить проведение дорогостоящих и трудоемких натурных экспериментов на угольных ТЭС.

Ключевые слова. Горение, граничные условия, компьютерное моделирование, низкосортный уголь, пылеугольное топливо, реагирующая смесь, топочная камера, численный эксперимент.

Information about authors:
Askarova A.S. - SRI ETP of Al-Farabi Kazakh national university, professor, Dr.of Phys.-math.Sci., Aliya.Askarova@kaznu.kz;
Bolegenova S.A. - Al-Farabi Kazakh national university, professor, Dr.of Phys.-math.Sci., Saltanat.Bolegenova@kaznu.kz
Safarik P. - Czech Technical University in Prague, professor, Cd. of Tech.Sci., pavel.safarik@fs.cvut.cz
Bolegenova S.A. - Al-Farabi Kazakh national university, Sen. Lecturer, PhD, bolegenova.symbat@kaznu.kz
Maximov V.Yu. - SRI ETP of Al-Farabi Kazakh national university, scientific engineer, PhD, postdoctoral student, valeriy.maximov@kaznu.kz
Бекетаева М.Т. - SRI ETP of Al-Farabi Kazakh national university, scientific engineer, PhD, Beketayeva.m@gmail.com
Nugymanova A.O. - Al-Farabi Kazakh national university, Lecturer, PhD Student, aijoka01@gmail.com

N E W S
OF THE NATIONAL ACADEMY OF SCIENCES OF THE REPUBLIC OF KAZAKHSTAN
PHYSICO-MATHEMATICAL SERIES
ISSN 1991-346X **https://doi.org/10.32014/2018.2518-1726.12**
Volume 6, Number 322 (2018), 15 – 22

Maulen Nassurlla [1,2], N.Burtebayev [1,2], Zh.K. Kerimkulov [1], T. Suzuki [3], S.B. Sakuta [4], Marzhan Nassurlla [1,2], R. Khojayev [1,2]

[1]Institute of Nuclear Physics, Almaty, Kazakhstan;
[2]Al-Farabi Kazakh National University, Almaty, Kazakhstan;
[3]Saitama University, Saitama, Japan;
[4]National Research Center "Kurchatov Institute", Moscow, Russia
nburtebayev@yandex.ru

INVESTIGATION OF DEUTERON SCATTERING BY ^{7}Li NUCLEI AT ENERGY OF 14.5 MeV

Abstract. The pronounced cluster structure of lithium isotopes is an excellent test for verification the various theoretical nuclear models. The study of the cluster exchange mechanism in direct nuclear reactions opens new possibilities in determining the structures of these nuclei. The sets of parameters of optical potentials available in the literature are vary, that can lead to ambiguity in determining the cluster spectroscopic factors of the studied nuclei. Therefore, an experimental study of the scattering process at energy E = 14.5 MeV has been carried out in order to obtain an independent global systematic of optical potentials for the d + ^{7}Li system. New experimental data were obtained in the current paper on the elastic and inelastic scattering of deuterons on ^{7}Li nuclei at energy E = 14.5 MeV. In addition to this, in our analysis we used experimental data on elastic scattering, measured previously at deuteron energies from 7 to 28 MeV. The analysis of the differential cross sections was performed within the framework of the optical model. The optimal parameters of optical potentials for the studied nucleus are established. The obtained parameters in this work will be used in the analysis of the data on inelastic scattering of deuterons and the ^{7}Li (d, t) reaction to refine the structural characteristics of lithium isotopes.

Keywords: lithium nuclei, differential cross sections, elastic and inelastic scattering, optical potential.

Introduction

The systematics of the proton and α-particles scattering by light nuclei at an energy of about 10 MeV/nucleon showed an anomalous growth of cross sections at the large angles. It was shown [1–5], that other mechanism, such as exchange processes, make a significant contribution to the formation of a scattering cross section in this energy region.

At present, scattering of α-particles by ^{6}Li and ^{7}Li nuclei, having a pronounced cluster structure, was systematically studied. The anomalous large-angle scattering (ALAS), observed in [1-4], can be only described with taking into account the contribution of the cluster exchange mechanism, which is physically indistinguishable from potential scattering. Therefore, in a number of papers [4, 6], with taking into account this mechanism, it was possible not only to obtain more reliable parameters of the optical potentials, but also to extract the values of cluster spectroscopic factors from the cross sections analysis at large angles. In particular, the values of the spectroscopic factors were obtained for the configurations d +α, ^{3}He + t and t + α, and not only for the ground, but also for the excited states of the ^{6,7}Li nuclei from the analysis of the scattering of ^{3}He and α - particles. A systematic analysis of the deuteron scattering on ^{6}Li nuclei, performed in [4] in wide energy range, confirmed the possibility of describing the behavior of the cross sections at backward angles by the exchange mechanism. The differential cross sections of the transfer of the alpha cluster were obtained taking into account the channels coupling and the spectroscopic factor for the configuration of the ^{6}Li nucleus as d + α, which is close to 1.

In the present work, to determine the parameters of the optical potential for the ^{7}Li nucleus, the scattering of deuterons was studied not only at 14.5 MeV, but also at other energies using literature data.

Experimental method and results of the measurements

The experimental angular distribution of the deuteron elastic scattering by ^{7}Li was measured at the energy of 14.5 MeV. The deuteron beam was extracted from the U150-M isochronous cyclotron of the Institute of Nuclear Physics (Almaty, Kazakhstan).

The scattering chamber [7], used in the experiment, allowed to measure both in the region of small scattering angles (from 3° to 22°) and in the wide angular range (10°≤θ≤170°). The metal lithium with 90% enrichment of ^{7}Li was used as a target. It was manufactured by thermal evaporation of lithium on a thin alundum (Al_2O_3) film (30 μg/cm^2) in vacuum. After deposition, the target was transferred to a scattering chamber without breaking the vacuum. The target thickness was determined by weighing, as well as the energy losses of α-particles from the radioactive source ^{241}Am-^{243}Am-^{244}Cm and ^{239}Pu. The ^{7}Li target thickness was determined as 0.393 ± 0.030 mg/cm^2.

For the registration and identification of nuclear reactions products the Δ*E*-*E* method was used [8]. Thin surface-barrier silicon detectors with thicknesses of 100 or 50 μm (for small angles) and 30 μm (for large angles) were used as a Δ*E* counters. As the *E* counter, a surface barrier silicon detector with a thickness of 2 mm was used.

The angular distribution of the deuteron elastic scattering by ^{7}Li nuclei was measured in the angular range of 18^0–128^0 with step 2^0. The systematic error in the cross sections is related with the uncertainty of the target thickness (6–9%), of the solid angle of the spectrometer (1%) and of the calibration of the current integrator (<1%). The statistical errors of the analyzed data are 1-5% and reached 6-15% only in the minimum of the cross sections.

The energy resolution of the registration system (~ 150 keV) allowed to reliably identify all low-lying levels of the ^{7}Li nucleus. Typical spectrum of deuterons is shown in Fig. 1a. The transitions to states with excitation energies E_x = 0.478 MeV (1/2$^-$) and 4.65 MeV (7/2$^-$) as well as the elastic peak (3/2$^-$) were observed in the spectrum of deuterons. The peaks corresponding to the excited states of the ^{12}C (4.43 MeV) and ^{16}O (6.09 MeV) nuclei (due to the presence of carbon and oxygen impurities in the target) were not reliably separated from the 4.65 MeV state of the ^{7}Li nucleus. Thus, this state was excluded from further analysis.

The energy peaks located higher from the ground state of ^{7}Li belong to the nuclei of oxygen, aluminum (from the substrate) and carbon. The presence of carbon is due to its deposition on the target during the experiment.

When analyzing elastic scattering at small angles, the contribution of impurities was taken into account using literature data on the elastic scattering of deuterons by ^{12}C, ^{16}O nuclei at an energy of *E* = 13.6 MeV [9].

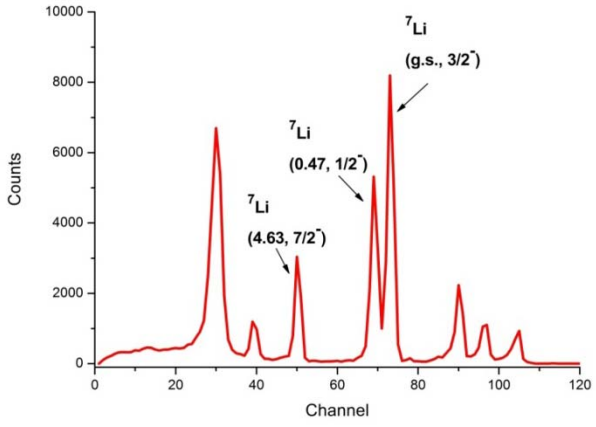


Fig. 1a. Energy spectrum of deuterons scattered by ^{7}Li (θ_{lab} = 70^o) at the energy of 14.5 MeV.

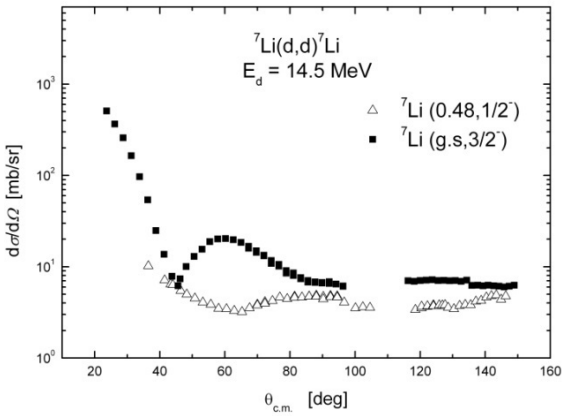


Fig. 1b. Differential scattering cross sections of deuterons scattered by ^{7}Li at *E* = 14.5 MeV.
Black squares - elastic scattering cross sections;
white triangles - inelastic scattering cross sections.

The Fig. 1b shows measured differential cross sections for deuteron scattering by ^{7}Li at the energy of 14.5 MeV. As it can be seen from figure, the diffraction structure is characteristic for the measured

angular distributions. In contrast to deuteron scattering on ^{6}Li nuclei [4], for which a significant rise of the cross sections is observed in the backward hemisphere, the scattering cross sections for ^{7}Li gradually decrease with increasing of the scattering angle. This fact confirms the absence of deuteron cluster in the ^{7}Li nucleus. It should be noted that the experimental data measured at the energy of 14.5 MeV are in good agreement within experimental errors with literature data for the energy of 14.7 MeV [10].

Analysis of elastic scattering of deuterons by the optical model and discussion of the results

The differential cross sections of the elastic scattering were calculated in the framework of the optical model (OM) [11]. The parameters of the phenomenological optical potential (OP) were found by comparing the calculated angular distribution with the experimental data. The potential is used in the Woods-Saxon parameterization, which well reproduces the distribution of nuclear matter:

$$U(r) = -Vf(r) + i4a_D W_D \frac{df_w(r)}{dr} + V_{so}(\frac{h}{m_\pi c})^2 \frac{1}{r}\frac{d}{dr} f_{so}(r)(LS) + V_C(r) \quad (1)$$

The first two terms are responsible for the nuclear central interaction with surface absorption. The third term is the spin-orbit potential. V and W_D are the depths of the real and imaginary parts of the optical potential with surface absorption. V_{SO} is the depth of the real part of the spin-orbit potential. Radial dependence $f_i(\mathrm{r})$ is described by the Woods-Saxon form factor with a reduced radius r_i and diffuseness a_i, where i is R, D or SO:

$$f_i = \left[1 + \exp((r - r_i A^{1/3}) / a_i)\right]^{-1}, \quad (2)$$

V_C (r) is the Coulomb potential of uniformly charged sphere with radius R_C:

$$V_C(r) = Z_p Z_t \mathrm{e}^2/R_C, \text{ when } r > R_C, \quad (3)$$

where Z_p, Z_t are the charges of the incident projectile (p) and the target (t). In our calculations $R_C = r_C A^{1/3}$, where r_C =1.30 fm.

The parameters of the potential corresponding to the best description of the experimental cross sections were found by minimizing the value of χ^2:

$$\chi^2 = \frac{1}{N}\sum_{i=1}^{N}\left[\frac{\sigma^T(\theta_i) - \sigma^E(\theta_i)}{\Delta\sigma^E(\theta_i)}\right]^2, \quad (4)$$

where N - the number of experimental points in the angular distribution, σ^T and σ^E - the calculated and measured values of the differential scattering cross section at angle θ_i, $\Delta\sigma^E$- the uncertainty of the σ^E value.

It should be noted that choosing this potential as optimal, we follow the physically reasonable value of the volume integral of the real part defined as:

$$J_V = (1/A_p A_t)\int V(r) 4\pi r^2 \mathrm{d}r, \quad (5)$$

where A_p and A_t are the mass numbers of the incident projectile and the target nucleus. Its value should be close to the corresponding value of the nucleon-nucleon ·interaction potential, which is approximately equal to 400 MeV fm^3 [12].

It is well known that the parameters of the optical potentials have discrete and continuous uncertainties [13]. Therefore, to eliminate the discrete uncertainty of the real part of the potential, the potential energy dependence is often used.

For this purpose, global systematic of the OP parameters for the d + ^{7}Li system was performed in a wide energy range using literature data. For this purpose, experimental data on the elastic scattering of

deuterons on ^{7}Li measured at energies 7–12 MeV [14, 15], 14.7 MeV [10], 25 MeV [16] and 27.7 [17] were used.

It has been established that with energy increasing of the incident particle, the discrete uncertainty is eliminated, for example, at energies above 12 MeV/nucleon. According to this, first of all, the experimental data were analyzed at energies of 28 and 25 MeV. As starting parameters, the values from [18] were used, which were established by the global systematic of the optical potentials for the elastic scattering of deuterons in the energy range 20–90 MeV for the atomic mass range from A = 12 to A = 238.

The search of the parameters of the optical potential was carried out by fitting the calculated angular distributions with the experimental data using the FRESCO code [19]. To eliminate the discrete uncertainty in determining the optical parameters, the radii of the real (r_V) and imaginary (r_W) parts of the potentials were fixed. The theoretical calculations were fitted to experimental data by varying the 4 remaining OP parameters (V_R, W_D, a_R and a_D). The fitting of the calculated cross sections to the experimental data was performed at the maximum angular range. The diffuseness's (a), established in such approach, strongly depend on the energy at low energies (see Fig.2a) which probably reflect the effects of the resonances in the d + ^{7}Li system (set A in the Table). It can be seen from the figure 2a, the diffuseness's become constant with energy increasing.

The second set of OP (B) is obtained using these fixed values a_R = 0.9 fm and a_W = 0.75 fm. Next, to clarify the dynamics of eliminating discrete ambiguity, the dependence of χ^2 on the depth of the real part for this set was studied. The depth values varied from 30 MeV to 250 MeV with step of 10 MeV. The results are shown in figure 2b. It can be seen that several minimums are observed at low energies. This indicates the presence of discrete families of OPs. With energy increasing, the number of minimums decreases up to one.

As one would expect, with the increase in the energy of the incident particles, the discrete uncertainty of the depth of the real part of the potential was eliminated. The results of the description of the experimental angular distributions of the elastic scattering of deuterons on the studied nucleus are presented in Fig. 3. Since the experimental data on the elastic scattering at energies of 14.7 and 14.5 MeV are virtually indistinguishable within the limits of the error, the graphs show data at 14.7 MeV (these data cover more large angles).

As a comparison, Fig. 3 shows the calculations of the angular distributions of elastic scattering performed using the optimal OP sets from [20] (Set C); [21] (Set D) (see Table).

As can be seen from the Fig. 3, the theoretical calculations performed using the OP sets from this work better describe the differential cross sections for the elastic scattering of deuterons in the investigated energy range.

The optimal OP parameters established in this work will be used in the data analysis of the inelastic scattering of deuterons and ^{7}Li(d, t) reactions at energy of 14.5 MeV to clarify the structure of lithium isotopes.

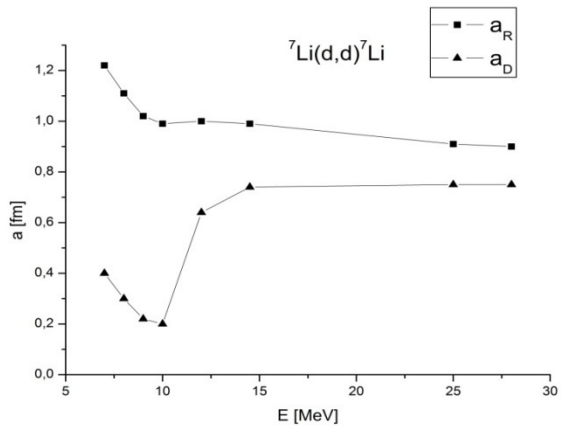


Fig.2a. The energy dependence of the OP diffuseness's for the deuteron scattering by ^{7}Li.

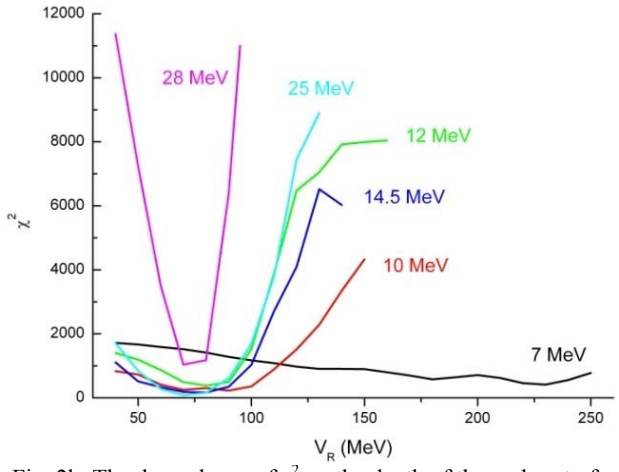


Fig. 2b. The dependence of χ^2 on the depth of the real part of the potential.

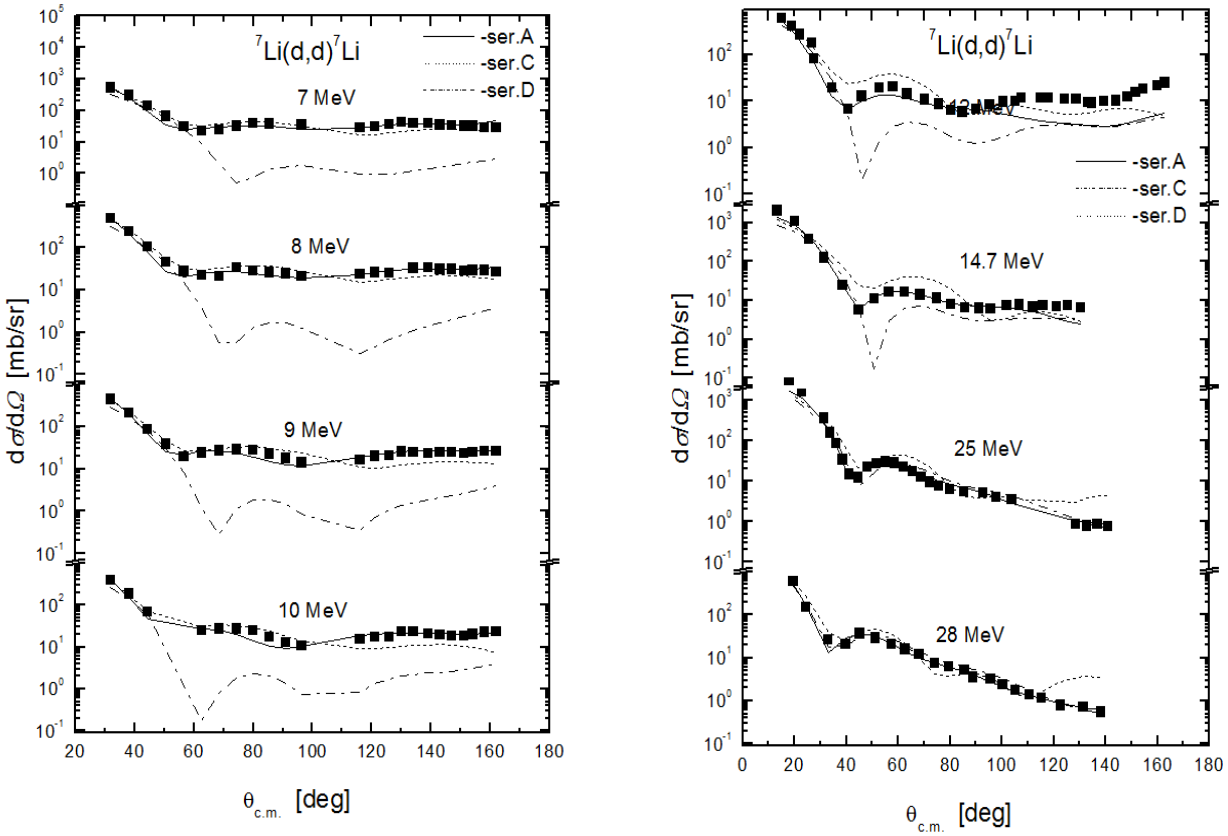


Fig. 3. Differential cross sections of the elastic scattering of deuterons on ^{7}Li nucleus at different energies. Symbols - experimental data, solid lines - OM calculations with set A; dotted lines - calculations with set C; dash-dotted lines - calculations with set D.

Table – Optical potential parameters for elastic scattering ^{7}Li + *d*

E_d, MeV	Set	V_R (MeV)	r_R (fm)	a_R (fm)	W_D (MeV)	r_D (fm)	a_D (fm)	V_{SO} (MeV)	r_{SO} (fm)	a_{SO} (fm)	r_C (fm)
7	A	62.96	1.17	1.22	9.08	1.325	0.4	6.76	1.07	0.66	1.3
	B	89.7	1.17	0.9	3.99	1.325	0.75	6.76	1.07	0.66	1.3
	C	66.0	1.35	0.9	4.5	2.37	0.3	8.0	0.86	0.25	1.3
	D	77.78	1.173	0.809	14.21	1.327	0.551	3.703	1.23	0.813	1.69
8	A	67.53	1.17	1.105	12.47	1.325	0.304	6.76	1.07	0.66	1.3
	B	91.62	1.17	0.9	4.36	1.325	0.75	6.76	1.07	0.66	1.3
	C	65.0	1.35	0.88	4.9	2.3	0.3	8.0	0.86	0.25	1.3
	D	77.62	1.173	0.809	14.13	1.327	0.551	3.702	1.23	0.813	1.69
9	A	72.65	1.17	1.02	19.22	1.325	0.217	6.76	1.07	0.66	1.3
	B	94.31	1.17	0.9	4.87	1.325	0.75	6.76	1.07	0.66	1.3
	C	62.0	1.35	0.86	6.0	2.15	0.3	8.0	0.86	0.25	1.3
	D	77.46	1.173	0.809	14.05	1.327	0.551	3.702	1.23	0.813	1.69
10	A	73.41	1.17	0.988	23.29	1.325	0.195	6.76	1.07	0.66	1.3
	B	95.85	1.17	0.9	4.65	1.325	0.75	6.76	1.07	0.66	1.3
	C	61.5	1.35	0.83	7.2	2.18	0.3	8.0	0.86	0.25	1.3
	D	77.29	1.173	0.809	13.974	1.327	0.551	3.702	1.23	0.813	1.69
12	A	68.84	1.17	1.0	11.08	1.325	0.64	6.76	1.07	0.66	1.3
	B	72.35	1.17	0.9	6.21	1.325	0.75	6.76	1.07	0.66	1.3
	C	64.0	1.35	0.79	10.5	2.1	0.3	8.0	0.86	0.25	1.3
	D	76.96	1.173	0.809	13.815	1.327	0.551	3.702	1.23	0.813	1.69
14.7	A	73.97	1.17	0.986	9.57	1.325	0.74	6.76	1.07	0.66	1.3
	B	73.08	1.17	0.9	7.95	1.325	0.75	6.76	1.07	0.66	1.3
	C	62.0	1.35	0.73	12.0	2.0	0.3	8.0	0.86	0.25	1.3
	D	76.49	1.173	0.809	13.60	1.327	0.551	3.702	1.23	0.813	1.69

Продолжение таблицы

E_d, MeV	Set	V_R (MeV)	r_R (fm)	a_R (fm)	W_D (MeV)	r_D (fm)	a_D (fm)	V_{SO} (MeV)	r_{SO} (fm)	a_{SO} (fm)	r_C (fm)
25	A	81.14	1.17	0.91	14.37	1.325	0.75	6.76	1.07	0.66	1.3
	B	75.94	1.17	0.9	10.7	1.325	0.75	6.76	1.07	0.66	1.3
	C	57.0	1.35	0.72	12.9	1.94	0.3	8.0	0.86	0.25	1.3
	D	74.60	1.173	0.809	12.78	1.327	0.551	3.70	1.23	0.813	1.69
28	A	75.32	1.17	0.9	10.28	1.325	0.75	6.76	1.07	0.66	1.3
	B	75.32	1.17	0.9	10.28	1.325	0.75	6.76	1.07	0.66	1.3
	C	55.62	1.35	0.72	12.9	1.94	0.3	8.0	0.86	0.25	1.3
	D	74.02	1.173	0.809	12.54	1.327	0.551	3.70	1.235	0.813	1.69

Conclusions

Differential cross sections of the deuteron elastic scattering by ^{7}Li were measured at the energy of 14.5 MeV in the angular range from 18^0 to 128^0. In addition to the angular distribution of elastic scattering, measured by us, in the optical-model analysis, we used other experimental data obtained earlier in the energy range of 7-28 MeV. As result, the optimal parameters of optical potentials were found, which correctly describe the angular distributions at the different beam energies in the full angular range.

It is shown that the diffusion parameter (a) strongly depends on the energy up to E_d = 15 MeV, and at higher energies this parameter practically does not change its values.

The dependence of χ^2 on the depth of the real potential shows that several minimums are observed at low energies and at the higher energies (E_d > 20 MeV) the number of minimums decreases up to one. This indicates the elimination of a discrete ambiguity in determining the real part of the potential.

REFERENCE

[1] Bachelier D, Bernas M, Boyard JL, Harney HL, Jourdain JC, Radvanyi P, Roy-Stephan M, Devries R (1972) Exchange effect in the 166 MeV α-particle elastic scattering on ^{6}Li, Nuclear Physics A, 195:361-368. DOI: 10.1016/0375-9474(72)91064-0 (in English).

[2] Goldberg VZ, Gridnev KA, Hefter EF, Novatskii BG (1975) Exchange effects in the scattering of α-particles and deuterons by ^{6}Li, Physical Letters B, 58:405-407. DOI: 10.1016/0370-2693(75)90572-9 (in English).

[3] Bragin VN, Burtebaev N, Duysebaev A, Ivanov GN, Sakuta SB, Chuev VI, Chulkov LV (1986) The role of exchange effects in the elastic scattering of α-particles and ^{3}He ions on ^{6}Li nuclei, Yad.Fiz., 44: 312-319. DOI: 10.1134/S1063778812040138 (in Russian).

[4] Sakuta SB, Artemov SV, Burtebaev N, Kerimkulov ZhK, Novatsky BG, Stepanov DN, Yarmukhamedov R (2009) The Role of Channels Coupling and Exchange Mechanisms with Deuteron in Anomalous Scattering of α-particles on ^{6}Li, Physics of Atomic nuclei, 72:2046-2055. (in English).

[5] Goldberg DA, Smith SM, Burdzik GF (1974) Refractive behavior in intermediate-energy alpha scattering, Phys Rev C, 10:1362. DOI: 10.1103/PhysRevC.10.1362 (in English).

[6] Sakuta SB, Burtebayev N, Burtebayeva JT, Duisebayev A et al. (2014) The channel coupling and triton cluster exchange effects in scattering of ^{3}He ions on ^{6}Li nuclei, Acta Phys Pol B, 45:1853-1863. DOI: 10.5506/APhysPolB.45.1853 (in English).

[7] Artemov SV, Bazhazhin AG, Baktybaev MK, Burtebaev N, Duysebaev A, Duysebayev BA, Zarifov RA, Kadyrzhanov KK, Karakhodzhaev AA, Sakhiev SK, Satpayev NK, Sargaskaev AM, Seytimbetov AM (2006) Bulletin of the National Academy of Sciences of Kazakhstan [Izvestija Akademii Nauk Kazahstana serija fizicheskaja] 6:61-64. (in Russian) **https://doi.org/10.32014/2018.2518-1726**

ISSN 2518-1726 (Online), ISSN 1991-346X (Print)

[8] Artemov SV, Bazhazhin AG, Burtebaev N, Baktybaev M K, Karakhodzhayev A A, Nam IV, Nebesny AF, Radiuk GA, Yakushev VP (2009) Instruments and Experimental Technique [Pribory i Tehnika Jeksperimenta] 1:1668-170. (in Russian).

[9] Vereshchagin AN, Korostova IN, Sokolov LS, Tokarevskii VV, Chernov IP (1969) News of the National Academy of Sciences of Kazakhstan [Izvestija Akademii Nauk Kazahstana serija fizicheskaja] 32:573. (in Russian) **https://doi.org/10.32014/2018.2518-1726**

ISSN 2518-1726 (Online), ISSN 1991-346X (Print)

[10] Matsuki S, Yamashita S, Fukunaga K, Nguyen DC, Fujiwara N, Yanabu T (1969) Elastic and inelastic scattering of 14.7 MeV deuterons and of 29.4 MeV alpha-particles on ^{6}Li and ^{7}Li, Jpn Phys J, 26:1344. DOI: 10.1143/JPSJ.26.1344 (in English).

[11] Hodgson PE (1967) The Optical Model of the Nucleon-Nucleus Interaction, Annual Review on nuclear Science, 17:1-32. DOI: 10.1146/annurev.ns.17.120167.000245 (in English).

[12] Barret R (1977) Nuclear sizes and structure, Clarendon Press, Oxford. ISBN-10: 0198512724.

[13] Goldberg DA and Smith SM (1972) Criteria for the Elimination of Discrete Ambiguities in Nuclear Optical Potentials, Phys Rev Lett, 29:500-504. DOI: 10.1103/PhysRevLett.29.500 (in English).

[14] Abramovich SN, Guzjovskii BYa, Dzyuba BM, Zvenigorods AG, Trusillo SV, Sleptsov GN (1976) Physics of Atomic nuclei [Yad. Fiz.] 40:842. (in Russian).

[15] Bingham HG, Zander AR, Kemper KW, Fletcher NR (1971) Elastic scattering of deuterons by ^{6}Li and ^{7}Li at 8.0–12.0 MeV, Nuclear Physics A, 173:265-272. DOI: 10.1016/0375-9474(71)90344-7 (in English).

[16] Burtebayev N, Burtebayeva JT, Duisebayev A et al. (2015) Mechanism of the ^{7}Li(d,t) reaction at 25 MeV energy of deuterons, values of spectroscopic factors and asymptotic normalization coefficients for the ^{7}Li → ^{6}Li+n vertex, Acta Phys Pol B, 46:1037-1054. DOI: 10.5506/APhysPolB.46.1037 (in English).

[17] Slobodrian RJ (1962) Scattering of 27.7 MeV deuterons on beryllium and boron, Nuclear Physics, 32:684-694. DOI: 10.1016/0029-5582(62)90370-X (in English).

[18] Daehnick WW et al (1980) Global optical model potential for elastic deuteron scattering from 12 to 90 MeV, Phys Rev C, 21:2253. DOI: 10.1103/PhysRevC.21.2253 (in English).

[19] Thompson IJ (1988) Coupled reaction channels calculations in nuclear physics, Comput Phys Rep, 7:167-212. DOI: 10.1016/0167-7977(88)90005-6 (in English).

[20] Avrigeanu M, Oertzen W, Fischer U, Avrigeanu V (2005) Analysis of deuteron elastic scattering on ^{6,7}Li up to 50 MeV, Nucl Phys A, 759:327. DOI: 10.1016/j.nuclphysa.2005.05.153 (in English).

[21] Yinlu H, Qingbiao S (1997) Analysis and Prediction of the Cross Sections of d + ^{6}Li and d + ^{7}Be Reactions for Energy up to 30 MeV, INDC (CPR), 042/L:27. (in English).

Маулен Насурлла [1,2], Н. Буртебаев [1,2], Ж. К. Керимкулов [1], Т. Сузуки [3], С.Б. Сакута [4], Маржан Насурлла [1,2], Р. Ходжаев [1,2]

[1]Ядролық физика институты, Алматы, Қазахстан;
[2] аль-Фараби атындағы КазҰУ, Алматы, Қазахстан;
[3] Сайтама Университеті, Сайтама, Жапония;
[4]ҰЗО «Курчатов институты», Москва, Рессей

ЭНЕРГИСЫ 14.5 МэВ ДЕЙТРОНДАРДЫҢ ^{7}Li ЯДРОЛАРЫНАН ШАШЫРАУЫН ЗЕРТТЕУ

Аннотация. Литий изотоптарының айқын кластерлік құрылымы - әр түрлі теориялық ядролық модельдерді тексеру үшін өте жақсы сынақ болып табылады. Тікелей ядролық реакциялардағы кластерлік алмасу механизмін зерделеу осы ядролардың құрылымын анықтау үшін жаңа мүмкіндіктер ашады. Әдебиеттегі оптикалық потенциалдар параметрлерінің жиынтығы әртүрлі, бұл зерттеліп жатқан ядролардың кластерлік спектроскопиялық факторларын анықтаған кезде бірмәнділіктің болмауына әкелуі мүмкін. Сондықтан, d + ^{7}Li жүйесі үшін оптикалық потенциалдардың тәуелсіз глобалдық жүйелігін алу үшін E = 14.5 МэВ энергиясында шашырау процесіне эксперименттік зерттеу жүргізілді. Жұмыс барысында E = 14.5 МэВ энергиясы жағдайында ^{7}Li ядроларында дейтрондардың серпімді және серпімсіз шашырауы бойынша жаңа эксперименттік деректер алынды. Сонымен қатар біздің сараптамадада дейтрондардың 7 - 28 МэВ аралығында серппімді шашырауының эксперименталдық мәндері қамтылды. Ядроның оптикалық моделінің шеңберінде энергиялардың кең ауқымында серпімді шашыраудың дифференциалдық қималарына талдау жүргізілді. Нәтижесінде, зерттелген ядро үшін оптикалық потенциалдардың оңтайлы параметрлері анықталды. Осы жұмыста алынған параметрлер келешекте литий изотоптарының құрылымдық сипаттамаларын нақтылауға қажетті дейтрондардың серпімсіз шашырауы және ^{7}Li (d, t) реакцияларының бойынша деректерді талдау кезінде пайдаланылады.

Түйін сөздер: литий ядросы, дифференциалдық қима, серпімді және серпімсіз шашыраулар, оптикалық потенциал.

УДК 524.1; 539.14/.17

Маулен Насурлла [1,2], Н. Буртебаев [1,2], Ж. К. Керимкулов [1], Т. Сузуки [3], С. Б. Сакута [4], Маржан Насурлла [1,2], Р. Ходжаев [1,2]

[1]Институт ядерной физики, Алматы, Казахстан;
[2]КазНУ им. аль-Фараби, Алматы, Казахстан;
[3]Университет Сайтама, Сайтама, Япония;
[4]Национальный Исследовательский Центр «Курчатовский институт», Москва, Россия

ИССЛЕДОВАНИЕ РАССЕЯНИЯ ДЕЙТРОНОВ НА ЯДРАХ ^{7}Li ПРИ ЭНЕРГИИ 14.5 МэВ

Аннотация. Выраженная кластерная структура изотопов лития является отличным тестом для проверки различных теоретических ядерных моделей. Изучение механизма обмена кластерами в прямых ядерных реакциях открывает новые возможности определения структур этих ядер. Имеющиеся в литературе наборы параметров оптических потенциалов различны, что может привести к неоднозначности при определении кластерных спектроскопических факторов исследуемых ядер. Поэтому для получения независимой глобальной систематики оптических потенциалов для системы d + ^{7}Li проведено экспериментальное исследование процесса рассеяния при энергии E = 14.5 МэВ. В работе получены новые экспериментальные данные по упругому и неупругому рассеянию дейтронов на ядрах ^{7}Li при энергии E = 14.5 МэВ. В добавление к этому, в нашем анализе были использованы экспериментальные данные по упругому рассеяния, измеренные ранее при энергиях дейтронов от 7 до 28 МэВ. Анализ дифференциальных сечений проводился в рамках оптической модели. Установлены оптимальные параметры оптических потенциалов для исследуемого ядра. Полученные в данной работе параметры в дальнейшем будут использованы при анализе данных по неупругому рассеянию дейтронов и реакций ^{7}Li (d, t) для уточнения структурных характеристик изотопов лития.

Ключевые слова: ядра лития, дифференциальное сечение, упругое и неупругое рассеяние, оптический потенциал.

Information about authors:

Nassurlla Maulen - al-Farabi Kazakh National University, Almaty, Kazakhstan, PhD student, nespad@mail.ru, experimental data processing and analyzing of data;

Burtebayev Nassurlla - Institute of Nuclear Physics, Almaty, Kazakhstan, Deputy General Director for Science and Education, Doctor of physical and mathematical sciences, Professor of Physics, nburtebayev@yandex.ru, setting and planning tasks;

Kerimkulov Zhambul Kuanyshbekovich - Institute of Nuclear Physics, Almaty, Kazakhstan, Senior Researcher, candidate of physical and mathematical sciences, zhambul-k@yandex.ru, experimental data processing and analyzing of data;

Suzuki Takechi - Saitama University, Saitama, Japan, Professor of Physics, suzuki@siva.ne.phy.saitama-u.ac.jp, setting and planning tasks;

Sakuta Stanislav Borisovich - National Research Center “Kurchatov Institute”, Moscow, Russia, Doctor of physical and mathematical sciences, professor, sbsakuta@mail.ru, setting and planning tasks;

Marzhan Nassurlla - Institute of Nuclear Physics, Almaty, Kazakhstan, Senior Researcher, PhD, morzhic@mail.ru, analysis of experimental data;

Khojayev Romazan - Institute of Nuclear Physics, Almaty, Kazakhstan, engineer, ramazan_inp@mail.ru, obtaining experimental data.

N E W S
OF THE NATIONAL ACADEMY OF SCIENCES OF THE REPUBLIC OF KAZAKHSTAN
PHYSICO-MATHEMATICAL SERIES
ISSN 1991-346X **https://doi.org/10.32014/2018.2518-1726.13**
Volume 6, Number 322 (2018), 23 – 27

UDC 515.1; 004.932

N.G.Makarenko[1], ChoYong-beom[2], A.B.Yessenaliyeva[1]

[1]Institute of information and computational technologies, Kazakhstan, Almaty
[2]Konkuk University, South Korea, Seoul
a.esenalieva@mail.ru

RIEMANNIAN METRIC FOR TEXTURE RECOGNITION

Abstract. The article discusses the recognition of textures on digital images by methods of computational topology and Riemannian geometry. Topological properties of patterns are represented by segments (barcodes) obtained by filtering by the level of photometric measure. Beginning of barcode encodes level at which topological property appears (connected component and/or "hole"), and its end - level at which the property disappears. Barcodes are conveniently parameterized by coordinates of their ends in rectangular coordinate system "birth" and "death" of topological property. Such representation in form of a cloud of points on plane is called a persistence diagram (PD). In the article show that texture class recognition results are significantly better compared to other vectorization methods of PD.

Keywords: Riemannian metric, persistence diagram, probability density function, persistent image (PI).

To describe the patterns of digital images, we use TDA - Topological Data Analysis [1,2]. TDA does not require any a priori assumptions about nature of data source and allows to extract new knowledge from changingshape of neighborhoods of points in space of features.

The approach is associated with persistent images [3], using Riemannian metric to calculate distances between persistence diagrams (PD) is based on analogy, which originates in quantum mechanics (Figure 1).

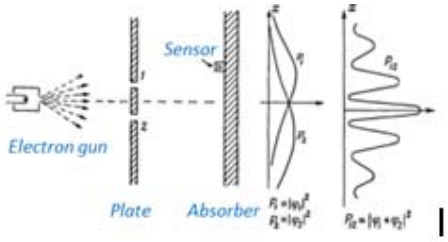


Figure 1 - Experience with two slots[4]

Electronsfromgunpass through plate with two slots.Incasewithoutabsorber, wave interference is described by joint distribution P_{12}. Presence of detector leads to double-humped distribution $P_1 + P_2 \neq P_{12}$.

In quantum mechanics introduces a probability amplitude $P_i = |\varphi_i|^2$ .Assuming that amplitudes from two slits add up, we obtain:

$$P_{12} = |\varphi_1 + \varphi_2|^2 = |\varphi_1|^2 + |\varphi_2|^2 + 2|\varphi_1\varphi_2|^2 = P_1 + P_2 + 2|\varphi_1\varphi_2|^2 \quad (1)$$

Probability density defined on persistence diagrams does not form vector space. But if we introduce additive probability amplitudes, then we can transfer them to Hilbert unit sphere. The distance on such

sphere does not depend on the choice of beginning of coordinates and number of points in compared diagrams.

The approach that realizes this idea is based on positive definite multiscalekernel [5,6].It is reliedon the vector representation of persistence diagram in the form ofpersistent image (PI).Since each PD consists of set of points in 2D, we start by creating a two-dimensional probability density function (pdf) using Gaussian kernel with zero mean and variance σ^2.For each probability density function, we calculate the representation in the form of square root $\phi(x) = \sqrt{pdf}$. In this case, persistence diagram, as element of geodesic γ between the compared diagrams X and Y, can be written as:

$$\gamma(s) = (1-s)x + s\phi(x), \tag{2}$$

where x - point on diagram X, $\phi(x)$ is corresponding point of diagram Y, and $s \in [0,1]$ parametrizes the geodesic.

Without loss of generality we assume that all probability density functions lie in $[0,1]^2$. The analyzed space of probability density functions is:

$$\mathrm{P} = \{p : [0,1]\times[0,1] \to \Re\} \ \forall x, y \mid p(x,y) \geq 0, \int_0^1 \int_0^1 p(x,y)dxdy = 1\} \tag{3}$$

Transition from pdf to probability amplitudes is closely related to so-called Fisher-Rao information metric.For discrete probability space Fisher metric can be considered simply as Euclidean metric bounded by positive "quadrant" of unit sphere after corresponding change of variables.Consider Euclidean space $y = (y_0, \ldots, y_n) \in R^{N+1}$. The metric will be defined by quadratic form:

$$h = \sum_{i=0}^{N} dy_i dy_i \ , \tag{4}$$

where dy_i is 1-forms, which form basis in co-tangent space.

Denote by $\frac{\partial}{\partial y_j}$ basis vectors in the tangent space, so that:

$$dy_j\left(\frac{\partial}{\partial y_k}\right) = \delta_{jk} \, . \tag{5}$$

Define N -dimensional unit sphere embedded in the $(N+1)$-dimensional Euclidean space as:

$$\sum_{i=0}^{N} y_i^2 = 1 \tag{7}$$

This embedding induces metric on sphere, which follows directly from Euclidean metric of surrounding space.Introducevariable change $p_i = y_i^2$.

The equation of sphere then takes the form of a condition of the probability normalization:

$$\sum_i p_i \ = 1 \, , \tag{8}$$

and metric becomes:

$$h = \sum_i dy_i \, dy_i \ = \sum_i d\sqrt{p_i} d\sqrt{p_i} \ = \frac{1}{4}\sum_i \frac{dp_i dp_i}{p_i} = \frac{1}{4}\sum_i p_i d(\log p_i) d(\log p_i) \, . \tag{9}$$

The last expression represents a quarter of Fisher's information metric[7]. Probabilities are parametric functions of the manifold of variables θ, thus $p_i = p_i(\theta)$. Then we obtain the induced metric on parametric manifold:

$$h = \frac{1}{4}\sum_i p_i d(\log p_i(\theta)) d(\log p_i(\theta)) = \frac{1}{4}\sum_{jk}\sum_i p_i(\theta)\frac{\partial \log p_i(\theta)}{\partial \theta_j}\frac{\partial \log p_i(\theta)}{\partial \theta_k} d\theta_j d\theta_k, \quad (10)$$

or in coordinate form Fisher's information metric is determined by the tensor:

$$g_{ik}(\theta) = 4h_{jk}^{fisher} = 4h\left(\frac{\partial}{\partial \theta_j}, \frac{\partial}{\partial \theta_k}\right). \quad (11)$$

Geodesic in Fisher metric is difficult to compute. Therefore, we will use representation proposed in the paper [8]. It strongly simplifies subsequent calculations. Instead of probabilities, we will consider the space:

$$\Psi = \{\psi : [0,1]\times[0,1] \to \Re\} \mid \psi \geq 0, \text{и} \int_0^1 \int_0^1 \psi^2(x,y)dxdy = 1\} \quad (12)$$

For any two tangent vectors $v_1, v_2 \in T_\psi(\Psi)$, Fisher-Rao metric is defined asscalar product in Hilbert space:

$$\langle \upsilon_1, \upsilon_2 \rangle = \int_0^1 \int_0^1 \upsilon_1(x,y)\upsilon_2(x,y)dxdy. \quad (13)$$

It implies that representation in the form of square root $\psi = \sqrt{p}$ makes space a unit Hilbert sphere with a given metric in the form ofscalar product.For two points ψ_1, ψ_2 in such space, geodetic distance between them is defined as:

$$d_H(\psi_1, \psi_2) = \cos^{-1}\left(\langle \psi_1, \psi_2 \rangle\right), \quad (14)$$

where in calculating the scalar product of two points ψ_1, ψ_2 we normalize scalar product using standard Frobenius norm. Computational complexity for such distances between persistence diagrams increasesas $O(K^2)$, for $K \times K$ discretization on $[0,1]^2$. Increasing of K leads to increasing ofaccuracy of determination of distances, but increases computational complexity.

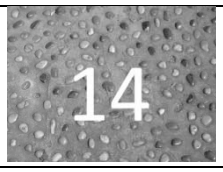
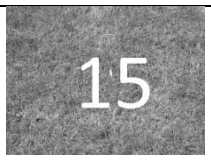
Numerical results. For the experiment, we chose the value of resolutionparameter of persistent images $K = 200$. Fromstandard image database [9] four texture classes were selected, two of which contain vegetation images and two - images of inanimate nature.Each class contains 40 images.

In experiments,we calculated the average value and variance of Riemannian distances of PI both within each class and pairwise for all pairs of texture classes.When calculating the distance between two classes, Riemannian distances between all possible pairs of PI textures of two compared classes are computed.

The results are shown in the table.Thus, firstly, all distances between PI of textures within one class are computed.Then, all pairwise distances between PI of textures of different classes are calculated. The average distances between the corresponding PI for Betti 0 and Betti 1 within the class are less than corresponding distances to PI from another class.

However, variances within classes are quite high. Therefore, there may occur cases when Riemannian distance between two arbitrarily taken textures from different classes may be less than the average distance inside the class.For practical use, it is usually necessary to determine the aboutness of not two separately taken textures, but belonging of the considered sample to certain class of textures: grass, stones, water, etc.In such a case, it is appropriate for classes building ofaveraged topological features.Average distances and variance within each of the classes are shown in the table.

Table-Mean and variance of Riemannian distance between all pairs of persistent images (PIs) of textures of 4 classes

				
	$<\beta_0>=10.1$ $\sigma(\beta_0)= 4.4$ $<\beta_1>=8.9$ $\sigma(\beta_1)= 4.5$	$<\beta_0>=11.7$ $\sigma(\beta_0)= 3.9$ $<\beta_1>=12.9$ $\sigma(\beta_1)= 2.9$	$<\beta_0>=29.0$ $\sigma(\beta_0)= 4.4$ $<\beta_1>=23.3$ $\sigma(\beta_1)= 5.3$	$<\beta_0>=25.3$ $\sigma(\beta_0)= 5.2$ $<\beta_1>=13.9$ $\sigma(\beta_1)= 3.8$
		$<\beta_0>=11.7$ $\sigma(\beta_0)= 4.7$ $<\beta_1>=7.8$ $\sigma(\beta_1)= 3.1$	$<\beta_0>=32.8$ $\sigma(\beta_0)= 2.9$ $<\beta_1>=26.0$ $\sigma(\beta_1)= 3.0$	$<\beta_0>=29.3$ $\sigma(\beta_0)= 4.8$ $<\beta_1>=19.0$ $\sigma(\beta_1)= 2.3$
			$<\beta_0>=19.1$ $\sigma(\beta_0)= 11.7$ $<\beta_1>=13.9$ $\sigma(\beta_1)= 6.6$	$<\beta_0>=38.3$ $\sigma(\beta_0)= 4.7$ $<\beta_1>=22.6$ $\sigma(\beta_1)= 4.8$
				$<\beta_0>=23.6$ $\sigma(\beta_0)= 15.1$ $<\beta_1>=16.1$ $\sigma(\beta_1)= 7.7$

In the lines and columns there are 4 classes of textures. The numbers indicate the texture class number. Diagonal elements correspond to distances between PIs of textures inside the class.Off-diagonal elements correspond to pairwise distances between PIs of textures of the two correspondingtoline and columnofclasses. Mean value and variance are calculated separately for Riemannian distances of PI of Betti 0 and Betti 1.

Conclusion. Obtained results show that the described approach allows, bypassing large computational complexities, to classify reliably the textures even without the use of machine learning methods.

The work has been done due to support of grant №AP05134227 of MES of RK.

REFERENCES

[1] Edelsbrunner H., Harer J., *Computational Topology, An Introduction*, Amer. Math. Soc. 2009. 241 p.

[2] Ghrist R. W. Elementary applied topology. Seattle : Createspace, 2014

[3] M.N. Kalimoldayev, N.G. Makarenko, I.T. Pak, A.B. Yessenaliyeva Texture recognition by the methods of topological data analysis // Open Engineering. №6. 2016. P.326-334.

[4] Фейнман Р., Лейтон Р., Сэндс М. Фейнмановские лекции по физике. М.:Мир, 1976. - С. 207.

[5]Anirudh R. et al., A Riemannian Framework for Statistical Analysis of Topological Persistence Diagrams, ArXiv e-prints, 1605.08912, 2016: 09.2016.

[6] H. Adams, S. Chepushtanova, T. Emerson, et al. A stable vector representation of persistent homology // arXiv preprint arXiv:1507.06217: 09.2016.

[7] Amari S., Nagaoka H. Methods of information geometry. - American Mathematical Society, 2000. Vol.191. 13 p.

[8] A. Srivastava, I. Jermyn, and S. Joshi. Riemannian analysis of probability density functions with applications in vision // IEEE Conference on Computer Vision and Pattern Recognition. 2007. P. 1–8.

[9]Data base of textures:http://www.cfar.umd.edu/~fer/website-texture/texture.htm

[10] Askarova AS, Bolegenova SA, etc (2017) Investigation of aerodynamics and heat and mass transfer in the combustion chambers of the boilers PK-39 and BKZ-160, News of the National Academy of Sciences of the Republic of Kazakhstan-Series physico-mathematical, 2:27-38.DOI: https://doi.org/10.32014/2018.2518-1726.

УДК 515.1; 004.932

Н.Г. Макаренко[1], ЧоЙонг-беом[2], А.Б.Есеналиева[1]

[1]Институт информационных и вычислительных технологий КН МОН РК;
[2]Университет Конкук, Южная Корея, Сеул

РИМАНОВА МЕТРИКА ДЛЯ РАСПОЗНАВАНИЯ ТЕКСТУР

Аннотация. В статье обсуждается распознавание текстур на цифровых изображениях методами вычислительной топологии и римановой геометрии. Топологические свойства паттернов представлены отрезками (баркодами), полученными при фильтрации по уровню фотометрической меры. Начало баркода кодирует уровень на котором появляется топологическое свойство (компонента связности и/или «дыра»), а его конец – уровень на котором свойство исчезает. Баркоды удобно параметризовать координатами их концов в прямоугольной системе координат «рождение» и «смерть» топологического свойства. Такое представление в форме облака точек на плоскости, называют диаграммой персистентности (ДП). В статье показано, что результаты распознавания классов текстур существенно лучше, по сравнению с другими способами векторизации ДП.

Ключевые слова: Риманова метрика, диаграмма персистентности, функция плотности вероятности, персистентное изображение (ПИ).

УДК 515.1; 004.932

Н.Г. Макаренко[1],ЧоЙонг-беом[2], А.Б.Есеналиева[1]

[1]Ақпараттық және есептеуіш технологиялар институты;
[2]Конкук Университеті, Оңтүстік Корея, Сеул

ТЕКСТУРАЛАРДЫ ТАНУ ҮШІН РИМАНМЕТРИКАСЫ

Аннотация. Мақалада сандық бейнелердегі текстураларды есептеу топология және Риман геометриясы әдістерімен тану талқыланады. Паттерлердің топологиялық қасиеттері фотометриялық өлшем деңгейі бойынша сүзу кезінде алынған кесінділермен (баркодтармен) берілген.Баркодтың басы топологиялық сипат (байланыс компоненті және/немесе "тесік") пайда болатын деңгейді, ал оның соңы – сипат жоғалатын деңгейді кодтайды. Баркодтарды топологиялық қасиеттің "туу" және "өлім" координаттарының тікбұрышты жүйесіндегі олардың ұштарының координаттарын параметрлеуге ыңғайлы. Жазықтықтағы нүктелердің бұлт түріндегі мұндай көрініс персистенттік диаграмма (ПД) деп аталады. Мақалада басқа ДП векторизация әдістерімен салыстырғанда, текстураның сыныптарын тану нәтижелері айтарлықтай жақсы екендігі көрсетілген.

Түйін сөздер: Риман метрикасы, персистенттік диаграммасы, ықтималдықтығыздығы функциясы, персистенттібейнелер (ПБ).

Information about authors:

Makarenko N. D. – doctor of technical Sciences, chief researcher of the Institute of information and computing technologies;

Cho Yong-Beom – professor, PhD, Konkuk University, Korea, Seoul;

Esenaliev A. B. – PhD students, MSC Institute of information and computing technology, MES RK,

N E W S
OF THE NATIONAL ACADEMY OF SCIENCES OF THE REPUBLIC OF KAZAKHSTAN
PHYSICO-MATHEMATICAL SERIES
ISSN 1991-346X **https://doi.org/10.32014/2018.2518-1726.14**
Volume 6, Number 322 (2018), 28 – 36

UDC 517.948.34

M.K. Dauylbayev[1,4], N. Atakhan[2,4], A.E. Mirzakulova[3]

[1]al-Farabi Kazakh national university, Almaty, Kazakhstan;
[2]Kazakh state women's teacher training university, Almaty, Kazakhstan;
[3]Abay Kazakh national pedagogical university, Almaty, Kazakhstan;
[4]Institute of Information and Computational technologies, Almaty, Kazakstan
E-mail:atakhan-nilupar@mail.ru

ASYMPTOTIC EXPANSION OF SOLUTION OF GENERAL BVP WITH INITIAL JUMPS FOR HIGHER-ORDER SINGULARLY PERTURBED INTEGRO-DIFFERENTIAL EQUATION

Abstract. In this article we constructed an asymptotic expansion of the solution undivided boundary value problem for singularly perturbed integro-differential equations with an initial jump phenomenon m – th order. We obtain the theorem about estimation of the remainder term's asymptotic with any degree of accuracy in the smallparameter.

Key words: singular perturbation, the integro-differential equation, a small parameter, asymptotic expansion, the initial jump, the boundary layer.

Introduction

Singularly perturbed equations act as mathematical models in many applied problems related to diffusion, heat and mass transfer, chemical kinetics and combustion, heat propagation in thin bodies, semiconductor theory, gyroscope motion, quantum mechanics, biology and biophysics and many other branches of science and technology. In this paper we consider general undivided boundary-value problem for singularly perturbed linear integro-differential equations of n-th order, when the boundary conditions are not ordered with respect to the highest derivatives. At first the characteristic features of the problem under consideration are that the limiting unperturbed problem degenerates incompletely, i.e. the loss of boundary conditions imposed on the initial perturbed problem does not occur and secondly, the solution of the singularly perturbed problem as the small parameter tends to zero tends to the solution of the unperturbed equation with changed boundary conditions. The values of the initial jumps of the solution and of the integral terms are determined. A uniform asymptotic expansion of the solutions of the original singularly perturbed integro-differential boundary value problem with any degree of accuracy with respect to the small parameter is constructed. The solution of the above problems made it possible to extend the class of singularly perturbed integro-differential equations possessing the phenomena of initial jumps. The scientific novelty of the presented work is that the presence of integrals qualitatively changes the asymptotic representation of the solution of the corresponding integro-differential equations.

Note that other mathematical school of singularly perturbed equations in Kazakhstan and abroad investigate only boundary value problems, which does not have an initial jump. In our previous works in [1-10], we considered the initial and boundary value problems that are equivalent to the Cauchy problem with the initial jump for differential and integro-differential equations in the stable case.

Consider the following singularly perturbed integro-differential equation

$$L_\varepsilon y \equiv \varepsilon y^{(n)} + A_1(t)y^{(n-1)} + ... + A_n(t)y = F(t) + \int_0^1 \sum_{i=0}^{m+1} H_i(t,x)y^{(i)}(x,\varepsilon)dx \qquad (1)$$

with nonlocal boundary conditions

$$h_i y(t,\varepsilon) \equiv \sum_{j=0}^{m} \alpha_{ij} y^{(j)}(0,\varepsilon) + \sum_{j=0}^{l} \beta_{ij} y^{(j)}(1,\varepsilon) = a_i, \quad i = \overline{1,n}\ ,\ m < n-1,\ l < n-1, \quad (2)$$

where $\varepsilon > 0$ is a small parameter, α_{ij}, β_{ij}, $a_i \in R$ are known constants independent of ε and $\alpha_{im} \neq 0, i = \overline{1,n}.$

Assume that the following conditions hold:

(C1) Functions $A_i(t), F(t), i = \overline{1,n}$ are sufficiently smooth and defined on the interval $[0,1]$.

(C2) $A_1(t) \geq \gamma = const > 0,\ 0 \leq t \leq 1$.

(C3) Functions $H_i(t,x), i = \overline{0,m+1}$ are defined in the domain $D = \{0 \leq t \leq 1, 0 \leq x \leq 1\}$ and sufficiently smooth.

$$\text{(C4)}\ \overline{\Delta} = \begin{vmatrix} h_1 y_{10}(t) & ... & h_1 y_{n-1,0}(t) & \alpha_{1m} \\ ... & ... & ... & ... \\ h_n y_{10}(t) & ... & h_n y_{n-1,0}(t) & \alpha_{nm} \end{vmatrix} \neq 0\ ,$$

where $y_{i0}(t),\quad i = \overline{1,n-1}$ are the fundamental set of solutions of the following homogeneous differential equation

$$L_0 y(t) \equiv A_1(t)y^{(n-1)}(t) + ... + A_n(t)y\ (t) = 0.$$

(C5) $\lambda = 1$ is not an eigenvalue of the kernel $H(t,s,\varepsilon)$.

$$\text{(C6)}\ \overline{\omega} = \begin{vmatrix} 1+\overline{d}_{11} & \overline{d}_{12} & ... & \overline{d}_{1n} \\ \overline{d}_{21} & 1+\overline{d}_{22} & ... & \overline{d}_{2n} \\ ... & ... & ... & ... \\ \overline{d}_{n1} & \overline{d}_{n2} & ... & 1+\overline{d}_{nn} \end{vmatrix} \neq 0.$$

(C7) Number 1 is not an eigenvalue of the kernel $\overline{H}(t,s)$.

For the solution of the problem (1),(2) are valid the following limiting equalities:

$$\lim_{\varepsilon \to 0} y^{(j)}(t,\varepsilon) = \overline{y}^{(j)}(t),\ j = \overline{0,m-1},\ 0 \leq t \leq 1, \qquad (3)$$

$$\lim_{\varepsilon\to 0} y^{(m+j)}(t,\varepsilon) = \bar{y}^{(m+j)}(t),\ j=\overline{0,n-1-m},\ 0<t\le 1,$$

where $\bar{y}(t)$ is the solution of the degenerate problem, Δ_0 is the initial jump of the solution,

$$L_0\bar{y} \equiv A_1(t)\bar{y}^{(n-1)}(t) + \sum_{i=2}^{n} A_i(t)\bar{y}^{(n-i)}(t) = F(t) + \int_0^1 \sum_{i=0}^{m+1} H_i(t,x)\bar{y}^{(i)}(x)dx + \Delta_0 H_{m+1}(t,0),$$

$$h_i\bar{y}(t) \equiv \sum_{j=0}^{m} \alpha_{ij}\bar{y}^{(j)}(0) + \sum_{j=0}^{l} \beta_{ij}\bar{y}^{(j)}(1) = a_i - \alpha_{im}\Delta_0, \quad i=\overline{1,n}. \tag{4}$$

From (3) it follows that the solution $y(t,\varepsilon)$ of the general boundary value problem (1) and (2) converges to the solution $\bar{y}(t)$ of the modified degenerate problem (4) as $\varepsilon\to 0$. We note that the limits for $y^{(m+j)}(t,\varepsilon), j=\overline{0,n-1-m}$ are not uniform on the interval $0\le t\le 1$. They are uniform on the interval $0<t_0\le t\le 1$, where t_0 is sufficiently small but fixed number as $\varepsilon\to 0$. In the work will be constructed uniformly asymptotic expansion of the solution of the problem (1),(2) on the interval $0\le t\le 1$.

Since the solution of the problem (1) and (2) has the $m-$th order initial jump at the point $t=0$, we seek the asymptotic expansion of the solution of the problem (1), (2) in the next form:

$$y(t,\varepsilon) = y_\varepsilon(t) + \varepsilon^m w_\varepsilon(\tau),\quad \tau=\frac{t}{\varepsilon}, \tag{5}$$

where $y_\varepsilon(t)$ is a regular part of the asymptotic and $w_\varepsilon(\tau)$ is a boundary layer part, those can be represented in the form:

$$y_\varepsilon(t) = \sum_{i=0}^{\infty}\varepsilon^i y_i(t),\quad w_\varepsilon(\tau) = \sum_{i=0}^{\infty}\varepsilon^i w_i(\tau). \tag{6}$$

Substituting the series (5) into (1), we obtain the following equalities:

$$\begin{aligned}\varepsilon\left(y_\varepsilon^{(n)}(t) + \varepsilon^{m-n}\overset{(n)}{w}_\varepsilon(\tau)\right) + \sum_{i=1}^{n} A_i(t)\left(y_\varepsilon^{(i)}(t) + \varepsilon^{m-n+i}\overset{(i)}{w}_\varepsilon(\tau)\right) = \\ = F(t) + \int_0^1 \sum_{i=0}^{m+1} H_i(t,x)\left(y_\varepsilon^{(i)}(x) + \varepsilon^{m-i}\overset{(i)}{w}_\varepsilon\left(\frac{x}{\varepsilon}\right)\right)dx.\end{aligned} \tag{7}$$

By replacing the integral expression $s=\frac{x}{\varepsilon}$ on the right-hand side of the equation (7), we get the improper integral

$$\begin{aligned}J(t,\varepsilon) = \int_0^{\frac{1}{\varepsilon}} \sum_{i=0}^{m+1}\varepsilon^{m+1-i}H_i(t,\varepsilon s)\overset{(i)}{w}_\varepsilon(s)ds = \int_0^{\infty}\sum_{i=0}^{m+1}\varepsilon^{m+1-i}H_i(t,\varepsilon s)\overset{(i)}{w}_\varepsilon(s)ds - \\ - \int_{\frac{1}{\varepsilon}}^{\infty}\sum_{i=0}^{m+1}\varepsilon^{m+1-i}H_i(t,\varepsilon s)\overset{(i)}{w}_\varepsilon(s)ds.\end{aligned} \tag{8}$$

The improper integral in (8) converges and the second sum in (8) is vanished, because $O\left(\exp\left(-\gamma\frac{t}{\varepsilon}\right)\right)$ is less than any power of ε, as $\varepsilon \to 0$.

We write separately the coefficients depending on t and on τ we obtain the following equalities for $y_\varepsilon(t)$ and $w_\varepsilon(\tau)$:

$$\varepsilon y_\varepsilon^{(n)}(t)+\sum_{i=1}^{n}A_i(t)y_\varepsilon^{(n-i)}(t)=F(t)+\int_0^1\sum_{i=0}^{m+1}H_i(t,x)y_\varepsilon^{(i)}(x)dx+\int_0^\infty\sum_{i=0}^{m+1}\varepsilon^{m+1-i}H_i(t,\varepsilon s)\overset{(i)}{w}_\varepsilon(s)ds, \tag{9}$$

$$\overset{(n)}{w}_\varepsilon(\tau)+A_1(\varepsilon\tau)\overset{(n-1)}{w}_\varepsilon(\tau)+\varepsilon A_2(\varepsilon\tau)\overset{(n-2)}{w}_\varepsilon(\tau)+\ldots+\varepsilon^{n-1}A_n(\varepsilon\tau)w_\varepsilon(\tau)=0. \tag{10}$$

By the degree of ε formally expanding $H_i(t,\varepsilon s), i=\overline{0,m+1}$ into a Taylor series at the point $(t,0)$:

$$H_i(t,\varepsilon s)=H_i(t,0)+\varepsilon sH_i'(t,0)+\frac{(\varepsilon s)^2}{2!}H_i''(t,0)+\ldots+\frac{(\varepsilon s)^k}{k!}H_i^{(k)}(t,0)+\ldots\ \ i=\overline{0,m+1} \tag{11}$$

Use (11) in (9), equating coefficients of like powers of ε, for the regular part $y_k(t), k=0,1,2,\ldots.$ we arrive the following equalities:

$$A_1y_0^{(n-1)}(t)+\sum_{k=2}^{n}A_k(t)y_0^{(n-k)}(t)=F(t)+\int_0^1\sum_{i=0}^{m+1}H_i(t,x)y_0^{(i)}(x)dx+\int_0^\infty H_{m+1}(t,0)\overset{(m+1)}{w}_0(s)ds$$

where $\int_0^\infty H_{m+1}(t,0)\overset{(m+1)}{w}_0(s)ds=-H_{m+1}(t,0)\overset{(m)}{w}_0(0)$, denote by

$$\Delta_0(t)=H_{m+1}(t,0)\Delta_0,\ \Delta_0=-\overset{(m)}{w}_0(0). \tag{12_0}$$

for determining the coefficient $y_0(t)$, we obtain the integro-differential equation

$$A_1y_0^{(n-1)}(t)+\sum_{k=2}^{n}A_k(t)y_0^{(n-k)}(t)=F(t)+\int_0^1\sum_{i=0}^{m+1}H_i(t,x)y_0^{(i)}(x)dx+\Delta_0(t), \tag{13_0}$$

where $\Delta_0(t)$ is defined by formula (12_0).

For determining the coefficients $y_k(t), k=1,2,\ldots.$ we obtain the integro-differential equation

$$A_1(t)y_k^{(n-1)}(t)+\sum_{i=2}^{n}A_i(t)y_k^{(n-i)}(t)=F_k(t)+\int_0^1\sum_{i=0}^{m+1}H_i(t,x)y_k^{(i)}(x)dx+\Delta_k(t), \tag{13_k}$$

where

$$\Delta_k(t)=H_{m+1}(t,0)\Delta_k,\ \Delta_k=-\overset{(m)}{w}_k(0) \tag{12_k}$$

and $F_k(t)$ is known function, can be written as

$$F_k(t)=\int_0^{\infty}\sum_{j=1}^{k}\frac{s^j}{j!}H_{m+1}^{(j)}(t,0)\overset{(m+1)}{w}_{k-j}(s)ds+\int_0^{\infty}\sum_{i=1}^{k}\sum_{j=0}^{k-i}\frac{s^j}{j!}H_{m+1-i}^{(j)}(t,0)\overset{(m+1-i)}{w}_{k-i-j}(s)ds-y_{k-1}^{(n)}(t),$$

$$k=\overline{1,m+1} \tag{14}$$

$$F_k(t)=\int_0^{\infty}\sum_{j=1}^{k}\frac{s^j}{j!}H_{m+1}^{(j)}(t,0)\overset{(m+1)}{w}_{k-j}(s)ds+\int_0^{\infty}\sum_{i=1}^{m+1}\sum_{j=0}^{k-i}\frac{s^j}{j!}H_{m+1-i}^{(j)}(t,0)\overset{(m+1-i)}{w}_{k-i-j}(s)ds-y_{k-1}^{(n)}(t),$$

$$k>m+1.$$

The values $\Delta_k(t),\Delta_k, k\geq 0$ are called respectively *theinitial jumps of the integral terms and solutions.*

By the degree of ε formally expanding $A_i(\varepsilon\tau),\ i=\overline{1,n}$ into a Taylor series at the point 0:

$$A_i(\varepsilon\tau)=A_i(0)+\varepsilon\tau A_i'(0)+\frac{(\varepsilon\tau)^2}{2!}A_i''(0)+\ldots+\frac{(\varepsilon\tau)^k}{k!}A_i^{(k)}(0)+\ldots\ \ i=\overline{1,n}. \tag{15}$$

Use (15) in (10), equating coefficients of like power of ε on both sides (10), wev get the equations for the boundary layer functions $w_k(\tau),\quad k=0,1,2,\ldots$

$$\overset{(n)}{w}_0(\tau)+A_1(0)\overset{(n-1)}{w}_0(\tau)=0, \tag{16$_0$}$$

$$\overset{(n)}{w}_k(\tau)+A_1(0)\overset{(n-1)}{w}_k(\tau)=\Phi_k(\tau), \tag{16$_k$}$$

where $\Phi_k(\tau)$ is known function, can be written as

$$\Phi_k(\tau)=\begin{cases}-\sum_{j=1}^{k}\frac{\tau^j}{j!}A_1^{(j)}(0)\overset{(n-1)}{w}_{k-j}(\tau)-\sum_{m=0}^{k-1}\sum_{j=0}^{m}\frac{\tau^j}{j!}A_{k+1-m}^{(j)}(0)\overset{(n-1+m-k)}{w}_{m-j}(\tau),\ k=\overline{1,n-1},\\ -\sum_{j=1}^{k}\frac{\tau^j}{j!}A_1^{(j)}(0)\overset{(n-1)}{w}_{k-j}(\tau)-\sum_{m=k+1-n}^{k-1}\sum_{j=0}^{m}\frac{\tau^j}{j!}A_{k+1-m}^{(j)}(0)\overset{(n-1+m-k)}{w}_{m-j}(\tau),\quad k\geq n\end{cases} \tag{17}$$

To determineuniquely the terms $y_k(t)$ and $w_k(\tau)$ of the asymptotic, we use (5) in (6) and taking into account boundary condition (2)

$$\sum_{j=0}^{m}\alpha_{ij}[y_0^{(j)}(0)+\varepsilon y_1^{(j)}(0)+\ldots+\varepsilon^{m-j}(\overset{(j)}{w}_0(0)+\varepsilon\overset{(j)}{w}_1(0)+\ldots)]+$$

$$+\sum_{j=0}^{l}\beta_{ij}\left[y_0^{(j)}(1)+\varepsilon y_1^{(j)}(1)+\ldots+\varepsilon^{m-j}\left(\overset{(j)}{w}_0\left(\frac{1}{\varepsilon}\right)+\varepsilon\overset{(j)}{w}_1\left(\frac{1}{\varepsilon}\right)+\ldots\right)\right]=a_i\ ,\ i=\overline{1,n}\ . \tag{18}$$

In (18) $\overset{(j)}{w_k}\left(\frac{1}{\varepsilon}\right), k=0,1,...$ it is not take into account, it can not be compared than any degree of ε.

Equating the coefficients at zero degrees of ε in (18) and in view of (12_0), we have

$$h_i y_0(t) = a_i + \alpha_{im}\Delta_0, \qquad i=\overline{1,n}. \tag{19$_0$}$$

Thus, the main coefficient $y_0(t)$ of the regular part of the asymptotic and the initial jump of the solution Δ_0 are determined from the problem (13$_0$), (19$_0$).

For determining the coefficient $w_0(\tau)$, we have the initial condition $\Delta_0 = -\overset{(m)}{w_0}(0)$ from (13$_0$), (19$_0$). Finding the missed initial condition for coefficient $w_0(\tau)$ we reduce the order of theequation (16$_0$) byintergrating from τ to ∞ and by virtue of the conditions $\overset{(i)}{w_0}(\infty)=0, i=\overline{0,n-1}$. As a result, after $n-1-m$ -th step, we obtain equation $\overset{(m+1)}{w_0}(\tau)+A_1(0)\overset{(m)}{w_0}(\tau)=0$. From this equation as $\tau=0$, we determine the initial condition $\overset{(m+1)}{w_0}(0)=-A_1(0)\overset{(m)}{w_0}(0)$.Continuing this process lowering the degree of equation (16$_0$), we obtain the following initial conditions for $w_0(\tau)$:

$$\overset{(i)}{w_0}(0) = (-1)^{m+1-i}\frac{\Delta_0}{A_1(0))^{m-i}}, \quad i=\overline{0,n-1}. \tag{20$_0$}$$

Thus, the main coefficient $w_0(\tau)$ of the boundary layer part of the asymptotic is determined from the problem (16$_0$), (20$_0$).

Thus, the zeroth approximation of the asymptotic expansion is completely constructed.

In the k -th approximation, for determining the boundary conditions of the coefficient $y_k(t)$, $k=1,2,...$, we compare the coefficients of the same powersof the parameter ε. As a result, we obtain the following initial conditions for $y_k(t)$:

$$h_i y_k(t) = \begin{cases} \alpha_{im}\Delta_k - \sum_{j=1}^{k}\alpha_{i,m-j}\overset{(m-j)}{w_{k-j}}(0), & k=\overline{1,m}, \\ \alpha_{im}\Delta_k - \sum_{j=1}^{m}\alpha_{i,m-j}\overset{(m-j)}{w_{k-j}}(0), & k\geq m+1 \end{cases} \qquad i=\overline{1,n}. \tag{19$_k$}$$

From (13$_k$), (19$_k$) we determine $y_k(t), \Delta_k, k\geq 1$.

Now, we will be determine the initial conditions for the coefficient $w_k(\tau), k\geq 1$. In order to find themissingof the equation (16$_k$) by virtue of the conditions $\overset{(i)}{w_k}(\infty)=0, i=\overline{0,n-1}$. Then, we get the initial conditions for determining $w_k(\tau), k\geq 1$:

$$w_k^{(i)}(0)=\begin{cases}\dfrac{(-1)^{m-i+1}}{A_1^{m-i}(0))}\Delta_k+(-1)^{n-1-i}\displaystyle\int_0^{\infty}\sum_{j=n-1-m}^{n-2-i}\frac{s^j}{j!}(A_1(0))^{j-(n-1-i)}\Phi_k(s)ds, & i=\overline{0,m},\\ (-1)^{i-m+1}A_1^{i-m}(0)\Delta_k+(-1)^{n-i}\displaystyle\int_0^{\infty}\sum_{j=n-1-i}^{n-2-m}\frac{s^j}{j!}(A_1(0))^{j-(n-1-i)}\Phi_k(s)ds, & i\ge m+1\end{cases} \tag{20$_k$}$$

Thus, the k -th approximation of the asymptotic is completely constructed.

Theorem. *Let functions $A_i(t),F(t)\in C^{N+n-m}[a,b],i=\overline{1,n}$ and conditions(C2) - (C7) hold. Then for sufficiently small ε the boundary value problem (1) and (2) has an unique solution on the $0\le t\le 1$ and that is expressed by the formula*

$$y(t,\varepsilon)=\bar{y}_N(t,\varepsilon)+R_N(t,\varepsilon), \tag{21}$$

where $\bar{y}_N(t,\varepsilon)$ is defined by the formula

$$\bar{y}_N(t,\varepsilon)=\sum_{k=0}^{N}\varepsilon^k y_k(t)+\varepsilon^m\sum_{k=0}^{N+n-1-m}\varepsilon^k w_k(\tau),\quad \tau=\frac{t}{\varepsilon}, \tag{22}$$

and for the remainder term the estimates are valid

$$\left|R_N^{(i)}(t,\varepsilon)\right|\le C\varepsilon^{N+1},\quad i=\overline{0,n-1},\qquad 0\le t\le 1. \tag{23}$$

where $C>0$ is a some constant independent of ε .

Proof. We construct the N -th partial sum (22) of the expansion (5),(6).

The function $\bar{y}_N(t,\varepsilon)$ satisfies problem (1), (2) with accuracy of order $O(\varepsilon^{N+1})$, i.e.

$$L_\varepsilon\bar{y}_N(t,\varepsilon)=F(t)+\int_0^1\sum_{i=0}^{m+1}H_i(t,x)\bar{y}_N^{(i)}(x,\varepsilon)dx+O(\varepsilon^{N+1}), \tag{24}$$

$$h_i\bar{y}_N(t,\varepsilon)=a_i+O(\varepsilon^{N+1}),\quad i=1,n$$

Denote by $y(t,\varepsilon)=\bar{y}_N(t,\varepsilon)+R_N(t,\varepsilon)$. Then for the remainder $R_N(t,\varepsilon)$ we obtain the problem as follows

$$L_\varepsilon R_N(t,\varepsilon)=\int_0^1\sum_{i=0}^{m+1}H_i(t,x)R_N^{(i)}(x,\varepsilon)dx+O(\varepsilon^{N+1}), \tag{25}$$

$$h_iR_N(t,\varepsilon)=O(\varepsilon^{N+1}),\quad i=1,n\ .$$

We apply the asymptotic estimation of the solution of the problem (1),(2) to the problem (25). Then we obtain the estimates

$$\left|R_N^{(j)}(t,\varepsilon)\right| \le C\varepsilon^{N+1} + C\varepsilon^{N+1+m-j}\exp\left(-\gamma\frac{t}{\varepsilon}\right), \quad j=\overline{0,n-1}. \tag{26}$$

This means that estimates $R_N^{(m+1)}(t,\varepsilon)=O\left(\varepsilon^N\right),...,R_N^{(n-1)}(t,\varepsilon)=O\left(\varepsilon^{N-n+2+m}\right)$is valid at point $t=0$, i.e. The required estimates do not hold. To obtain the necessary estimates, we consider the equalities

$$y^{(m+1)}(t,\varepsilon)=y_N^{(m+1)}(t,\varepsilon)+R_N^{(m+1)}(t,\varepsilon),\ y^{(m+1)}(t,\varepsilon)=y_{N+1}^{(m+1)}(t,\varepsilon)+R_{N+1}^{(m+1)}(t,\varepsilon) \tag{27}$$

Hence, equating the right-hand sides of (27), we get

$$R_N^{(m+1)}(t,\varepsilon)=y_{N+1}^{(m+1)}(t,\varepsilon)-y_N^{(m+1)}(t,\varepsilon)+R_{N+1}^{(m+1)}(t,\varepsilon), \tag{28}$$

where $\bar{y}_{N+1}^{(m+1)}(t,\varepsilon)-\bar{y}_N^{(m+1)}(t,\varepsilon)=\varepsilon^{N+1}y_{N+1}^{(m+1)}(t)+\varepsilon^{N+n-1-m}\overset{(m+1)}{w}_{N+n-m}(\tau)$ and the remainder term $R_{N+1}^{(m+1)}(t,\varepsilon)$ in (28) satisfies the estimate $\left|R_{N+1}^{(m+1)}(t,\varepsilon)\right| \le C\varepsilon^{N+2}+C\varepsilon^{N+1}\exp\left(-\gamma\frac{t}{\varepsilon}\right)$. Thus, we obtain the required estimates: $\left|R_N^{(m+1)}(t,\varepsilon)\right| \le C\varepsilon^{N+1}$. Similarly, considering the equalities

$$y^{(n-1)}(t,\varepsilon)=y_N^{(n-1)}(t,\varepsilon)+R_N^{(n-1)}(t,\varepsilon),\ y^{(n-1)}(t,\varepsilon)=y_{N+n-1-m}^{(n-1)}(t,\varepsilon)+R_{N+n-1-m}^{(n-1)}(t,\varepsilon) \tag{29}$$

Hence, equating the right-hand sides of (29), we obtain

$$R_N^{(n-1)}(t,\varepsilon)=y_{N+n-1-m}^{(n-1)}(t,\varepsilon)-y_N^{(n-1)}(t,\varepsilon)+R_{N+n-1-m}^{(n-1)}(t,\varepsilon), \tag{30}$$

where

$$\bar{y}_{N+n-1-m}^{(n-1)}(t,\varepsilon)-\bar{y}_N^{(n-1)}(t,\varepsilon)=\varepsilon^{N+1}y_{N+1}^{(n-1)}(t)+...+\varepsilon^{N+n-1-m}\overset{(n-1)}{y}_{N+n-1-m}(t)+$$

$$+\varepsilon^{N+1}\overset{(n-1)}{w}_{N+n-m}(\tau)+...+\varepsilon^{N+n-1-m}\overset{(n-1)}{w}_{N+2(n-1-m)}(\tau),$$

The remainder term $R_{N+n-1-m}^{(n-1)}(t,\varepsilon)$ in (30) satisfies the estimate $\left|R_{N+n-1-m}^{(n-1)}(t,\varepsilon)\right| \le C\varepsilon^{N+n-m}+C\varepsilon^{N+1}\exp\left(-\gamma\frac{t}{\varepsilon}\right)$. Thus, we obtain the required estimates $\left|R_N^{(n-1)}(t,\varepsilon)\right| \le C\varepsilon^{N+1}$. Theorem is proved.

CONCLUSION

We investigatedasymptotic expansion of solution of general boundary value problem with initial jumps for higher-order singularly perturbed integro-differential equation with any degree of accuracy with respect to a small parameter have been constructed.

Acknowledgement

The authors were supported in parts by the MES RK grant

No. AP05132573.“Cellular neural networks with continuous/discrete time and singular perturbations.” (2018-2020)

of the Committee of Science, Ministry of Education and Science of the Republic of Kazakhstan.

REFERENCES

[1] Dauylbayev M.K., Atakhan N. The initial jumps of solutions and integral terms in singular BVP of linear higher order integro-differential equations // Miskolc Mathematical Notes. **2015**. Vol.16. No.2. P.747-761.

[2] Dauylbayev M.K., Dzhumabaev D.S., Atakhan N. Asymptotical representation of singularly perturbed boundary value problems for integro-differential equations// News of the National Academy of Sciences of the Republic of Kazakhstan. Physical-mathematical series.**2017**. Vol.312. No.2. P.18-26.

[3] Atakhan N. Asymptotical representation of singularly perturbed boundary value problems for integro-differential equations // The 5th Abu Dhabi University Annual International Conference Mathematical Science and its Applications. Abu Dhabi, **2017**. P.28-29.

[4] Kassymov K.A., Nurgabyl D.N. Asymptotic behavior of solutions of linear singularly perturbed general separated boundary-value problems with initial jump // Ukrainian Mathematical Journal. **2003**. Vol. 55. No.11. P. 1777-1792.

[5] Kassymov K.A., Nurgabyl D.N. Asymptotic estimates of solution of a singularly perturbed boundary value problem with an initial jump for linear differential equations // Differential Equations.**2004**. Vol. 40. No.5. P. 641-651.

[6] Dauylbaev M.K., Mirzakulova A.E. Asymptotic behavior of solutions of singular integro-differential equations // Journal of Discontinuity, Nonlinearity, and Complexity. **2016**. Vol. 5. No.2. P.147-154.

[7] Dauylbaev M.K., Mirzakulova A.E. Boundary-value problems with initial jumps for singularly perturbed integrodifferential equations// Journal of Mathematical Sciences. **2017**. –Vol. 222. No.3. P. 214-225.

[8] Mirzakulova A. E., Atakhan N. Construction of the solution of the boundary value problem for integro-differential equation with a small parameter in the highest derivatives // Bulletin of the Karaganda University. **2016.**Vol.84. No.4.P.99-103.

[9] Akhmet M., Dauylbaev M., Mirzakulova A. A singularly perturbed differential equation with piecewise constant argument of generalized type // Turkish Journal of Mathematics.**2018**. Vol.42. No.4.P.1680-1685.

[10] Atakhan N. Asymptotical convergence in singularly perturbed boundary value problems for integro-differential equations // Third international conference on analysis and applied mathematics. Almaty, **2016.** P.45.

М.Қ. Дауылбаев[1,4], Н. Атахан[2,4], А.Е. Мирзакулова[3]

[1]әл-Фараби атындағы Қазақ ұлттық университеті, Алматы, Қазақстан
[2]Қазақ мемлекеттік қыздар педагогикалық университеті, Алматы, Қазақстан
[3]Абай атындағы Қазақ ұлттық педагогикалық университеті, Алматы, Казахстан
[4]Ақпараттық және есептеуіш технологиялар институты, Алматы, Казахстан

ЖОҒАРҒЫ РЕТТІ СИНГУЛЯРЛЫ АУЫТҚЫҒАН ИНТЕГРАЛДЫ-ДИФФЕРЕНЦИАЛДЫҚ ТЕҢДЕУ ҮШІН ЖАЛПЫЛАНҒАН БАСТАПҚЫ СЕКІРІСТІ ШЕТТІК ЕСЕБІ ШЕШІМІНІҢ АСИМПТОТИКАЛЫҚ ЖІКТЕЛУІ

Аннотация. Мақаладасингулярлы ауытқығанинтегралды-дифференциалдық теңдеулер үшін*m*-ші ретті бастапқы секірісі бар бөлінбеген шеттік есепшешімінің асимптотикалық жіктелуі құрылды. Кіші параметр бойынша кезкелген дәлдікпен асимптотиканың қалдық мүшесін бағалау туралы теорема алынды.

Түйін сөздер: сингулярлы ауытқу, интегралды-дифференциалдық теңдеу, кіші параметр, асимптотика-лық жіктелу, бастапқысекіріс, шекаралық қабат.

М.К. Дауылбаев[1,4], Н. Атахан[2,4], А.Е. Мирзакулова[3]

[1]Казахский национальный университет имени аль-Фараби, Алматы, Казахстан
[2]Казахский государственный женский педагогический университет, Алматы, Казахстан
[3]Казахский национальный педагогический университет имени Абая, Алматы, Казахстан
[4] Институт информационных и вычислительных технологий, Алматы, Казахстан

АСИМПТОТИЧЕСКОЕ РАЗЛОЖЕНИЕ РЕШЕНИЯ ОБЩЕЙ КРАЕВОЙ ЗАДАЧИ С НАЧАЛЬНЫМИ СКАЧКАМИ ДЛЯ ВЫСШЕГО ПОРЯДКА СИНГУЛЯРНО ВОЗМУЩЕННОЕ ИНТЕГРО-ДИФФЕРЕНЦИАЛЬНОЕ УРАВНЕНИЕ

Аннотация. В статье построено асимптотическое разложение решений неразделенной краевой задачи с начальным скачком *m*-го порядка для сингулярно возмущенных интегро-дифференциальных уравнений. Полученатеорема об оценке остаточного члена асимптотики с любой степенью точности по малому параметру.

Ключевые слова: сингулярное возмущение, интегро-дифференциальное уравнение, малый параметр, асимптотическое разложение, начальный скачок, погранслой.

Information about authors:

Dauylbayev M.K. - al-Farabi Kazakh national university, Almaty, Kazakhstan, Institute of Information and Computational technologies, Almaty, Kazakstan;

Atakhan N. - Kazakh state women's teacher training university, Almaty, Kazakhstan, Institute of Information and Computational technologies, Almaty, Kazakstan;

Mirzakulova A.E. - Abay Kazakh national pedagogical university, Almaty, Kazakhstan

N E W S
OF THE NATIONAL ACADEMY OF SCIENCES OF THE REPUBLIC OF KAZAKHSTAN
PHYSICO-MATHEMATICAL SERIES
ISSN 1991-346X **https://doi.org/10.32014/2018.2518-1726.15**
Volume 6, Number 322 (2018), 37 – 43

UDC 517.925:62.50
IRSTI*27.29.17*

S.S.Zhumatov

Institute of Mathematics and Mathematical Modeling, Almaty, Kazakhstan
E-mail: sailau.math@mail.ru,sailau.math@gmail.com

ABSOLUTE STABILITY OF A PROGRAM MANIFOLD OF NON-AUTONOMOUS BASIC CONTROL SYSTEMS

Abstract. In this paper the inverse dynamics problemisstudied: for a given manifold restore a force field, which lies in the tangent subspace to manifold. One of the general inverse problems of dynamics is solved: the corresponding system of differential equations is but as well as the stability is considered. This inverse problem is very important for a variety of mathematical models mechanics.Absolutestability of a program manifold of non-autonomous basic control systems with stationary nonlinearity is investigated.Theproblem of stability of the basic control systems is considered in the neighborhood of a program manifold. Nonlinearitysatisfies to conditions of local quadratic relations. The sufficient conditions of the absolute stability of the program manifold have been obtained relatively to a given vector-function by means of construction of Lyapunov function, in the form "quadratic form plus an integral from nonlinearity". The obtained results are used to solve the problem of the synthesis of high-speed regulators.

Key words. Absolute stability, basic control systems, program manifold, Lyapunov function, local quadratic relation. high-speed regulators.

Introduction. The problem of constructing for systems of ordinary differential equations on a given integral curve was formulated by Yerugin in [1] and there was proposed a method for its solving. Later, this problem was developed by Galiullin, Mukhametzyanov, Mukharlyamov and others [2-19] to the problem of the construction of systems of differential equations by a given integral manifold, to solving of various inverse problems of dynamics, and to constructing of systems of program motion.The integral manifold is defined as the intersection of hypersurfaces.It should be noted that the construction of stable systems developed into an independent theory. A detailed survey of these works can be found in [2, 7, 16]. The works [2-5] are devoted to the construction of automatic control systems on the basis of a given manifold. In these works, control systems were constructed for a scalar nonlinear function $\varphi(\sigma)$, and sufficient conditions for absolute stability were established. The problem of the construction of automatic control systems for a vector nonlinear function with locally quadratic relations was solved in [6, 7].In [10, 12, 13], inverse problems of dynamics are considered in the presence of random perturbations, namely in the class of stochastic differential Ito equations. In [20 - 22], conditions for reducibility to a canonical form and conditions for the stability of a Cauchy problem were established, and the problem of the existence of periodic solutions of equations unresolved with respect to the higher derivative was investigated. Sufficient conditions for the asymptotic stability of the program manifold of degenerate automatic control systems were obtained in[8]. The problem of the exponential stability of the trivial solution was investigated in [23]. In [24, 25], exponential-stability conditions were established for automatic control systems of a certain class. The problem of the synthesis of asymptotically stable systems possessing a given property was posed in[26], where a method for the synthesis of feedback laws was also given. Questions of the stability of the trivial solution of systems with variable coefficients were considered in the works [26, 27].

In the present paper,we investigate the stability of a program manifold with respect to the given vector-function of non-autonomous basic control systems with stationary nonlinearity.

*Statement of the problem.*Note,what the general statement of the problem is as follows:

To construct a material system describing by ordinary differential equations

$$\dot{x} = f(t,x,u),\ t \in I = [0,\infty[, \tag{1}$$

where $x \in R^n$ is the state vector of the object; $u \in R^r$ - control vector; $f \in R^n$ is a continuous vector-function, on a given $(n-s)$-dimensional program manifold $\Omega(t) \equiv \omega(t,x) = 0,\ \omega \in R^s$

The method of solving this problem consists in finding a set of right parts of the desired systems satisfying the equality on the manifold.

Definition 1.*A set* $\Omega(t)$ *is called an integral manifold of equation (1) if, from that* $\omega(t_0,x_0) \in \Omega(t_0)$ *follows* $\omega(t,x(t,t_0,x_0)) \in \Omega(t)$ *for all* $t \geq t_0$.

The concepts of an integral manifold and a program manifold have the same meaning, for convenience, we will use the term program manifold.

Taking into account the necessary and sufficient conditions for the manifold $\Omega(t)$ to be integral for the system (1), we get:

$$\dot{\omega} = \frac{\partial \omega}{\partial t} + \frac{\partial \omega}{\partial x} f(t,x,u) = F(t,\omega,u). \tag{2}$$

Here $F \in R^S$ is the Erugin vector-function [1] satisfying the condition $F(t,0,u) \equiv 0.$

We will introduce for consideration a class Ξ of continuously-differentiable at times t and bounded on a norm matrices.

Suppose that the right-hand side of the system (1) can be represented in the form

$$\dot{x} = f(t,x) - B(t)\xi, \quad \xi = \varphi(\sigma), \quad \sigma = P^T(t)\omega, \quad t \in I = [0,\infty), \tag{3}$$

where $x \in R^n$ is a state vector of the object, $f \in R^n$ is a vector-function, satisfying to conditions of existence of a solution $x(t) = 0$, $B \in \Xi^{n\times r}$, $P \in \Xi^{s\times r}$ are matrices, $\omega \in R^s\,(s \leq n)$ is a vector, $\xi \in R^r$ is the control vector-function of the deviation from the given program, satisfying to conditions of local quadratic connection

$$\varphi(0) = 0 \wedge \varphi^T(\sigma)\theta(t)(\sigma - K^{-1}(t)\varphi(\sigma)) > 0, \quad \forall\, \sigma \neq 0, \tag{4}$$

$$K_1(t) \leq \frac{\partial \varphi(\sigma)}{\partial \sigma} \leq K_2(t),$$

$$[\theta = diag\|\theta_1,\ldots,\theta_r\|] \in \Xi^{r\times r}, \quad K(t) = diag\|k_1(t),\ldots,k_r(t)\|, \quad [K(t) = K^T(t) > 0] \in \Xi^{r\times r},$$

$$[K_i(t) = K_i^T(t) > 0 \quad i = 1,2] \in \Xi^{r\times r}.$$

Note that the following estimate

$$\frac{\beta_1}{\nu_2}\|\omega\|^2 \leq \|\varphi\|^2 \leq \frac{\beta_2}{\nu_1}\|\omega\|^2 \ (\tag{5}$$

can be obtained from the condition (4), where $\beta_1,\nu_1;\beta_2,\nu_2$ are the smallest, largest eigenvalues of matrices $P\theta P^T$, θK^{-1}.

The given program $\Omega(t)$ is exactly realized only if the initial values of the state vector satisfy the condition $\omega(t_0, x_0) = 0$. However, this condition cannot be exactly satisfied, because of always there exist initial and permanent acting perturbations. Therefore, the conditions of the stability of the program manifold $\Omega(t)$ with respect to the vector function ω should be additionally required in the construction of systems of program motion.

On the basis of relation (2) and our assumption, choosing the Yerugin function as $F = -A(t)\omega,\ A \in \Xi^{s\times s}$ we obtain the following system with respect to vector-function ω [2, 3]:

$$\dot{\omega} = -A(t)\omega - H(t)B(t)\xi, \quad \xi = \varphi(\sigma), \quad \sigma = P^T(t)\omega, \quad t \in I = [0, \infty), \tag{6}$$

where $H = \dfrac{\partial \omega}{\partial x}$ is the Jacobi matrix, nonlinearity $\varphi(\sigma)$ satisfies also to generalized conditions (4), (5).

Definition 2.*A program manifold $\Omega(t)$ is called absolutely stable with respect to vector-function ω if it is asymptotically stable in whole for solution of equations (6) for all $\omega(t_0, x_0)$ and the function $\varphi(\sigma)$ satisfying conditions (4), (5).*

Statement of the problem.*To get the condition of absolute stability of a program manifold $\Omega(t)$ of the non-autonomous basic control systems in relation to the given vector-function ω.*

Sufficient conditions of the program manifold's absolute stability. First, we consider a linear system of differential equations with respect to a vector function ω:

$$\dot{\omega} = -A(t)\omega, \quad t \in I = [0, \infty). \tag{7}$$

For this system, we have

Theorem1 [27].*Suppose that there exists $L(t) = L^T(t) > 0$ and $-\dot{V}|_{(7)} = W$.*

Then for the asymptotic stability of a program manifold $\Omega(t)$, it is necessary and sufficient that the following relations hold

$$V = \omega^T L(t)\omega > 0,$$
$$W = \omega^T G(t)\omega > 0,$$

where V, W have the following properties

$$l_1\|\omega\|^2 \le V \le l_2\|\omega\|^2, \tag{8}$$

$$g_1\|\omega\|^2 \le W \le g_2\|\omega\|^2, \tag{9}$$

where l_1, l_2, g_1, g_2 are positive constants.

In the space, X_n we choose the region $G(R)$ as follows

$$G(R) = (t, x) : t \ge 0 \wedge \|\omega(t, x)\| < R < \infty. \tag{10}$$

Basic theorem. *If there exists a real, continuous and differentiable function $V(t, \omega)$ in region (10) that is definitely positive and admits a higher limit as a whole, such that*

$$-\frac{dV}{dt}|_{(6)} = W(t, \omega)$$

will be a definite-positive function for all values of ω, then the program manifold $\Omega(t)$ is absolutely stable.

Theorem 2. *Suppose that there exist matrices*

$$L(t) = L^T(t) > 0 \in \Xi^{s\times s},\ \beta = diag(\beta_1,\ldots,\beta_r) > 0$$

and non-linear function $\varphi(\sigma)$ *satisfies the conditions (4), (5). Then, for the absolute stability of the program manifold* $\Omega(t)$ *with respect to the vector function* ω *it is sufficient performing of the following conditions*

$$l_1\|\omega\|^2 \le V \le l_2\|\omega\|^2, \tag{11}$$

$$g_1\|\omega\|^2 \le -\dot{V} \le g_2(\|\omega\|^2, \tag{12}$$

where l_1, l_2, g_1, g_2 *are positive constants.*

*Proof.*Let there exist $L(t) = L^T(t) > 0 \in \Xi^{s\times s},\ \beta = diag(\beta_1,\ldots,\beta_r) > 0,$ then for the system (6) we can construct a Lyapunov function of the form

$$V(\omega,\xi) = \omega^T L(t)\omega + \int_0^\sigma \varphi^T \beta\, d\sigma > 0. \tag{13}$$

Taking into account the property (4), making the substitution

$$\varphi(\sigma) = h\sigma \quad (0 \le [h = h^T] \le k) \quad k = \min_t K(t),$$

we obtain the estimate

$$l_1(t)\|\omega\|^2 \le V \le l_2(t)]\omega]^2, \tag{14}$$

where

$$l_1(t) = l^{(1)}(t) + \lambda_1(t), \quad l_2(t) = l^{(2)}(t) + \lambda_2(t);$$

$$\lambda_1(t)\|\omega\|^2 \le \int_0^\sigma \varphi^T(\sigma)\beta\, d\sigma \le \lambda_2(t)\|\omega\|^2.$$

Here $l^{(1)}, \lambda_1, l^{(2)}, \lambda_2$ are the smallest and largest eigenvalues of matrices $L, \Lambda,\ \Lambda = P(t)H(t)\beta P^T(t)$. The diagonal elements of the matrix Λ are divided by the number 2. On the basis of property (5) the derivative of the function (13) takes the form

$$-\dot{V} = \omega^T G\omega + 2\omega^T G_1\xi + \xi^T G_2\xi > 0, \tag{15}$$

where

$$G(t) = -\dot{L} + A^T(t)L(t) + L(t)A(t);$$

$$G_1(t) = \frac{1}{2}A^T(t)P(t)\beta + L(t)H(t)B(t) + \frac{1}{2}\beta\dot{P}^T(t);$$

$$G_2(t) = \beta P^T(t)H(t)B(t).$$

Due to the fact that $-\dot{V} > 0$ the following estimates hold

$$q_1\left(\|\omega\|^2 + \|\xi\|^2\right) \le z^T Qz \le q_2\left(\|\omega\|^2 + \|\xi\|^2\right), \tag{16}$$

where

$$z = \begin{Vmatrix} \omega \\ \xi \end{Vmatrix}, \quad Q = \begin{Vmatrix} G & G_1 \\ G_1^T & G_2 \end{Vmatrix},$$

q_1, q_2 are the smallest and largest eigenvalues of matrix Q.

Taking into account the estimates (5) from (16), we get

$$\eta_1(t)\|\omega\|^2 \le -\dot{V} \le \eta_2(t)\|\omega\|^2, \tag{17}$$

$$\eta_1 = q_1\left(1 + \frac{\beta_1}{\nu_2}\right); \quad \eta_2 = q_2\left(1 + \frac{\beta_2}{\nu_1}\right).$$

On the basis of (11), (17), the inequalities are valid

$$l_2^{-1}V_0 \exp[-\int_{t_0}^{t}\alpha_1(\tau)d\tau] \le \|\omega\|^2 \le l_1^{-1}V_0 \exp[-\int_{t_0}^{t}\alpha_2(\tau)d\tau], \tag{18}$$

where

$$\alpha_1(t) = \frac{\eta_2(t)}{l_1(t)}; \quad \alpha_2(t) = \frac{\eta_1(t)}{l_2(t)}.$$

Assume that

$$\alpha_1 = \sup_{t\in I}[-\alpha_1(t)] \wedge \alpha_2 = \inf_{t\in I}[-\alpha_2(t)];$$

$$l_1 = \inf_{t\in I} l_1(t) \wedge l_2 = \sup_{t\in I} l_2(t).$$

from inequality (18) we obtain

$$Ł_2^{-1}V_0 \exp[\alpha_1(t-t_0)] \le \|\omega\|^2 \le l_1^{-1}V_0 \exp[\alpha_2(t-t_0)].$$

Whence follows the next estimates which hold on the sphere R:

$$\|\omega\|^2 \le R^2 \exp[\alpha_2(t-t_0)]. \tag{19}$$

*Conditions of the synthesis of high-speed regulators.*The obtained results we use to solve of the problem of the syntewsis of high-speed regulators.

Let $t = t_0^*$. Then from (19) we get

$$R^2 \exp[\alpha_2(t_0^* - t_0)] = \varepsilon^2$$

or solving with respect to $t_0^* - t_0$:

$$t_0^* - t_0 = \alpha_2^{-1} \ln\frac{\varepsilon^2}{R^2}. \tag{20}$$

The control time $t_\rho = t^* - t_0$ on the basis of (20) is defined as follows

$$t_\rho = \alpha_2^{-1} \inf_{\omega_0} \ln\frac{\varepsilon^2}{R^2}. \tag{21}$$

The solution of the problem of the synthesis of high-speed regulators follows from inequalities

$$\alpha_2^{-1} \inf_{\omega_0} \ln \frac{\varepsilon^2}{R^2} \le t_s .$$

here t_s is the specified time.

*Funding.*This resultsare supported by grant of the Ministry education and science of Republic Kazakhstan No. AP 05131369for 2018-2020 years.

REFERENCES

[1] Erugin N.P. Construction all the set of systems of differential equations, possessing by given integral curve.Prikladnaya Matematika i Mecanika, **1952**, 6, 659-670. (In Russ.)

[2] Galiullin A.S., Mukhametzyanov I.A., Mukharlyamov R.G. A survey of investigating on analytic construction of program motion's systems. Vestnik RUDN. **1994**, No. 1., 5-21. (In Russ.)

[3]Mukametzyanov I.A. On stability of a program manifold. I. //Differential Equations. **1973.** Vol.9. No 5. P. 846-856. (In Russ.)

{4] Mukametzyanov I.A. On stability of a program manifold. II. //Differential Equations. **1973.** Vol. 9. No 6. P. 1057-1048. (In Russ.)

[5] Mukametzyanov I.A., Saakyan A.O. On some sufficient conditions for the absolute stability of nonlinear integral manifold, in:Problems of Mechaniks of Controlled Motion. Perm. **1979.** P.137-144. (In Russ.)

[6]Maygarin B.G. Stability and quality of process of nonlinear automatic control system, Alma-Ata. Nauka.**1981**. (In Russ.)

[7] Zhumatov S.S., Krementulo B.B., Maygarin B.G. Lyapunov's second method in the problems of stability and control by motion. Almaty. Gylym.**1999.**(In Russ.)

[8] Zhumatov S.S. Stability of a program manifold of control systems with locally quadratic relationsw // Ukrainian Mathematical Journal. **2009.** Vol.61. No 3. P.500-509.
https://doi.org/10.1007/s 11253-008-0224-y

[9] Zhumatov S.S. Exponential stability of a program manifold of indirect control systems // Ukrainian Mathematical Journal. **2010.** Vol.62. No 6. P.907-915.
https://doi.org/10.1007/s 11253-010-0399-2

[10]Tleubergenov M.T. On the inverse stochastic reconstruction problem // Differential Equations. **2014.** Vol. 50. No 2. P. 274-278.
https://doi.org/10.1134/s 0012266 11402 0165

[11] Mukharlyamov R.G. Simulation of Control Processes, Stability and Stabilization of Systems with Program Contraints. Journal of Computer and Systems Sciences International. **2015.** 54, No.1., 13--26.

[12] Tleubergenov M.T., Ibraeva G.T. On the restoration problem with degenerated diffusion// in TWMS Journal of pure and applied mathematics: **2015.**Vol. 6, Issue: 1. P. 93-99.

[13] Vasilina G.K., Tleubergenov M.T. Solution of the problem of stochastic stability of an integral manifold by the second Lyapunov method // Ukrainian Mathematical Journal. **2016.** Vol.68. No 1. P.14-28.
https://doi.org/10.1007/s 11253-016-1205-6

[14] Llibre J., Ramirez R. Inverse Problems in Ordinary Differential Equations and Applications. Springer International Publishing Switzerland. **2016**.

[15] Zhumatov S.S. On an instability of the indirect control systems in the neighborhood of program manifold// Mathematical Journal.-Almaty, **2017**. Vol.17. No 1. P.91-97.

[16] Zhumatov S.S. Frequently conditions of convergence of control systems in the neighborhoods of program manifold // Journal of Mathematical Sciences, **2017.**226(3), 260-269. DOI 10.1007/s10958-017-3532-z.

[17] Zhumatov S.S. On a program manifrold's stability of one contour automatic control systems// Open Engineering, **2017.**Vol. 7, Issue 1, Pages 479-484, ISSN (Online) 2391-5439, DOI: https://doi.org/10.1515/eng-2017-0051.

[18]. Zhumatov S.S. Instability of the control systems with non-stationary nonlinearities//,in Functional analysis in interdisciplinary applications (FAIA2017), AIP Conf. Proc.**2017.**1880, 060007-1–060007-5; doi: 10.1063/1.5000661Published by AIP Publishing. 978-0-7354-1560-7/30.00

[19] Zhumatov S.S. On instability of a program manifold of basic control systems// inSpringer Proceedings in Mathematics & Statistics, Eds.: Kalmenov T.S., Nursultanov E.D., Ruzhansky M.V., Sadybekov M.A. – Springer, **2017**. – Vol. 216. – P. 467-474.

[20] Samolenko A.M., Yakobetc V.P.. On the reducibility of a degenerate linear system to the central canonical form // Dopovedi Nath.Akad Nauk Ukr. **1993**. No 4. P.10-13.(In Russ.)

[21] Yakobetc V.P. On some properties of degenerate systems // Ukrainian Mathematical Journal. **1997.** Vol.49. No 9. P.1278-1296.(In Russ.)

[22] Yakobetc V.P. On the structure of the general solution of degenerate linear system of second-order differential equations // Ukrainian Mathematical Journal. **1998.** Vol.50. No 2. P.292-298.(In Russ.)

[23] Demidovich V.P. Lectures on the mathematical theory of stability. M.: Fizmatgiz. **1967.**(In Russ.)

[24] Yakubovich V.A/ Methods of teory of abswolute stability. In: Methods for Investigation of Nonlinear Automatic Control Systems. M.: Nauka. **1975.** P. 74-180. (in Russ.)
[25] Voronov A.A. Stability, Controllability and Observablity. M.: Nauka. **1979.** (in Russ.)
[26] Letov A.M. Mathematical theory of control processes, M.: Nauka, **1981.** (in Russ.)
[27] Zubov V.I. Lectures on control theory. M.: Nauka. **1975.**(In Russ.)

УДК 517.925:62.50
МРНТИ 27.29.17

С.С. Жуматов

Институт математики и математического моделирования, Алматы, Казахстан

АБСОЛЮТНАЯ УСТОЙЧИВОСТЬ ПРОГРАММНОГО МНОГООБРАЗИЯ НЕ АВТОНОМНЫХ ОСНОВНЫХ СИСТЕМ УПРАВЛЕНИЯ

Аннотация. В статье рассматривается обратная задача динамики: для заданного многобразия воостанавливается поле сил, которые расположены на перпендикулярной полуплоскости к многообразию. Решается более общая задача динамики: исследуются устойчивость систем соответствующих дифференциальных уравнений. Эти обратные задачи очень важны для различных моделей механических систем. Исследуется абсолютная устойчивость программного многообразия не автономных основныхсистем управления со стационарными нелинейностями. Условия устойчивости основных систем исследованы вокрестности заданного программного многообразия. Нелинейности удовлетворяют условиям локальной квадратичной связи. Достаточные условия абсолютной устойчивости программного многообразия, относительно заданной вектор-функции, получены с помощью построения функции Ляпунова, «кадратичная форма плюс интеграл от нелинейности». Полученные результаты использованы длярешениязадачисинтеза быстродействующихрегуляторов.

Ключевые слова. Абсолютная устойчивость, основная система управления, прогораммное многообразие, функция Ляпунова, локальная квадратичная связь, быстродействующие регуляторы.

УДК 517.925:62.50
МРНТИ 27.29.17

С.С.Жұматов

Математика және математикалық моделдеу институты, Алматы, Қазақстан

АВТОНОМДЫ ЕМЕС НЕГІЗГІ БАСҚАРУ ЖҮЙЕЛЕРІНІҢ БАҒДАРЛАМАЛЫҚ КӨПБЕЙНЕСІНІҢ АБСОЛЮТ ОРНЫҚТЫЛЫҒЫ

Аннотация. Мақалада динамиканың кері есебі зерттеледі: яғни, берілген көпбейне үшін, көпбейнеге перпендикуляр жазықшада жататын күш өрісі тұрғызылады. Динамиканың жалпы есебі шешіледі: яғни сәйкес дифференциалдық теңдеулер жүйесінің орнықтылығы зерттейледі. Бұл кері есеп механиканың түрлі математикалық моделдері үшін өте маңызды. Стацонар бейсызықты автономды емес негізгі басқару жүйелерінің абсолют орнықтылығы зерттеледі. Негізгі басқару жүйелерінің орнықтылығы бағдарламалық көпбейненің маңайында қарастырылады. Бейсызықтықтар локалды квадраттық байланыстарды қанағаттандырады. Бағдарламалық көпбейненің берілген вектор-функция бойынша абсолют орнықтылығының жеткіліктк шарттары “квадраттық форма қосу бейсызықтын интеграл” түріндегі Ляпунов функциясын құру арқылы алынады. Алынған нәтижелер тез жылдамдықты реттегіштерді синтездеу есебіне қолданылды.

Түйін сөздер. Абсолюттік отнықтылық, негізгі басқару жүйелері, бағдарламалық көпбейне, Ляпунов функциясы, локалді квадраттық байланыс, жоғары жылдамдықты реттегіш.

Information about authors:
Zhumatov Sailaubay Sagimbaevich - Institute of Mathematics and Mathematical Modeling, chief scientific researcher, Doctor of Physical and Mathematical Sciences, sailau.math@mail.ru,sailau.math@gmail.com

N E W S
OF THE NATIONAL ACADEMY OF SCIENCES OF THE REPUBLIC OF KAZAKHSTAN
PHYSICO-MATHEMATICAL SERIES
ISSN 1991-346X **https://doi.org/10.32014/2018.2518-1726.16**
Volume 6, Number 322 (2018), 44 – 48

МРНТИ: 41.29.21; 41.29.25
UDK 524

A. Amangeldyieva[1], D.Kairatkyzy[2], T.Konysbayev[3]

[1] Al Farabi National University, "V.G. Fesenkov Astrophysical Institute ", JSC "National Center for Space Research and Technology", Aerospace Committee of the MDAI of the Republic of Kazakhstan;
[2]Ionosphere Institute, JSC "National Center for Space Research and Technology" Aerospace Committee of the MDAI of the Republic of Kazakhstan;
[3]"V.G. Fesenkov Astrophysical Institute ", JSC "National Center for Space Research and Technology", Aerospace Committee of the MDAI of the Republic of Kazakhstan;
[1] akkumyssss@gmail.com; [2]kairatkyzy.dina@mail.ru;[3]talgar_777@mail.ru;

ON THE NONSTATIONARY PARAMETER OF STATE FOR DARK MATTER

Abstract: The purpose of work is a conclusion non-stationary the equations of a condition of WIMP-gas for their various models - ideal gas (Mendeleyev-Clapeyron equation), non-ideal gas (equations type of Van der Waals and Dieterichi). It is shown that their general dependence on time has the negative power. Therefore, the closer to the Planck time, and also to the time of WIMP particles birth, the more state of non-baryonic substance differ from the dust-like one.

Keywords: dark matter (DM), WIMP-gas, nonstationary equation of a state, early epochs of the Universe, Mendeleyev-Clapeyron equation, Van der Waals equations, equations of Dieterichi, fluctuation of temperature of relic radiation.

Introduction

It is known that the Universe (approximately) for 73% consists of dark energy, for 23% – of dark matter and about 4% of baryonic substance – primary plasma and radiation. [1, 2]. We will note that there is a classification of dark matter - "hot", "warm" and "cold". The last is most preferable from the cosmological point of view. The class of WIMPs is among candidates for a role of particles of cold DM. The peculiarity of WIMP is that their concentration gives not only the necessary contribution to the total Universe energy balance, but also effectively describes the galaxies halos of dark matter.

At the same time, it should be noted that the most of previous articles were based on idea of halos' static character. But later the idea that density of dark matter in halos may depend on time also began to develop and, therefore, dark matter possesses by dynamic properties [3 - 10].

For searching the distribution of relic radiation in of WIMP-particles gas, we will remind that small periodic indignations in the continuous environment represent sound waves. If these changes adiabatic, the speed of a sound wave is described by expression $\upsilon = \sqrt{\left(\partial P/\partial\rho\right)_S}$. Since the entropy of the Universe is highly constant, later expression can be used for study the substance of non-baryonic matter (in our case - gas of WIMP-particles). So, first we will consider gas of WIMP-particles as ideal gas, and then – as real gas in which parameter of state is nonstationary.

So, due to the thermodynamic balance of WIMP-particles with particles of baryonic plasma the approximate condition takes place $T_{BM} \propto\sim T_{DM}$ [11]. Therefore Mendeleyev-Clapeyron equation takes on the form.

$$P_{DM}V = \frac{m_{DM}}{\mu} RT_{BM} . \quad (1)$$

It's convenient to rewrite (1) as

$$P_{DM} = \rho_{DM} \frac{R}{\mu} T_{BM} \quad (2)$$

with explicit expression of gas density at a given temperature. In our case, therefore, we are talking about baryonic matter in the form of ultrarelativistic plasma. For the standard cosmological Friedman model filled with relativistic gas (ultrarelativistic plasma), there is an approximate relation connecting the temperature of primary relativistic gas with the age of Universe. It is:

$$T_{BM} \sim t^{-1/2} . \quad (3)$$

From (2) and (3) it follows that equation of state of ideal gas, generally speaking, has the form $P_{DM} = \bar{\omega}_{DM}(t) \cdot \rho_{DM} = \rho_{DM} \frac{R}{\mu} T_{BM}(t)$. So, taking into account the dependence (3), its state parameter depends on time in the same way, i.e.

$$\bar{\omega}_{DM}(t) = \frac{R}{\mu} T_{BM}(t) \sim t^{-1/2} . \quad (4)$$

(ii) Van der Waals gas. Consider now the case of filling the Universe by real gas consisting of molecules and described by the Van der Waals equation of state. (Another version of the description of non-ideal dark matter is given in articles [12-15].) If the temperature is measured in degrees, then, according to [16], it takes on the form

$$\left(P_{DM} + N^2 \frac{\tilde{a}}{V^2} \right) \left(V - N\tilde{b} \right) = NkT_{BM} , \quad (5)$$

in which $\tilde{a}$ and $\tilde{b}$ are the constant values that describe gas of WIMP - particle, k is Boltzmann constant. Recall that the physical meaning of parameter $\tilde{a}$ that it describes the interaction of substance molecules, parameter $\tilde{b}$ is responsible for accounting their sizes.

Let us rewrite (5) in the form that convenient for study of state parameter, -

$$P_{DM} \left(1 + \nu^2 \frac{\tilde{a}}{P_{DM} V^2} \right) \left(1 - \nu \frac{\tilde{b}}{V} \right) = \rho_{DM} \frac{R}{\mu} T_{BM} . \quad (6)$$

Here - μ is the molar mass of substance, R - universal gas constant. Now our task is to combine (6) with (3) and find an explicit dependence of the real gas state parameter on time. For further explanation, we assume that $\tilde{b} / V \ll 1$. This condition describes the real property of the gas from the WIMP particles, in which its current size is essentially larger than size of all molecules themselves. In addition, it makes sense to assume that the interaction of molecules is not too large. After these simplifications, the state parameter of real gas can be express as a function of temperature as follows

$$\omega_{DM}(T_{BM}) = \frac{R}{\mu} \left(1 - \nu^2 P_{DM} \tilde{a} \Big/ \frac{m^2}{\mu^2} R^2 T_{BM}{}^2 \right) T_{BM} \quad (7)$$

or as the function of time of two components

$$\omega_{DM}(t) = \begin{matrix} \bar{\omega}_{DM}(t) \propto t^{-1/2} > 0 \\ \omega'_{DM}(t) \propto t^{-3/2} < 0 \end{matrix} . \quad (8)$$

iii) Real gas Dieterichi links the main thermodynamic quantities in the gas. It has the form,

$$P = R\frac{T}{(V-\beta)}\cdot\exp\left(-\frac{\alpha}{RTV}\right), \tag{9}$$

were P - pressure, - molar volume, T - absolute temperature, respectively. In addition, there is a mass of gas, and its molar mass. Finally, α parameter characterizes the interaction of gas molecules, and β parameter describes the size of molecules. This equation can be rewritten as

$$P = \rho\left(\frac{k}{\mu}\right)\cdot\frac{T}{\left(1-\frac{b}{V}\right)}\cdot\exp\left(-\frac{\alpha}{\frac{m}{\mu}kVT}\right). \tag{10}$$

For it examining introduce some additional conditions. First, we assume that $\frac{b}{V} << 1$ as before. Beside in exponent we'll use the equation of state of an ideal gas - $PV = \frac{m}{\mu}kT$. As a result, after a series of calculations for dusty matter and ultra relativistic matter, we obtain, respectively

$$\omega_D(t) + \left(\frac{k}{\mu}\right)\cdot t^{-1/2}\cdot\exp\left(-\frac{\bar{\alpha}}{\omega_D(t)\cdot t^2}\right)\approx 0, \tag{11}$$

$$\omega_D(t) + \left(\frac{k}{\mu}\right)\cdot t^{-1/2}\cdot\exp\left(-\frac{\tilde{\alpha}}{\omega_D(t)\cdot t}\right)\approx 0. \tag{12}$$

Here are the $\bar{\alpha}$ and $\tilde{\alpha}$ coefficients that proportional to k/μ. For further calculation introduce new constants $\bar{\alpha} = k/\mu, \tilde{\alpha} = k/\mu$. In addition assume that $-\frac{\bar{\alpha}}{\omega(t)\cdot t^2} << 1$ and $-\frac{\tilde{\alpha}}{\omega(t)\cdot t} << 1$. After decomposing exponential expressions into series according to the above specified parameters, we obtain the needed state parameters

$$\bar{\omega}_D = \frac{A}{2\sqrt{t}} \pm \frac{1}{2}\sqrt{\frac{A^2}{t} - \frac{4B}{t^{5/2}}}, \tag{13}$$

$$\breve{\omega}_D = \frac{A}{2\sqrt{t}} \pm \frac{1}{2}\sqrt{\frac{A^2}{t} - \frac{4B}{t^{3/2}}}, \tag{14}$$

Based on the previously obtained results, we find that the temperature fluctuations of cosmic microwave background (CMB) which corresponds to the real gas model Dieterichi. So, we find the peculiar velocity (in the notations (4))

$$\upsilon' = \upsilon'_D(t) = \bar{\omega}_D(t) \propto t^{-3/2}t_0 \tag{15}$$

and the corresponding fluctuation of the relic radiation

$$\left(\frac{\delta T}{T}\right)_D = \upsilon'_D(t)\cdot\cos\theta \propto t^{-3/2}\cdot\cos\theta. \tag{16}$$

Note that the temperature fluctuations of the relic radiation for different observed angles were previously considered in [17].

Conclusion

In conclusion, we note the findings.

i) The nonstationary equations of state of WIMP gas for their various models - ideal gas (Mendeleev-Clapeyron equation), non-ideal gas (Van der Waals and Dieterichi equations) are derived.

(ii) It's shown that total dependence of the state parameter on time has a power form with a negative state indicator. Therefore, the closer to Planck time, as well as to the time of birth of the WIMP particles, the indicator of substance state increases. This fact shows that the state of non-baryonic matter in the earliest Universe is significantly different from dust-like matter.

The work was carried out within the framework of Project No. AP05134454 "Evolution of the perturbations in the density of dark matter in a very early Universe", financed by the JSC "National Center for Space Research and Technology", Aerospace Committee of the MDAI of the Republic of Kazakhstan.

Author express their gratitude to professor L.M.Chechin for the problem forwarding and the permanent supporting for its solving.

Note that by comparing (1) with the speed of sound, it is easy to see that it depends on time like (4), i.e. as $\upsilon = \overline{\upsilon}_{MC}(t) = \overline{\omega}_{DM}(t) \propto t^{-1/2}$. This velocity is the peculiar velocity of the relic radiation, which changes the rate of Hubble expansion. (Interpretation of this type of fluctuations is given in [2]). Consequently, the temperature fluctuation of the relic radiation in the epoch is described by the expression (as we usually assume $c = 1$) $\left(\frac{\delta T}{T}\right)_{MC} = \overline{\upsilon}_{MC}(t) \cdot \cos\theta \propto t^{-1/2} \cdot \cos\theta$. So the variation of the temperature of the relic radiation will depend not only on the angle of observation, but also on the time.

REFERENCES

[1] Dolgov AD,Sazhin MV, Zeldovich YaB**(1990)** Basic of Modern Cosmology, Editions Frontiers. Paris,

[2] Byrd GG, Cernin AD, Valtonen MJ**(2007)**Cosmology,Foundations and Frontiers. M. 485.

[3] Hahn O, Carollo CM, Porciani C, Dekel A. The Evolution of Dark Matter Halo Properties in Clusters, Filaments, Sheets and Voids. arXiv: 0704.2595 [astro-ph].

[4]Tormen G, Bouchet FR,White SDM. the Structure and Dynamic Evolution of the Dark Matter Halos. arXiv:astro-ph/9603132.

[5] Diemand J, Kuhlen M, Madau P. Formation and evolution of galaxy dark matter halos and their substructure. arXiv:astro-ph/0703337.

[6] Bertone G, Merritt D**(2005)**Time-Dependent Models For Dark Matter at the Galactic Center, Phys. Rev. 72, 103 502.

[7]Tulin S, Hai-Bo Yu, ZurekKM**(Feb,2012)** Oscillating Asymmetric Dark Matter. arXiv: 1202.0283v2 [hep-ph].13.

[8] Merritt D**(2004)** Evolution of the Dark Matter Distribution at the Galactic Center, Phys. Rev. Lett., 92, 201, 304.

[9] Reed D, Governato F, Verde L**(2005)** Evolution of the Density Profiles of Dark Matter Haloes. MNRAS, 357, 82.

[10] Mirzagholi L, Vikman A**(2015)** Imperfect, Dark Matter. JCAP, 6, 028; doi:10.1088/1475-7516/2015/06/028.

[11] Dolgov AD, Thermodynamics and kinetics of elementary particles in cosmology. arXiv: 1302.v1 [astro-ph.CO]

[12] Jantsch RCS, Christmann MHB, Kremer GM**(2016)** The van der Waals fluid and its role in cosmology.IJMP D Vol. 25, No. 2 1650031.

[13]Vardiashvili G, Halstead E, PoltisR, Morgan A, Tobar DD **(2017)** Inflationary Constraints on the Van der Waals equation of state. arXiv170100748V.

[14] Brevik I, Elizalde E, Odintsov SD, Timoshkin AV**(2012)** Inflationary universe in terms of a Van der Waals viscous fluid. arXiv1205.3544Q.

[15] Quevedo H, Ramirez A, Antonio **(2012)** a geometric approach to the thermodynamics of the Van der Waals system. arXiv1205.3544Q.

[16] Landau LD, Lifshitz EM **(1964)** Statistical physics, M., Nauka, p. 568

[17] Starobinskii AA**(1988)**Relic radiation temperature fluctuations at intermediate angles, Pis'ma v Astronomicheskii Zhurnal,394-403. (In Russ.).

[18] Kabylbekov K. A., Ashirbaev H. A., Abdrakhmanova Kh.K., Dzhumagalieva A. I., Kadyrbekova J. B. Organization of the performance of the laboratory work "Modeling the electric field of a system consisting of a dielectric square and a long charged conductor" by using MATLAB software package. Proceedings of the National Academy of Sciences of the Republic of Kazakhstan, Series of Physics and Mathematics, Almaty 2017, No. 4, pp 252-259. **https://doi.org/10.32014/2018.2518-1726.** ISSN 2518-1726 (Online), ISSN 1991-346X (Print)

А. Амангельдиева[1], Д. Қайратқызы[2], Т. Қонысбаев[3]

[1] Әл-Фараби атындағы қазақ Ұлттық университеті, «В.Г. Фесенков атындағы Астрофизикалық Институт», «Ұлттық ғарыштық зерттеулер мен технологиялар орталығы» АҚ, ҚР ҚАӨМ Аэроғарыш комитеті;
[2]Ионосфера институты, «Ұлттық ғарыштық зерттеулер мен технологиялар орталығы» АҚ, ҚР ҚАӨМ Аэроғарыш комитеті;
[3]«В.Г. Фесенков атындағы астрофизикалық институт », «Ұлттық ғарыштық зерттеулер мен технологиялар орталығы» АҚ, ҚР ҚАӨМ Аэроғарыш комитеті

ҚАРАҢҒЫ МАТЕРИЯ ҮШІН БЕЙСТАЦИОНАР КҮЙ ПАРАМЕТРІ

Аннотация: Жұмыстың мақсаты - WIMP газының стационарлық емес теңдеулерін түрлі модельдер үшін - идеал газ (Менделеев-Клапейрон теңдеуі), идеал емес газ (Van der Waals және Diterich теңдеулерінің түрі)теңдеулерін есептеу. Олардың жалпы уақыт тәуелділігі теріс екендігі көрсетілген. Сондықтан Планк уақыты мен Вимп бөлшектерінің пайда болу уақыты неғұрлым жақын болған сайын барионды емес бөлшектердің күйі шаңтәріздестен соғұрлым өзгешелене түседі.

Түйін сөздер: қара материя (DM), WIMP газы, стационарлық емес теңдеуі, Әлемнің ерте дәуірі, Менделеев-Клапейрон теңдеуі, Ван дер Ваалс теңдеулері, Дитерери теңдеуі, Фондық сәуленің температуралық ауытқуы.

А. Амангельдиева[1], Д. Кайраткызы[2], Т. Конысбаев [3]

[1] Национальный университет имени аль-Фараби, « Астрофизический институт имени В.Г. Фесенкова» , АО «Национальный центр космических исследований и технологий», Аэрокосмический комитет МОАП Республики Казахстан;
[2]Институт Ионосферы, АО «Национальный центр космических исследований и технологий», Аэрокосмический комитет МОАП Республики Казахстан;
[3]«Астрофизический институт имени В.Г. Фесенкова», АО «Национальный центр космических исследований и технологий», Аэрокосмический комитет МОАП Республики Казахстан;

О НЕСТАЦИОНАРНОМ ПАРАМЕТРЕ СОСТОЯНИЯ ТЕМНОЙ МАТЕРИИ

Аннотация: Целью работы является вывод нестационарных уравнений состояния WIMP-газа для их различных моделей - идеального газа (уравнение Менделеева-Клапейрона), неидеального газа (типа уравнений Ван-дер-Ваальса и Дитеричи). Показано, что их общая зависимость от времени характеризуется отрицательным показателем. Поэтому чем ближе к времени Планка, так и к времени рождения WIMP-частиц, тем больше состояние небарионного вещества отличается от пылеподобного.

Ключевые слова: темная материя (DM), WIMP-газ, нестационарное уравнение состояния, ранние эпохи Вселенной, уравнение Менделеева-Клапейрона, уравнение Ван-дер-Ваальса, уравнение Дитеричи, флуктуация температуры реликтового излучения.

N E W S
OF THE NATIONAL ACADEMY OF SCIENCES OF THE REPUBLIC OF KAZAKHSTAN
PHYSICO-MATHEMATICAL SERIES
ISSN 1991-346X **https://doi.org/10.32014/2018.2518-1726.17**
Volume 6, Number 322 (2018), 49 – 53

UDC: 510.5

S.A. Badaev[1], B.S. Kalmurzayev[1], D.K. Kabylzhanova[1], K.Sh.Abeshev[2]

[1]Al Farabi Kazakh National University, Almaty, Kazakhstan;
[2]Almaty Management University, Almaty, Kazakhstan
Serikzhan.Badaev@kaznu.kz, birzhan.kalmurzayev@gmail.com,
dkabylzhanova@gmail.com, k.abeshev@almau.edu.kz

UNIVERSAL POSITIVE PREORDERS

Abstract. In this paper, we investigate universal objects in the class of positive preorders with respect to computable reducibility, we constructed a computable numbering of this class and proved theorems on the existence of a universal positive lattice and a universal weakly precomplete.

Keywords. Computable reducibility on preorders, universal positive preorder, computable numbering, positive lattice, positive linear order, weakly pre-complete positive preorder.

The class of positive (computably enumerable) equivalences, which is a proper subclass of the class of positive preorders, first appeared in the paper of Yu.L. Ershov [1]. In recent decades, interest has increased in a research of positive equivalences and positive preorders with respect to natural computable reducibility(see, for example, [2], [3], [4] and [5]). In the class of positive equivalences, universal objects are well-described (see [3], [4]), while universal preorders are poorly known, though attract huge interest, both from the point of view of the computability theory, and for applications in theoretical computer science. Our work is devoted to studying of positive preorders defined on the set of natural numbers ω with respect to computable reducibility, defined as follows: positive preorder P is computably reduced to positive preorder Q (symbolically, $P \leq_c Q$), if for any $x, y \in \omega$ there exists a computable function f such that $x\ P\ y$ if and only if $f(x)\ Q\ f(y)$. A positive preorder is universal if any positive preorder reduces to it. The first references to universal preorders meet in the work of Italian mathematicians F. Montagny and A. Sorbi, [6].

We follow the standard notation from the book [7]: Post's numbering of computably enumerable (c.e.) sets is denoted by$\{W_x\}_{x\in\omega}$, φ_e denotes the partially computable function of the Kleene's numbe$e$, and the standard coding of pairs of natural numbers is denoted by $\langle\cdot,\cdot\rangle$. Through $l(\cdot)$ and $r(\cdot)$ we denote computable functions that, by the code of the pair, restore its left and right components.

We briefly recall the basic concepts and introduce some notations used below in the paper.

Let S be some at most countable set, then an arbitrary map of the natural numbers ω to the set S is called the numbering of the family S. The numbering of the family S of c.e. sets are called computable if the set $\{\langle x, y\rangle : x \in \nu(y)\}$is c.e. set.

A positive preorder P is called universal in the class of preorders K if $P \in K$ and $Q \leq_c P$ for any $Q \in K$. For the preorder P, we denote by $[x]_P$ the equivalence class of x with respect to P, i.e.$[x]_P = \{y : x\ P\ y\ \&\ y\ P\ x\}$.

By Id we denote the identical preorder$\{(x, x) : x \in \omega\}$, and by $\mathcal{P}_1$ we denote the family of all positive preorders.

If$\leq$ is a partial ordering on some set M and x, y are elements of this set, then $z \in M$ is called the least upper bound of the elements $x, y(\sup(x, y))$, if $x, y \leq z$ and $\forall t\ [x, y \leq t \to z \leq t]$, and $z \in M$ is called the greatest lower bound of the elements $x, y(\inf(x, y))$, if $z \leq x, y$ and $\forall t\ [t \leq x, y \to t \leq z]$. It is clear that $x \leq y$ if and only if $\sup(x, y) = y$ and$\inf(x, y) = x$.

Recall that a lattice is a partially ordered set, where any two elements have the smallest upper bound and greatest lower bound.

If A is a set of natural numbers and n is a number, then $A \upharpoonright n$ denotes the set $A \cap \{0,1, \dots, n\}$.

Proposition 1. There is a computable numbering α of the family$\mathcal{P}_1$.

Proof. Fix some computable approximation $\{W_x^s\}_{s\in\omega}$ of the setW_x. We will construct a computable approximation $\{\alpha^s\}_{s\in\omega}$ of the numbering α and the computable function $b(x,s)$as follows:

CONSTRUCTION

Stage 0.For any $x \in \omega$we set$\alpha^0(x) = Id$ and $b(x,0) = n$for some$n > 1$.Go to the nextstage.

Stage $s+1$.For all$x \le s$do the following: if$W_x^s \upharpoonright b(x,s)$is a preorder, then we assume$\alpha^{s+1}(x) = \alpha^s(x) \cup W_x^s \upharpoonright b(x,s)$and $b(x,s+1) = b(x,s) + 1$.We call this stage an "expanding stage" for $\alpha\ (x)$. Go to the end of the stage.

End of stage.For all$x, s \in \omega$we assume that

- $\alpha^{s+1}(x) = \alpha^s(x)$,if$\alpha^{s+1}(x)$is undefined;
- $b(x,s+1) = b(x,s)$,if$b(x,s+1)$is undefined.

Go to the next stage.

Lemma 1. If for $\alpha(x)$ there are infinitely many "expanding stages", then $\alpha(x) = W_x$.

Theproofisobvious.

Lemma2.$\alpha(x)$is a positive preorder for any $x \in \omega$.

Proof. Obviously, $\alpha^s(x)$is a positive preorder at each stage s.There are two cases: the first, when for $\alpha(x)$there are infinitely many "extending stages".For this case, by Lemma 1, $\alpha(x) = W_x$. This can happen only when $W_x$is a positive preorder. And if the "expanding stages" are the finite number, then by construction

$$\alpha(x) = \lim_s \alpha^s(x) = \alpha^{s'}(x),$$

where$s'$is the last "expanding stage". It follows from the reasoning above that $\alpha^{s'}(x)$is a positive preorder.

Lemma 3. The numbering α is a computable numbering of the family $\mathcal{P}_1$

Proof. First, we prove that α is a numbering of the family$\mathcal{P}_1$.Let $P \in \mathcal{P}_1$be an arbitrary positive preorder. Since P is c.e. set, there exists x such that $P = W_x$. Since W_x is a preorder, then for $\alpha(x)$there are infinitely many "extending stages".Therefore, by Lemma 1, $\alpha(x) = W_x = P$. Moreover, the fact that α is a computable numbering follows from the construction efficiency.

Corollary 1. The numbering α constructed in Proposition 1 is universal in the class $Com(\mathcal{P}_1)$of computable numberings of the family $\mathcal{P}_1$.

Proof. Let β be an arbitrary computable numbering of the family $\mathcal{P}_1$. Since $\mathcal{P}_1$ is thefamilyof c.e. sets, then $\beta \le W_x$. Let $\beta \le W_x$via the function f. Then $\beta \le \alpha$via the function f. Indeed, for any$x\beta(x)$ is a positive preorder. Since $\beta(x) = W_{f(x)}$ and $W_{f(x)}$is a positive preorder, then $W_{f(x)} = \alpha\big(f(x)\big)$.

Corollary 2. There is a universal positive preorder with respect to computable reducibility $\le_c$.

Proof. We construct the preorder U as follows:

$$x\ U\ y \Leftrightarrow l(x) = l(y) \,\&\, r(x)\ \alpha\big(l(x)\big)\ r(y)$$

We show that any positive preorder P is computably reduced to U. Since P is a positive preorder, then there exists e such that $\alpha(e) = P$.Consequently,$P \le_c U$by the function $f(x) = \langle e, x\rangle$. ■

Consider special types of positive preorders:

We say that a positive preorder P is a positive linear order if the factor set $\omega/ER(P)$with the preorder $\le_P$given by the rule:

$$[x]_{ER(P)} \le_P [y]_{ER(P)} \Leftrightarrow \exists x' \exists y' \big(x' \in [x]_{ER(P)} \& y' \in [y]_{ER(P)} \& x'P\ y'\big)$$

is linearly ordered.

Definition. A positive prelattice is called a positive preorder P whose factor structure $(\omega/ER(P), sup, inf, \leq_P)$ is a lattice and the functions sup andinf are partially computable. Here by $ER(P)$ denotes the greatest equivalence, which is contained in P, i.e. $ER(P) = \{(x, y): xPy \ \& \ yPx\}$.

Positive linear preorders, which are a frequent case of positive prelattices, are defined similarly. We say that a positive preorder P is a positive linear preorder if the factor set $\omega/ER(P)$ with the preorder $\leq_P$ given by the rule:

$$[x]_{ER(P)} \leq_P [y]_{ER(P)} \Leftrightarrow \exists x' \exists y' \left(x' \in [x]_{ER(P)} \& y' \in [y]_{ER(P)} \& x' P \, y'\right)$$

is a linearly ordered set. The existence of a universal prelattice in the class of positive linear preorders was proved in [8].

Theorem 1. Let $\mathcal{R}$ be the family of all positive prelattices.Thereexists a family $\mathcal{T} \subseteq \mathcal{R}$ and a computable numbering of the family $\mathcal{T}$ such that for any $R \in \mathcal{R}$ there exists a prelattice $T \in \mathcal{T}$ for which $R \leq_c T$.

Proof. If R is a prelattice and $T \subseteq R$, then by $[T]$ we denote the closure of the set T with respect to sup and inf. Note that for any prelattice R and any finite set $T \subseteq R$, the closure $[T]$ is also a finite set.

Let π be the computable numbering of the family of all positive preorders and let $\{\pi^s(x)\}_{s\in\omega}$ be the computable approximation of the preorder $\pi(x)$. We construct an approximation of the computable numbering α and the family $\mathcal{T}$ as follows:

CONSTRUCTION

Stage 0. We define$\alpha^0(x) = Id$for all$x \in \omega$.The set $\{0,1\}$ is declared the effective range of the $\pi^0(x)$.Go to the next stage.

Stage$2s + 1$. Consider the following cases:

Case 1. If the effective range of $\pi^s(x)$ is a prelattice, then copy the effective range to $\alpha^{s+1}(x)$, i.e.we select the fresh (has never been used up to this point) elements a_i for all i from the effective range of $\pi^s(x)$.The smallest number $n$which is outside the effective range of $\pi^s(x)$ is added to the effective range of$\pi^{s+1}(x)$. Go to the next stage.

Case 2.If the effective range of $\pi^s(x)$ is not a prelattice, then add all the supandinf elements of the effective range of $\pi^s(x)$ to the effective rangeof $\pi^{s+1}(x)$, if there are any. Go to the next stage.

Stage$2s + 2$. Choose the least number $n \notin range\{a_i\}$and declare this element equivalent to the element 0. Go to the next stage.

Lemma1. For any $x \in \omega, \alpha(x)$ is a positive prelattice.

It's not so hard to show, since we copy only the positive prelattice to the elements a_i, and all other elements are equivalent to 0. Forarbitraryelementsx, y:

1) If$x = a_i$and$y = a_j$for some i, j, thensup$(x, y) = \sup(a_i, a_j) = a_k$where$k = \sup(i, j)$in the effective range of$\pi(x)$.

2) If$x = a_i$and$y \notin range\{a_i\}$,thensup$(x, y) = \sup(a_i, 0) = a_k$where$k = \sup(i, 0)$in the effective range of$\pi(x)$.

3) If$x \notin range\{a_i\}$and$y = a_j$forsome$j$, this case is similar to case 2.

4) If$x, y \notin range\{a_i\}$, thensup$(x, y) = \inf(x, y) = 0$.

For infwe carry out a similar reasoning.

Lemma2. If $\pi(x)$ is a positive prelattice, then atstages$2s + 1$, case 1 is repeated infinitely often and case 2 cannot be repeated infinitely times without case 1.

The proof follows from the remark about the finiteness of the closure of finite sets.

Lemma 3. Forany $R \in \mathcal{R}$, there exists a number x such that $R \leq_c \alpha(x)$.

Proof. Since R is a positive prelattice, then by Lemma 2 case 1 is performed infinitely often and all elements of R. will enter the effective range R. Therefore, reducibility is carried out bythe function $f(x) = a_x$. ■

Theorem 2. In the class of positive prelattices, there is a universal prelattice.

Proof. Let α be the computable numbering of the family T of positive prelattices. We construct a positive prelattice$U$ as follows:

$$x\ U\ y \Leftrightarrow l(x) < l(y) \vee \left[l(x) = l(y) \& r(x)\ \alpha\big(l(x)\big)\ r(y)\right].$$

If P is a positive prelattice, then $P \leq_c \alpha(e)$ for some e and $\alpha(e) \leq_c U$ by the function$f(x) = \langle e, x\rangle$. It remains to prove that the positive preorder U is a positive prelattice. Let$[x]_U$ and $[y]_U$be two different equivalence classes. If$l(x) = l(y)$ then the sup and inf of these classes coincide with the sup and inf in the positive prelattice $\alpha\big(l(x)\big)$. If$l(x) < l(y)$, then$\sup([x]_U, [y]_U) = [y]_Uand\inf([x]_U, [y]_U) = [x]_U$. The case when $l(x) > l(y)$ is similar to the previous case. ■

The following special type of positive preorders is weakly precomplete positive preorders.

Definition. A positive preorder P is called weakly precomplete [8], if for any total function φ_e, there exists an element $x_e$such that $\varphi_e(x_e) P x_e$.

Note that the concept of weaklyprecompleteness for positive preorders is identical to this concept for positive equivalences, which was originally introduced in [9]and found to be very useful in the study of positive equivalences (see review [4]).

Theorem 3. For any positive preorder Pthere is a weakly precomplete positive preorder Q such that $P \leq_c Q$.

Proof. Let P be an arbitrary positive preorder. We construct a positive preorder Q as follows:

CONSTRUCTION

Stage 0. Let$Q^0 = P \oplus Id$. Go to the next stage.

Stages $s+1$.Let$l(s) = e$.We work withφ_e.

Let$x_e = 2e + 1$.

1) If$\varphi_e^s(x_e) \uparrow$,then$Q^{s+1} = Q^s \cup P^{s+1} \oplus Id$.

2) If$\varphi_e^s(x_e) \downarrow$, then$Q^{s+1} = Q^s \cup P^{s+1} \oplus Id \cup \{(\varphi_e(x_e), x_e), \big(x_e, \varphi_e(x_e)\big)\}$we reflexively and transitively close. Go to the next stage.

By construction, it is easy to see that the preorder Q is positive and the computable function $f(x) = 2x$performs the reduction $P \leq_c Q$. ■

Corollary. In the class of positive preorders $\mathcal{P}_1$there exists a universal weakly precomplete preorder.

The research of all authors was carried out with the financial support of the Ministry of Education and Science of the Republic of Kazakhstan, grant AP05131579 “Positive preorders and computable reducibility on them, as a mathematical model of databases”

REFERENCES

[1] Ershov Y. L. Positive equivalences //Algebra and Logic. **1971**. Vol. 10. №. 6. P. 378-394.

[2] Gao S., Gerdes P. Computably enumerable equivalence relations //StudiaLogica. **2001**. Vol. 67. №. 1. P. 27-59.

[3] Andrews U., Lempp S., Miller J.S., Ng K.M., San Mauro L., and Sorbi A. Universal computably enumerable equivalence relations // The Journal of Symbolic Logic. **2014**. Vol. 79. №. 1. P. 60-88.

[4] Andrews U., Badaev S., and Sorbi A. A survey on universal computably enumerable equivalence relations // Computability and Complexity. Springer, Cham, **2017**. P. 418-451.

[5] Kalmurzayev B.S., Bazhenov N.A., Embeddability of m-degrees into equivalence relations in the Ershov hierarchy // News of the national academy of sciences of the Republic of Kazakhstan, physico-mathematical series. **2018**. Vol. 317, No. 1, P.14-17.https://doi.org/10.32014/2018.2518- 1726

[6] Montagna F. and Sorbi A. Universal recursion theoretic properties of r.e. preordered structures // Journal of Symbolic Logic. **1985**. Vol.50, № 2. P.397-406.

[7] Soare R.I. Recursively enumerable sets and degrees: A study of computable functions and computably generated sets. – Springer Science & Business Media, **1999**. 439p.

[8] Bazhenov N.A., Kalmurzaev B.S. On dark computably enumerable equivalence relations //Siberian Mathematical Journal. **2018**. Vol. 59. №. 1. P. 22-30.

[9] Badaev S. A. On weakly pre-complete positive equivalences //Siberian Mathematical Journal. **1991**. Vol. 32. №. 2. P. 321-323.

С.А. Бадаев[1], Б.С. Калмурзаев[1], Д.К. Кабылжанова[1], К.Ш. Абешев[2]

[1]Казахский национальный университет им.аль-Фараби, Алматы, Казахстан;
[2]AlmatyManagementUniversity, Алматы, Казахстан

УНИВЕРСАЛЬНЫЕ ПОЗИТИВНЫЕ ПРЕДПОРЯДКИ

Аннотация.В работе исследуются универсальные объекты в классе позитивных предпорядков относительно вычислимой сводимости, строится вычислимая нумерация этого класса, доказываются теоремы о существовании универсальной позитивной решеткии универсального слабо предполного предпорядка.

Ключевыеслова. Вычислимая сводимость на предпорядках, универсальный позитивный предпорядок, вычислимая нумерация, позитивная решетка, позитивный линейный порядок, слабо предполный позитивный предпорядок.

С.А.Бадаев[1], Б.С. Калмурзаев[1], Д.К. Кабылжанова[1], К.Ш. Абешев[2]

[1]әл-Фараби атындағы Қазақ ұлттық университеті, Алматы, Қазақстан;
[2]Алматы менеджмент университеті, Алматы, Қазақстан

УНИВЕРСАЛ ПОЗИТИВ ЖАРТЫ РЕТТЕР

Аннотация. Жұмыста позитив жарты реттер класында есептелімділік көшірулер бойынша универсал объектілері зерттеледі, бұл кластың есептелімді нөмірлеуі құрылады, универсал позитив торлар және универсал жартылай толық жарты реттер табылатындығы туралы теорема дәлелденеді.

Ключевые слова. Жарты реттердегі есептелімді көшіру, универсал позитив жарты рет, есептелімді нөмірлеу, позитив тор, позитив сызықтық рет, жартылай толық позитив жарты рет.

Information about the authors:

Badaev S.A. – al-Farabi Kazakh National University, Almaty, Kazakhstan; Institute of Information and Computational Technologies, Almaty, Kazakhstan.https://orcid.org/0000-0003-0444-2394;

Kalmurzayev B.S. – al-Farabi Kazakh National University, Almaty, Kazakhstan; Institute of Information and Computational Technologies, Almaty, Kazakhstan.https://orcid.org/0000-0002-4386-5915;

Kabilzhanova D.K. – al-Farabi Kazakh National University, Almaty, Kazakhstan; Institute of Information and Computational Technologies, Almaty, Kazakhstan.https://orcid.org/0000-0002-3996-3977;

AbeshevK.Sh. – Almaty Management University, Almaty, Kazakhstan; Institute of Information and Computational Technologies, Almaty, Kazakhstan.https://orcid.org/0000-0003-1140-7431.

N E W S
OF THE NATIONAL ACADEMY OF SCIENCES OF THE REPUBLIC OF KAZAKHSTAN
PHYSICO-MATHEMATICAL SERIES
ISSN 1991-346X **https://doi.org/10.32014/2018.2518-1726.18**
Volume 6, Number 322 (2018), 54 – 70

UDC 532.516

K. B. Jakupov

Kazak National University named after Al-Farabi
E-mail: jakupovKB@mail.ru

NATURAL FILTRATION EQUATIONS. FIASCO "OF DARCY'S LAW"

Abstract. The theory of natural filtration equations is given. The naturalness of the new filtration equations is that they are the exact consequences of the fundamental laws of physics, directly take into account the density and porosity of the soil, the viscosity and density of the filtration fluid, drainage, the influence of gravity, etc.the falsity of the traditional continuity equation in the filtration theory is Established. New filtration equations are derived from the equation of continuum dynamics in stresses, including the density and viscosity of the liquid and the porosity of the soil.Inadequacy of the modeling filter equations with the friction law of Newton. The efficiency of simulation of filtration by Jakupova equations based on the power laws of friction with odd exponents is numerically confirmed, with the use of which the calculations of filtration in the well, drainage under the influence of gravity, displacement of oil by water from the underground area through two symmetrically located pits are carried out.

Keywords: filtration, pressure, velocity, acceleration, equations.

Falsifications and contradictions of Darcy's law equations»

$$k\mathbf{v} = -gradp + \rho\mathbf{F},\ div\mathbf{v} = 0$$

with the fundamental laws of physics detailed in [1] and [13]. There is also considered the non-representativeness of the application of "Darcy's law" in the theory of spatial filtration. It shows the contradictions of equations "Darcy's law" law of friction and the second law of Newton. It is found that the spatial equations of "Darcy's law" correspond to potential flows, which contradicts the theory of viscous fluid. The equations of the "Darcy's law" do not comply with the law of conservation of energy. Based on the fact that the equations of the "Darcy's law" are composed of derivatives of the 1st order, contradictory problems of setting boundary conditions are revealed.

The equations of the Forchheimer model also contradict the laws of physics:

$$\rho_f \frac{\partial \mathbf{v}}{\partial t} = -\varphi[grad(p + \rho_f gz) + \rho_f \frac{\nu}{K}\mathbf{v} + \rho_f c_f K^{-\frac{1}{2}} |\mathbf{v}| \mathbf{v}],\ div\mathbf{v} = 0,$$

φ -coefficient of porosity of the medium, K – coefficient of permeability, ρ_f -is the fluid density, c_f - is the dimensionless coefficient of friction of Forchheimer. The viscosity of the liquid is also included ν.

The Forchheimer dynamics equation contains only local acceleration, but there is no transfer of medium particles $(\mathbf{v}\cdot\nabla)\mathbf{v}$, which is a *gross error*.

NumeroffC.H. as early as 1968г.paid attention to necessity of account acceleration (forces of inertia) in basic equalizations of theory of filtration and offered in [10] to use the next system of equalizations:

$$\frac{1}{g\sigma}\frac{\partial \mathbf{v}}{\partial t}+\frac{1}{(g\sigma)^2}(\mathbf{v}\cdot\nabla)\mathbf{v}+gradh+f(\mathbf{v})\mathbf{v}=0,\ div\mathbf{v}=0$$

The same Numeroff first specified on impermissibility of breaches of the second Newton's law, that says of : "mass on an acceleration equal to force". If an acceleration is equal to the zero, then on the first Newton's law a body accomplishes rectilineal motion with permanent speed or reposes. In the model of Нумерова, as well as in the model ofForchheimerthe first derivatives enter from the sought after functions, consequently, there is a problem of raising of regional terms.

1. Falseness of traditional equalization of indissolubility with the coefficient of porosity in the theory of filtration

In [3-5] and other in equalization of indissolubility is plug porosity

$$\frac{\partial s\rho}{\partial t}+div\rho\mathbf{v}=0$$

(In Wikipedia given in a form $\frac{\partial m\rho}{\partial t}+div\rho\mathbf{v}=0$). We will prove falsity of this equalization of indissolubility.

We will proceed from the fact that the elementary volume of the continuous medium is represented by the sum of the individual volumes of soil $\delta\tau_g$ and to the leakliquid $\delta\tau_f$: $\delta\tau=\delta\tau_g+\delta\tau_f$. Mass $\delta\tau$ is equal to the sum of the masses of soil and liquid $\delta m=\delta m_g+\delta m_f, \delta m_g=\rho_g\delta\tau_g, \delta m_f=\rho_f\delta\tau_f$..

Complete porosity is entered by attitude of volume $\delta\tau_f$ of pores toward a volume $\delta\tau$:

$$\mathrm{m}_0=\delta\tau_f/\delta\tau=\delta\tau_f/(\delta\tau_g+\delta\tau_f)$$

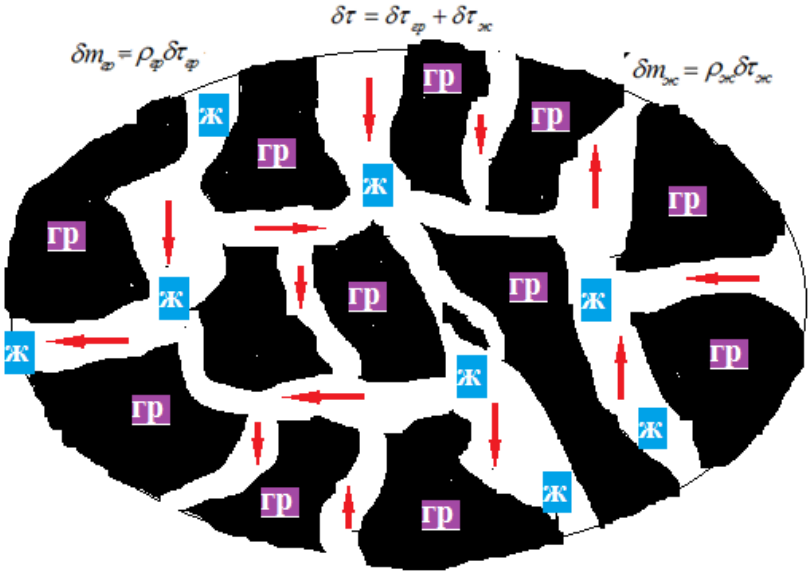
Effective porosity [3-5] there is a relation $s=\delta\tau_f/\delta\tau_g$.

1^0. Sources, flowsare contained in the volume of liquid $\delta\tau_f$.

Law of maintenance of mass taking into account by volume intensity J of flows and sources of liquid $\frac{d\delta m}{dt}=\delta m_f J$ we will present through the masses of soil and liquid (on a picture the elements of soil are drawn by a black):

$$\frac{d(\delta m_g+\delta m_f)}{dt}=\delta m_f J \qquad (1)$$

Soil, a liquid flows through the pores of that, is immobile and mass of him is permanent $\delta m_g=\rho_g\delta\tau_g=const.$ Therefore $\frac{d\delta m_g}{dt}=0.$



Consequently, from equalization (1) for a liquid it will be

$$\frac{d\delta m_f}{dt} = \delta m_f J,\ \frac{d\rho_f \delta\tau_f}{dt} = \rho_f \delta\tau_f J,\ \rho_f \frac{d\delta\tau_f}{dt} + \delta\tau_f \frac{d\rho_f}{dt} = \rho_f \delta\tau_f J$$

For locomotive $\mathbf{V}$ a formula takes place at a speed of liquid

$$\frac{d\delta\tau_f}{dt} = \delta\tau_f div\mathbf{v}.\ \text{Turns out}\ \rho_f \delta\tau_f div\mathbf{v} + \delta\tau_f \frac{d\rho_f}{dt} = \rho_f \delta\tau_f J.$$

From where equalization of indissolubility flows out with the intensity of sources and flows, being in a liquid

$$\rho_f div\mathbf{v} + \frac{d\rho_f}{dt} = \rho_f J$$

This equalization of indissolubility does not contain the coefficient of porosity, consequently, **does not coincide withequalizations of type**

$$\frac{\partial s\rho}{\partial t} + div\rho\mathbf{v} = 0$$

2^0. Sources and flows are contained in soil.

We have in this case

$$\frac{d\delta m}{dt} = \delta m_g J,\ \frac{d(\delta m_g + \delta m_g)}{dt} = \delta m_g J,\ \frac{d\delta m_g}{dt} = 0,$$

$$\frac{d\delta m_f}{dt} = \delta m_g J, \delta\tau_f(\rho_f div\mathbf{v} + \frac{d\rho_f}{dt}) = \rho_g \delta\tau_g J$$

Attributing both parts to the volume $\delta\tau_{gr}$ and using the effective coefficient of porosity, we get common equalization of indissolubility

$$\frac{d\rho_f}{dt} + \rho_f div\mathbf{v} = \frac{\rho_g J}{s}$$

For an incompressible liquid $\rho_f div\mathbf{v} = \frac{\rho_g J}{s}$, $div\mathbf{v} = \frac{\rho_g J}{s\rho_f}$.

Further passing to denotation $\rho_f \equiv \rho$, therefore $div\mathbf{v} = \frac{\rho_g J}{s\rho}$.

2.Natural equation of filtration in the soil

We will appeal to equalization of dynamics of individual volume of continuous environment in tensions [13]:

$$\frac{d}{dt}(\mathbf{v}\rho\delta\tau) = \mathbf{F}\rho\delta\tau + (\frac{\partial \mathbf{p}_x}{\partial x} + \frac{\partial \mathbf{p}_y}{\partial y} + \frac{\partial \mathbf{p}_z}{\partial z})\delta\tau$$

In a volume $\delta\tau = \delta\tau_g + \delta\tau_f$ soil is immobile, mass force only on motion ofliquid $\mathbf{F}\rho\delta\tau \Rightarrow \mathbf{F}\rho\delta\tau_f$,turns out therefore

$$\frac{d}{dt}[\mathbf{v}\rho(\delta\tau_g + \delta\tau_f)] = \mathbf{F}\rho\delta\tau_f + (\frac{\partial \mathbf{p}_x}{\partial x} + \frac{\partial \mathbf{p}_y}{\partial y} + \frac{\partial \mathbf{p}_z}{\partial z})\delta\tau$$

Soil is immobile $\frac{d}{dt}[\mathbf{v}\rho\delta\tau_g] = 0.$ Equalization assumes an air

$$\rho\delta\tau_f \frac{d\mathbf{v}}{dt} + \mathbf{v}\frac{d\rho\delta\tau_f}{dt} = \mathbf{F}\rho\delta\tau_f + (\frac{\partial \mathbf{p}_x}{\partial x} + \frac{\partial \mathbf{p}_y}{\partial y} + \frac{\partial \mathbf{p}_z}{\partial z})\delta\tau,$$

$$\rho\delta\tau_f \frac{d\mathbf{v}}{dt} + \mathbf{v}(\delta\tau_f \frac{d\rho}{dt} + \rho\frac{d\delta\tau_f}{dt}) = \mathbf{F}\rho\delta\tau_f + (\frac{\partial \mathbf{p}_x}{\partial x} + \frac{\partial \mathbf{p}_y}{\partial y} + \frac{\partial \mathbf{p}_z}{\partial z})\delta\tau,$$

where $\frac{d\delta\tau_f}{dt} = \delta\tau_f \mathrm{div}\mathbf{v}.$ Taking to $\delta\tau = \delta\tau_g + \delta\tau_f$,we get equalization of dynamics of liquid (filtrations) in soil in a general view

$$\frac{\delta\tau_f}{\delta\tau}[\rho\frac{d\mathbf{v}}{dt} + \mathbf{v}(\frac{d\rho}{dt} + \rho\mathrm{div}\mathbf{v})] = \mathbf{F}\rho\frac{\delta\tau_f}{\delta\tau} + (\frac{\partial \mathbf{p}_x}{\partial x} + \frac{\partial \mathbf{p}_y}{\partial y} + \frac{\partial \mathbf{p}_z}{\partial z}),$$

In equalization complete porosity is included in **natural way** " m_0 ":

$$\mathrm{m}_0 = \delta\tau_f / \delta\tau = \delta\tau_f / (\delta\tau_g + \delta\tau_f),\ 0 \le \mathrm{m}_0 \le 1,$$

$$\mathrm{m}_0[\rho\frac{d\mathbf{v}}{dt} + \mathbf{v}(\frac{d\rho}{dt} + \rho\mathrm{div}\mathbf{v})] = \mathrm{m}_0\rho\mathbf{F} + \frac{\partial \mathbf{p}_x}{\partial x} + \frac{\partial \mathbf{p}_y}{\partial y} + \frac{\partial \mathbf{p}_z}{\partial z}$$

The particles of liquid at motion in soil test braking from contiguity with the particulate matters of soil, that increases force of friction considerably $\mathbf{F}_{mpi} = -\frac{k}{s}\mathbf{v}_i$, $s = \frac{\delta\tau_f}{\delta\tau_g}$ – coefficient of porosity. (Blowing air or liquid through a millimeter-diameter tube requires a lot of effort compared to a centimeter-diameter tube.)

The theorem on the asymmetry of the stress tensor of a continuous medium, proved in [2-13], makes it possible to construct a wide spectrum of new rheological laws, from which it is possible to choose suitable models according to the flow velocity and the physical properties of the medium.

Let $u > 0$ and consider the frictional force proportional to the degree of velocity: $\mathbf{F}_{mp} = -\frac{k_u}{s}u^{m_u}\mathbf{i} - \frac{k_v}{s}v^{m_v}\mathbf{j} - \frac{k_w}{s}w^{m_u}\mathbf{k}$,whose projections on the *x* axis are equal to: $\mathbf{F}_1 = -\frac{k_u}{s}u_1^{m_u}\mathbf{i}$ on the plane y_1 , and $\mathbf{F}_2 = -\frac{k_u}{s}u_2^{m_u}\mathbf{i}$ on a plane y_2 .

Theincreasesofforceandspeedappearbetweenlayers:

$$\delta\mathbf{F} = \mathbf{F}_2 - \mathbf{F}_1,\ \delta\mathbf{F} = -\frac{k_{m_u}}{s}\delta u^{m_u}\mathbf{i},\ \delta u^{m_u} = u_2^{m_u} - u_1^{m_u} > 0,$$

thus $\delta\mathbf{F} \uparrow\downarrow \mathbf{i}$. The linear closeness of force is entered $\delta\mathbf{F}$ through a relation

$\mathbf{f} = \frac{\delta\mathbf{F}}{\delta y}, \delta\mathbf{F} = \delta y\mathbf{f}$. There is a vector on determination $\mathbf{p}_{yxcp} = \frac{\delta\mathbf{F}}{\delta x\delta z}$ tangent tension parallel and identically directed with forces of friction

$$\mathbf{p}_{yxcp} \uparrow\uparrow \delta\mathbf{F}, \mathbf{p}_{yxcp} \uparrow\uparrow \mathbf{f}.$$

Through the coefficient of proportion we have equalities $\mathbf{f} = k'\mathbf{p}_{yxcp},\ k' > 0,\ \mathbf{p}_{yxcp} \uparrow\downarrow \mathbf{i},$

$$k'\mathbf{p}_{yxcp}\delta y = -\frac{k_{m_u}}{s}\delta u^{m_u}\mathbf{i}.$$

This expression is multiplied by a thirl $\mathbf{i}$:

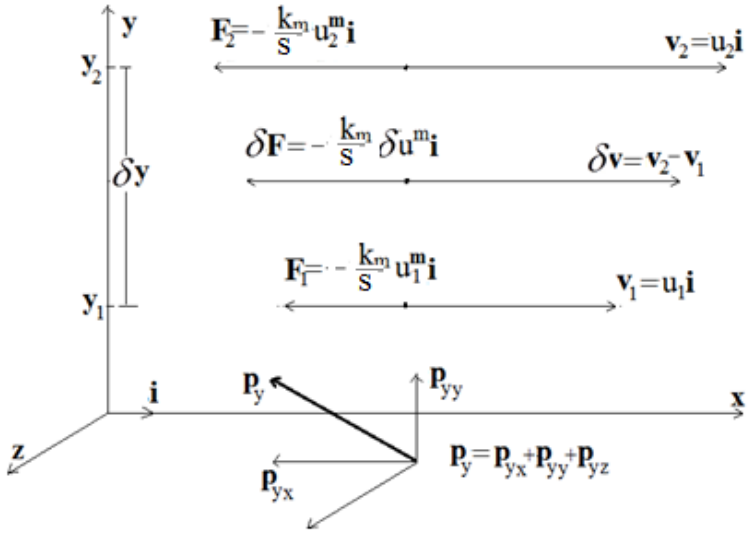
$$k'\delta y\mathbf{p}_{yxcp}\cdot\mathbf{i} = -\frac{k_{m_u}}{s}\delta u^{m_u}\mathbf{i}\cdot\mathbf{i}.$$

As a result $k'\mathbf{p}_{yxcp}\cdot\mathbf{i}\delta y = k'\left|\mathbf{p}_{yxcp}\right|\left|\mathbf{i}\right|\delta y\cos 180^o = k'p_{yxcp}\cdot 1\cdot\delta y\cdot(-1) = -k'p_{yxcp}\delta y,$

$$-\frac{k_{m_u}}{s}\delta u^{m_u}\mathbf{i}\cdot\mathbf{i} = -\frac{k_{m_u}}{s}\delta u^{m_u}\left|\mathbf{i}\right|\left|\mathbf{i}\right|\cdot\cos 0^o = -\frac{k_{m_u}}{s}\delta u^{m_u}\cdot 1\cdot 1\cdot 1 = -\frac{k_{m_u}}{s}\delta u^{m_u}$$

Equalities $-k'\delta yp_{yxcp} = -\frac{k_{m_u}}{s}\delta u^{m_u}$, $p_{yxcp} = \frac{k_{m_u}}{sk'}\frac{\delta u^{m_u}}{\delta y}$,в in a limit give tangent tension

$$p_{yx} = \lim_{\delta y\to 0} p_{yxcp},\ p_{yx} = \lim_{\delta y\to 0}\frac{k_{m_u}}{sk'}\frac{\delta u^{m_u}}{\delta y} = \frac{\mu_{m_u}}{s}\frac{\partial u^{m_u}}{\partial y},\ \mu_{m_u} = \frac{k_{m_u}}{k'}$$

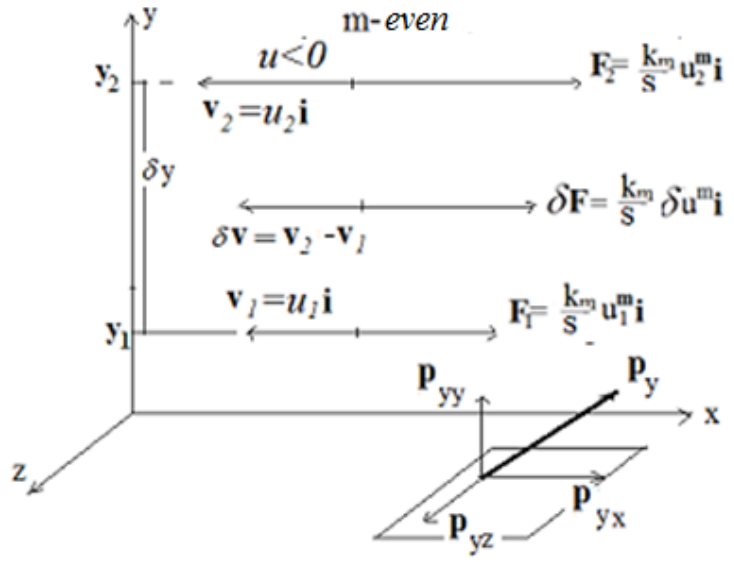


Generalizations of the got formula by transpositions of lower indexes give corresponding tangent tensions

$$p_{xy}=\frac{\mu_{m_v}}{s}\frac{\partial v^{m_v}}{\partial x},p_{xz}=\frac{\mu_{m_w}}{s}\frac{\partial w^{m_w}}{\partial x},p_{zx}=\frac{\mu_{m_u}}{s}\frac{\partial u^{m_u}}{\partial z},p_{yz}=\frac{\mu_{m_w}}{s}\frac{\partial w^{m_w}}{\partial y},p_{zy}=\frac{\mu_{m_v}}{s}\frac{\partial v^{m_v}}{\partial z}$$

Formulas are shown out, for the sake of simplicity, for $u>0,v>0,w>0$

and odd number indexes of degree $m=1;3;5;7;9...$. The same result turns out for an odd number m и $u\le 0,v\le 0,w\le 0$.



If *m- even* **and** $u<0,v<0,w<0$, **force of friction is equal**

$\mathbf{F}_{mp}=\frac{k_u}{s}u^{m_u}\mathbf{i}+\frac{k_v}{s}v^{m_v}\mathbf{j}+\frac{k_w}{s}w^{m_u}\mathbf{k}$ (look a picture), projections of that on an axis*x*equal:

$\mathbf{F}_1=\frac{k_u}{s}u_1^{m_u}\mathbf{i}$ on a plane y_1 and $\mathbf{F}_2=\frac{k_u}{s}u_2^{m_u}\mathbf{i}$ on a plane y_2.

The increases of force and speed appear between layers:

$\delta\mathbf{F} = \mathbf{F}_2 - \mathbf{F}_1,\ \delta\mathbf{F} = \frac{k_{m_u}}{s}\delta u^{m_u}\mathbf{i},\ \delta u^{m_u} = u_2^{m_u} - u_1^{m_u} > 0,$ thus

$\delta\mathbf{F} \uparrow\uparrow \mathbf{i}.$ The linear closeness of force is entered $\delta\mathbf{F}$ as a relation

$\mathbf{f} = \frac{\delta\mathbf{F}}{\delta y}, \delta\mathbf{F} = \delta y\mathbf{f}.$ There is a vector of tangent tension on determination

$\mathbf{p}_{yxcp} = \frac{\delta\mathbf{F}}{\delta x\delta z}$ parallel and identically directed with forces of friction

$$\mathbf{p}_{yxcp} \uparrow\uparrow \delta\mathbf{F}, \mathbf{p}_{yxcp} \uparrow\uparrow \mathbf{f}.$$

The coefficient of proportion gives equalities $\mathbf{f} = k'\mathbf{p}_{yxcp},\ k' > 0,\quad \mathbf{p}_{yxcp} \uparrow\uparrow \mathbf{i},$

$k'\mathbf{p}_{yxcp}\delta y = \frac{k_{m_u}}{s}\delta u^{m_u}\mathbf{i}.$

This expression is multiplied by a thirl $\mathbf{i}$: $k'\delta y\mathbf{p}_{yxcp}\cdot\mathbf{i} = \frac{k_{m_u}}{s}\delta u^{m_u}\mathbf{i}\cdot\mathbf{i}.$

As a result $k'\mathbf{p}_{yxcp}\cdot\mathbf{i}\delta y = k'\left|\mathbf{p}_{yxcp}\right|\left|\mathbf{i}\right|\delta y\cos 0^o = k'p_{yxcp}\cdot 1\cdot\delta y\cdot(1) = k'p_{yxcp}\delta y,$

$$\frac{k_{m_u}}{s}\delta u^{m_u}\mathbf{i}\cdot\mathbf{i} = \frac{k_{m_u}}{s}\delta u^{m_u}\left|\mathbf{i}\right|\left|\mathbf{i}\right|\cdot\cos 0^o = \frac{k_{m_u}}{s}\delta u^{m_u}\cdot 1\cdot 1\cdot 1 = \frac{k_{m_u}}{s}\delta u^{m_u}$$

Equalities $k'\delta yp_{yxcp} = \frac{k_{m_u}}{s}\delta u^{m_u},\quad p_{yxcp} = \frac{k_{m_u}}{sk'}\frac{\delta u^{m_u}}{\delta y}$,

in a limit give tangent tension

$$p_{yx} = \lim_{\delta y\to 0}\frac{k_{m_u}}{k'}\frac{\delta u^{m_u}}{\delta y} = \mu_{m_u}\frac{\partial u^{m_u}}{\partial y},\ \mu_{m_u} = \frac{k_{m_u}}{k'}$$

Id est the same result turns out.

Conclusion of viscid constituents of normal tensions

The analogical reasoning is set the formula of constituent $\mathbf{p}_{xx}^0$ normal tension $\mathbf{p}_{xx} = -p\mathbf{i} + \mathbf{p}_{xx}^0$.

Let forces of friction be equal: $\mathbf{F}_1 = -\frac{k_u}{s}u_1^{m_u}\mathbf{i}$ on a planex_1и $\mathbf{F}_2 = -\frac{k_u}{s}u_2^{m_u}\mathbf{i}$ on a plane

$x_2 = x_1 + \delta x$, $\delta\mathbf{F} = \mathbf{F}_2 - \mathbf{F}_1,\ \delta\mathbf{F} = -\frac{k_{m_u}}{s}\delta u^{m_u}\mathbf{i},\ \delta u^{m_u} = u_2^{m_u} - u_1^{m_u} > 0,$ thus $\delta\mathbf{F} \uparrow\downarrow \mathbf{i}.$

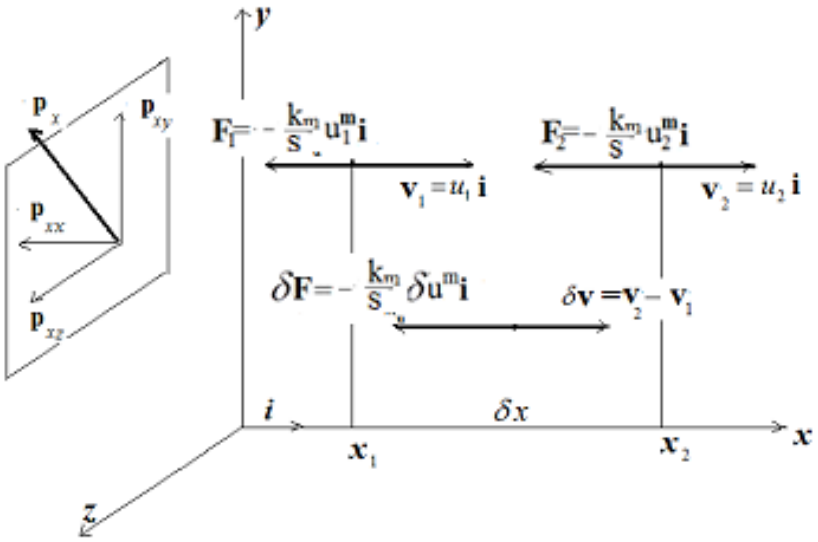
The linear closeness of force is equal: $\mathbf{f} = \frac{\delta\mathbf{F}}{\delta x}, \delta\mathbf{F} = \delta x\mathbf{f}.$

There is a vector of normal viscid tension on determination

$\mathbf{p}^0_{xxcp} = \dfrac{\delta \mathbf{F}}{\delta y \delta z}$ parallel and identically directed with forces of friction $\mathbf{p}^0_{xxcp} \uparrow\uparrow \delta\mathbf{F},\ \mathbf{p}^0_{xxcp} \uparrow\uparrow \mathbf{f}.$

The coefficient of proportion results in equalities $\mathbf{f} = k''\mathbf{p}^0_{xxcp},\ k'' > 0,$

$\mathbf{p}^0_{xxcp} \uparrow\downarrow \mathbf{i},\ k'\mathbf{p}_{xxcp}\delta x = -\dfrac{k_{m_u}}{s}\delta u^{m_u}\mathbf{i}.$



This expression is multiplied by a thirl $\mathbf{i}$: $k'\delta x\mathbf{p}_{xxcp}\cdot\mathbf{i} = -\dfrac{k_{m_u}}{s}\delta u^{m_u}\mathbf{i}\cdot\mathbf{i}.$

As a result $k'\mathbf{p}^0_{xxcp}\cdot\mathbf{i}\delta x = k'\left|\mathbf{p}^0_{xxcp}\right|\left|\mathbf{i}\right|\delta x\cos 180^o = k' p^0_{xxcp}\cdot 1\cdot\delta x\cdot(-1) = -k' p^0_{xxcp}\delta x,$

$-\dfrac{k_{m_u}}{s}\delta u^{m_u}\mathbf{i}\cdot\mathbf{i} = -\dfrac{k_{m_u}}{s}\delta u^{m_u}\left|\mathbf{i}\right|\left|\mathbf{i}\right|\cdot\cos 0^o = -\dfrac{k_{m_u}}{s}\delta u^{m_u}\cdot 1\cdot 1\cdot 1 = -\dfrac{k_{m_u}}{s}\delta u^{m_u}$ Equalities

$$-k'\delta x p^0_{xxcp} = -\frac{k_{m_u}}{s}\delta u^{m_u},\quad p^0_{xxcp} = \frac{k_{m_u}}{sk'}\frac{\delta u^{m_u}}{\delta x},$$

in a limit give viscid member of normal tension

$$p^o_{xx} = \lim_{\delta x\to 0}\frac{k_{m_u}}{sk''}\frac{\delta u^{m_u}}{\delta x} = \frac{\mu_{m_u}}{s}\frac{\partial u^{m_u}}{\partial x},\quad \mu_{m_u} = \frac{k_{m_u}}{k''} > 0$$

Generalizations of the got formula by transpositions of lower indexes give corresponding tensions

$$p^o_{yy} = \frac{\mu_{m_v}}{s}\frac{\partial v^{m_v}}{\partial y},\quad p^o_{zz} = \frac{\mu_{m_w}}{s}\frac{\partial w^{m_w}}{\partial z}$$

Obviously, complete normal tensions are the sum of hydrodynamic pressure andviscid constituents

$$p_{xx} = -p + p^o_{xx} = -p + \frac{\mu_{m_u}}{s}\frac{\partial u^{m_u}}{\partial y},\ p_{yy} = -p + p^o_{yy} = -p + \frac{\mu_{m_v}}{s}\frac{\partial v^{m_v}}{\partial y},$$

$p_{zz} = -p + p_{zz}^{o} = -p + \frac{\mu_{m_w}}{s}\frac{\partial w^{m_w}}{\partial z}$ Single values of indexes of degrees $m_u = 1, m_v = 1, m_w = 1$ correspond to the law of friction law of Newton. All exponentsmust be odd integers [2]. The filtrationequations for an incompressible fluid according to Newton's law of friction with coefficients of porosities in a gravity field have the form (x_3=z):

$$\mathrm{m}_0[\rho(\frac{\partial v_i}{\partial t} + \sum_{j=1}^{3} v_j \frac{\partial v_i}{\partial x_j}) + v_i \rho div\mathbf{v}] + \frac{\partial}{\partial x_i}(p + \mathrm{m}_0 \rho g z) = \frac{\mu}{s}\sum_{j=1}^{3}\frac{\partial^2 v_i}{\partial x_j^2}, i = 1,2,3,$$

relating to $\rho = const$ we find

$$\mathrm{m}_0[\frac{dv_i}{dt} + v_i \frac{\rho_g J}{s\rho}] = -\frac{\partial H}{\partial x_i} + \frac{\mu}{s\rho}\Delta v_i, i = 1,2,3, \quad H = \frac{p}{\rho} + \mathrm{m}_0 g z$$

The equations in dimensionless variables (with strokes) with the effective porosity coefficient in the continuity equation:

$$\mathrm{m}_0[\frac{dv'_i}{dt'} + v'_i \frac{\rho_g J\mathrm{L}}{s\rho \mathrm{U}}] = -\frac{\partial h'}{\partial x'_i} + \frac{\nu}{s\mathrm{UL}}\Delta v'_i, i = 1,2,3,$$

$$div\mathbf{v}' = \frac{\rho_g J\mathrm{L}}{s\rho\,\mathrm{U}} \qquad (2)$$

Thus, the equations include the density ρ_g soil and porosity coefficients, which emphasizes their significant effect on filtration.

Dimensionless numbers are formed:

$$\mathrm{m}_0[\frac{dv'_i}{dt'} + v'_i\,\mathrm{Ds}] = -\frac{\partial h'}{\partial x'_i} + \frac{1}{\mathrm{sRe}}\Delta v'_i,\ i = 1,2,3,\ div\mathbf{v}' = \mathrm{Ds},$$

$\mathrm{Ds} = \frac{\rho_g J\mathrm{L}}{s\rho\ \mathrm{U}}$, $\mathrm{Re} = \frac{\mathrm{UL}}{\nu}$ - Reynoldsnumber.

For sources and drains contained in liquid $\mathrm{Ds} = \frac{J\mathrm{L}}{\mathrm{U}}$. Here a new dimensionless number is given the name Darcy.

3. Modeling filtering equations with Newton's law of friction

In the well height*1000m*and wide*100m*with an impenetrable wall accumulated*200m*layer of water that displacesoil through the 800m layer of soil so that at the same time at the input upper boundary *y = 1000m* the water velocity is equal to 001m / s.The lower boundary of the soil, y = 0, is permeable to liquid.Filtration scales taken: cavity width *L = 100m,* water entry rate through the upper limit *U = 0.001 m / s,* water density $\rho_в = 1000кг/м^3$.Oil density $\rho_н = 880кг/м^3$, kinematic viscosity of water $\nu_в = 0.00556\cdot 10^{-4} м^2/с$, oil kinematic viscosity $\nu_н = 22.6\cdot 10^{-6} м^2/с$, effective soil porosity $s = 10^{-4}$, total porosity $\mathrm{m}_0 = 10^{-5}$. Power drain in the ground is such that $\mathrm{Ds} = -0,05$. Generalized density

$\rho=\alpha_в\rho_в+\alpha_н\rho_н$and kinematic viscosity of the medium $\nu=\alpha_в\nu_в+\alpha_н\nu_н$, $\alpha_в+\alpha_н=1$ calculated by the equation of transfer of water concentration [8]: $\frac{\partial\alpha_в}{\partial t}+\mathbf{v}\cdot\nabla\alpha_в=0$.

Dimensionless generalized density:

$$\rho'=\alpha_в+\alpha_н\rho_н/\rho_в,\quad \frac{\partial\alpha_в}{\partial t'}+\mathbf{v}'\cdot\nabla\alpha_в=0$$

The difference grid 100x200, a dimensionless time step, satisfies the condition of counting stability according to a semi-implicit five-point scheme without “circuit diffusion”.

Before the ground in the area from *800m* to *1000m* the numerical calculation was carried out using the dimensionless Navier equations

$$\rho'\frac{dv'_i}{dt'}=-\frac{\partial h'}{\partial x'_i}+\frac{1}{\mathrm{Re}_в}\Delta v'_i, i=1,2,3,\ \ div\mathbf{v}'=0$$

There is oil in the ground. Also used were the Navier equations, but with porosity coefficients:

$$\rho'\mathrm{m}_0\left(\frac{dv'_i}{dt'}+v'_i\,\mathrm{Ds}\right)=-\frac{\partial h'}{\partial x'_i}+\frac{1}{\mathrm{sRe}}\Delta v'_i, i=1,2,3,\ \ div\mathbf{v}'=\mathrm{Ds}$$

The location of water and oil at the initial moment of time is reflected in figure 1. Sticking conditions are set at the lateral boundaries of the well $\mathbf{v}'=0$, at the output boundary *y = 0,* the vertical derivatives of the velocity components are considered equal to 0. The water entering through the upper boundary *y = 1000m* begins to displace oil, the diagrams of the velocity vector fields in the cavity are shown in fig.2-4. The parabolic distribution of the velocity vector across the width of the channel in the ground can be seen in fig.2-4.Separately, fig. 5 shows a parabolic vertical velocity profile in one of the soil sections. The nonphysality of this flow in the ground is evident from the pictures of the velocity vector fields, which is the content of the output: the Navier equations (the old name is Navier-Stokes) are not a filtration model

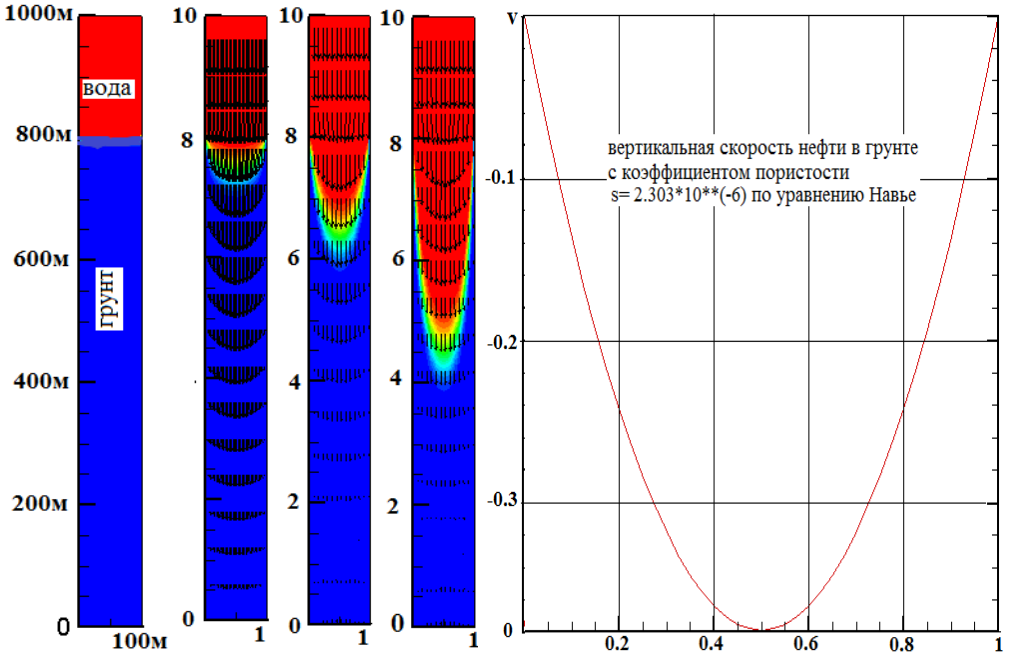


Fig.1 Fig.2 Fig.3 Fig.4 Fig..5

4.Filtration modeling by power friction equations

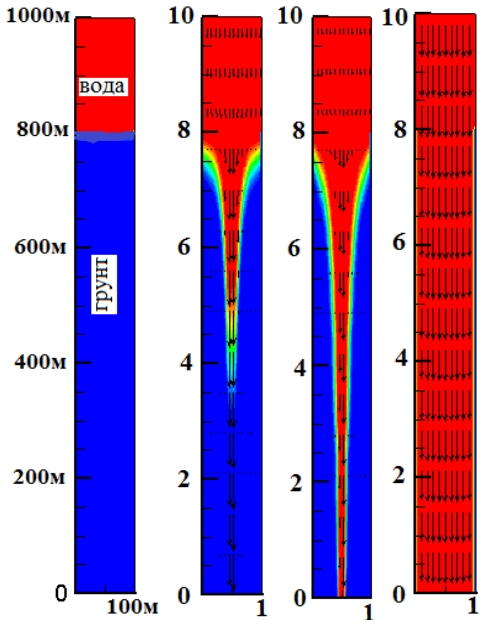


Fig.6 Fig.7 Fig.8

In this case, the dynamics of water over the ground is modeled by the equations of Jakupov's power laws of friction:

$$\rho\frac{dv_i}{dt}=-\frac{\partial p}{\partial x_i}+\sum_{j=1}^{3}\frac{\partial}{\partial x_j}\left(\frac{\alpha\mu}{m_i^{m_i-1}}\frac{\partial v_i^{m_i}}{\partial x_j}\right)+\rho F_i, i=1,2,3, div\mathbf{v}=0$$

This continuity equation corresponds to the fact that there are no sources and sinks in the flow of water. Equations in dimensionless variables have the form

$$\rho'\frac{dv'_i}{dt'}=-\frac{\partial h'}{\partial x'_i}+\sum_{j=1}^{3}\frac{1}{\mathrm{Re}}\frac{\partial}{\partial x'_j}\left[\left(\frac{\alpha U}{m_i}\right)^{m_i-1}m_i v_i'^{m_i-1}\frac{\partial v'_i}{\partial x'_j}\right], i=1,2,3, div\mathbf{v}'=0,\ \alpha=1\frac{сек}{м}$$

The exponents are determined in intervals[13-14:

$$|v_i|<1/9\ \ m_i=1;\ \ 1/9\le|v_i|<3/9\ \ m_i=3;\qquad(3)$$

$$3/9\le|v_i|<5/9\ \ m_i=5;\ \ 5/9\le|v_i|<7/9\ \ m_i=7;\ \ 7/9\le|v_i|\ \ m_i=9$$

The enormous resistance to movement (leakage) of a liquid in the soil is taken into account in the mass conservation equation and in the power law equations using the effective filtration coefficient s:

$$\rho'\mathrm{m}_0\left(\frac{dv'_i}{dt'}+v'_i Ds\right)=-\frac{\partial h'}{\partial x'_i}+\sum_{j=1}^{3}\frac{1}{s\,\mathrm{Re}}\frac{\partial}{\partial x'_j}\left[\left(\frac{\alpha U}{m_i}\right)^{m_i-1}m_i v_i'^{m_i-1}\frac{\partial v'_i}{\partial x'_j}\right], i=1,2,3,$$

$$div\mathbf{v}'=\mathrm{Ds},\quad \mathrm{Ds}=\frac{\rho_g JL}{s\rho U}$$

Initially, the determination of the degrees in the ground was carried out according to the distribution (3). The computed velocity fields in FIG. 6 and 7 show the unsuitability of this velocity distribution in the ground.In the ground, the equations of the vertical velocity proved to be effective as a constant exponent

$$m_i=17, i=1,2,3:$$

$$\rho' m_0 \left(\frac{dv'_i}{dt'} + v'_i Ds\right) = -\frac{\partial h'}{\partial x'_i} + \sum_{j=1}^{3} \frac{1}{s\,\mathrm{Re}} \frac{\partial}{\partial x'_j}\left[\left(\frac{\alpha U}{17}\right)^{16} 17 v'^{16}_i \frac{\partial v'_i}{\partial x'_j}\right], i = 1,2,3,$$

$$div\mathbf{v}' = Ds \tag{4}$$

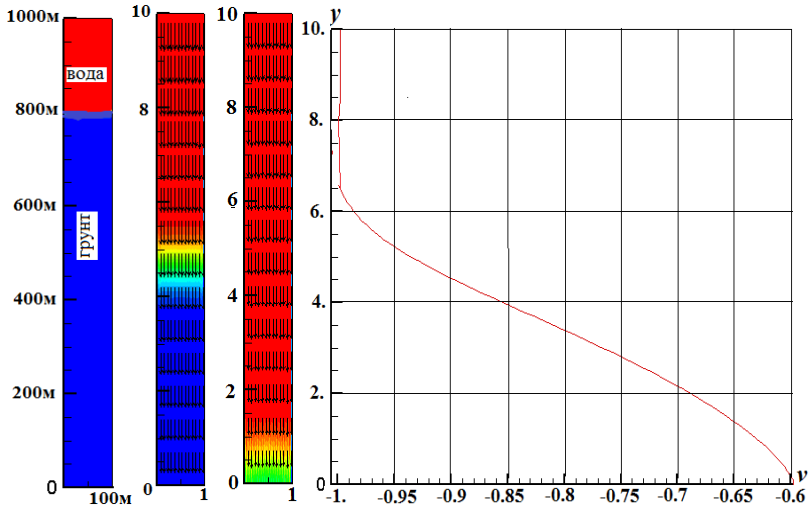


Fig.9 Fig.10 Fig.11

Fig.8 shows the field of the vector of water flow rates simply in the channel without soil, without sources-effluents, calculated according to the power law (2) and (4).

On fig. 9 and 10 show the velocity vector fields at different points in time. The difference from the parabolic velocity fields obtained by the Navier equations (-Stok`s) is simply amazing !!!Fig. 11 represents the vertical velocity distribution$_v$the height of the channel in the cross section x = 50 m.

On fig. 12 shows the trapezoidal profile of the vertical velocity v in the horizontal plane of the water y = *900m,* on fig. 13 in the horizontal plane of the soil y = *400m.*

In the ground, the speed of the filtration fluid is significantly reduced, as can be seen on fig. 9, 10, 11.

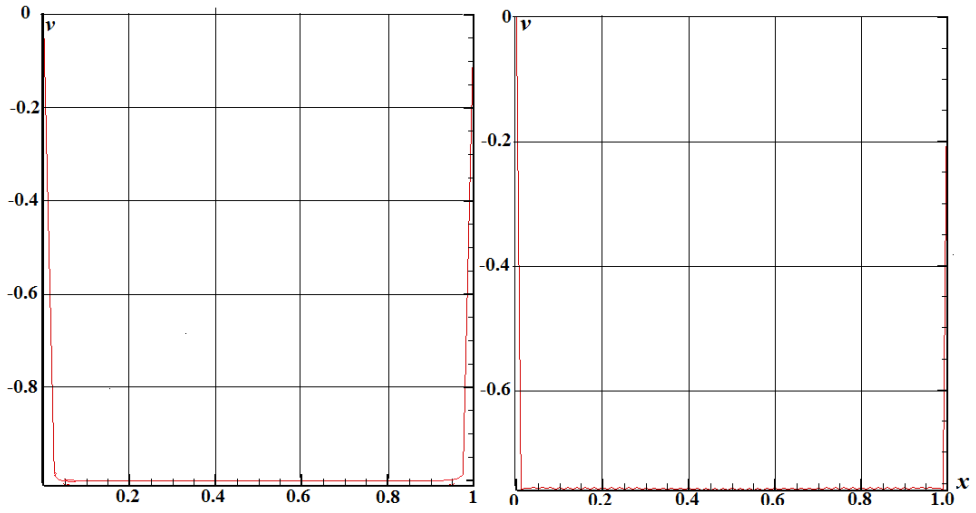


Fig.12 Fig.13

5. Filtration (drainage) under the action of gravity

Above are the results of calculations of forced filtration, when the upper limit of the channel *y=1000m* water is supplied at a speed of *0.001 m/s*.

In this paragraph, the numerical calculations of liquid filtration (drainage) under the influence of gravity, which acts on water from 800m to 1000m (the number of Froude is equal to $Fr=U^2/(gL)$), in the soil the effect of gravity is taken into account by the coefficient of total porosity "$m_0 = 0.00001$". In addition to the above scale, the scale of the speed adopted $U = 0.01\sqrt{gL}$, $g = 9.81 м/с^2$.

Before the ground $800м \le y \le 1000м$ the equations with the Froude number are applied (for the above reasons):

$$\rho' \frac{du'}{dt'} = -\frac{\partial p'}{\partial x'_i} + \sum_{j=1}^{2} \frac{1}{\mathrm{Re}} \frac{\partial}{\partial x'_j} [(\frac{\alpha U}{m_u})^{m_u - 1} m_u u'^{m_u - 1} \frac{\partial u'}{\partial x'_j}],$$

$$\rho' \frac{dv'}{dt'} = -\frac{\partial p'}{\partial x'_2} + \sum_{j=1}^{2} \frac{1}{\mathrm{Re}} \frac{\partial}{\partial x'_j} [(\frac{\alpha U}{m_v})^{m_v - 1} m_v v'^{m_v - 1} \frac{\partial v'}{\partial x'_j}] - \frac{\rho'}{Fr}, div\mathbf{v}' = 0, \ \alpha = 1 \frac{сек}{м}$$

In the ground, equations with a constant exponent are applied $m_i = 17, i = 1,2,3$ and with a total porosity coefficient are applied:

$$\mathrm{m}_0 \rho' (\frac{du'}{dt'} + u' Ds) = -\frac{\partial p'}{\partial x'_1} + \sum_{j=1}^{2} \frac{1}{s\,\mathrm{Re}} \frac{\partial}{\partial x'_j} [(\frac{\alpha U}{17})^{16} \cdot 17 \cdot u'^{16} \frac{\partial u'}{\partial x'_j}],$$

$$\mathrm{m}_0 \rho' (\frac{dv'}{dt'} + v' Ds) = -\frac{\partial p'}{\partial x'_2} + \sum_{j=1}^{2} \frac{1}{s\,\mathrm{Re}} \frac{\partial}{\partial x'_j} [(\frac{\alpha U}{17})^{16} \cdot 17 \cdot v'^{16} \frac{\partial v'}{\partial x'_j}] \frac{\rho' \mathrm{m}_0}{Fr}, div\mathbf{v}' = Ds$$

Fig.14 and 15 are the fields of the velocity vector at different times of oil displacement by water. Fig. 16 represents the distribution of the vertical velocity *v* along the height of the channel in the cross section x=50 m. the Difference between the distributions of the vertical velocities in Fig. 11 and Fig.16 the obvious.

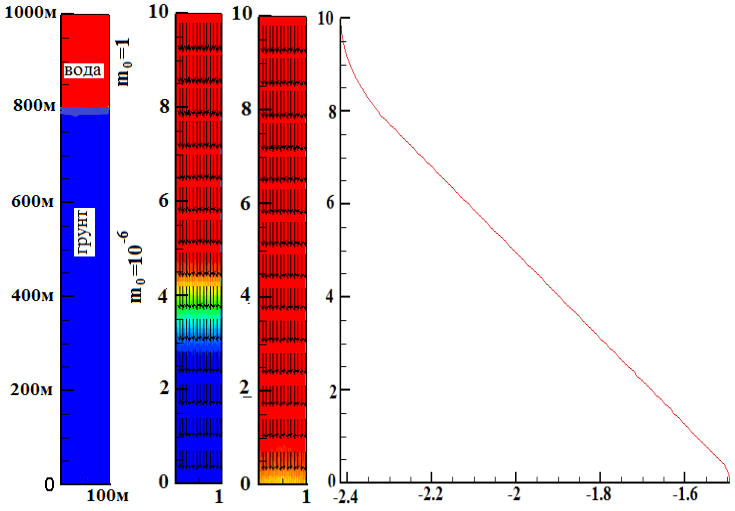


Fig.14 Fig.15 Fig..16

6. Displacement of oil by water from underground volume

Before the ground in the water are applied (for the above reasons) equations with the number of Froude:

$$\rho'\frac{du'}{dt'}=-\frac{\partial p'}{\partial x'_i}+\sum_{j=1}^{2}\frac{1}{\mathrm{Re}}\frac{\partial}{\partial x'_j}[(\frac{\alpha U}{m_u})^{m_u-1}m_u u'^{m_u-1}\frac{\partial u'}{\partial x'_j}],$$

$$\rho'\frac{dv'}{dt'}=-\frac{\partial p'}{\partial x'_2}+\sum_{j=1}^{2}\frac{1}{\mathrm{Re}}\frac{\partial}{\partial x'_j}[(\frac{\alpha U}{m_v})^{m_v-1}m_v v'^{m_v-1}\frac{\partial v'}{\partial x'_j}]-\frac{\rho'}{Fr},div\mathbf{v}'=0,\ \alpha=1\frac{сек}{м}$$

Water is supplied to the well at a rate U=2m / s (figures.17). In linear scale taken as the diameter of the borehole *L=1 m.* the Reynolds Number is equal to $\mathrm{Re}=\alpha_в\,\mathrm{Re}_в+\alpha_н\,\mathrm{Re}_н$, where $\mathrm{Re}_в$ - Reynolds number for water, $\mathrm{Re}_н$ - Reynolds number for oil.In the ground, equations with a constant exponent are applied $m_u=11,\ m_v=11$ and with the ratio of **total porosity** $\mathrm{m}_0=0.000021$:

$$\rho'\mathrm{m}_0(\frac{du'}{dt'}+u'Ds)=-\frac{\partial p'}{\partial x'_1}+\sum_{j=1}^{2}\frac{1}{s\,\mathrm{Re}}\frac{\partial}{\partial x'_j}[(\frac{\alpha U}{11})^{10}\cdot 11\cdot u'^{10}\frac{\partial u'}{\partial x'_j}],$$

$$\rho'\mathrm{m}_0(\frac{dv'}{dt'}+v'Ds)=-\frac{\partial p'}{\partial x'_2}+\sum_{j=1}^{2}\frac{1}{s\,\mathrm{Re}}\frac{\partial}{\partial x'_j}[(\frac{\alpha U}{11})^{10}\cdot 11\cdot v'^{10}\frac{\partial v'}{\partial x'_j}]-\frac{\mathrm{m}_0\rho'}{Fr},\ div\mathbf{v}'=\mathrm{Ds}$$

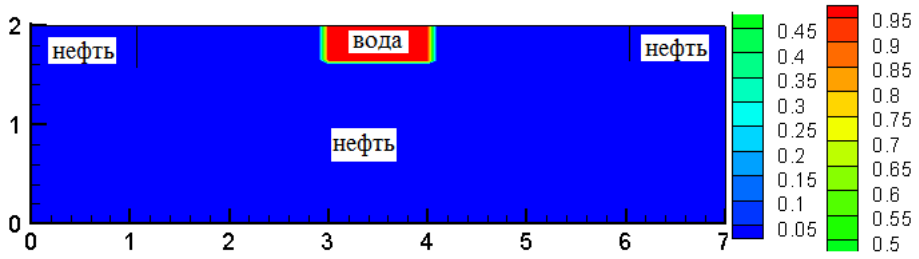


Fig.17

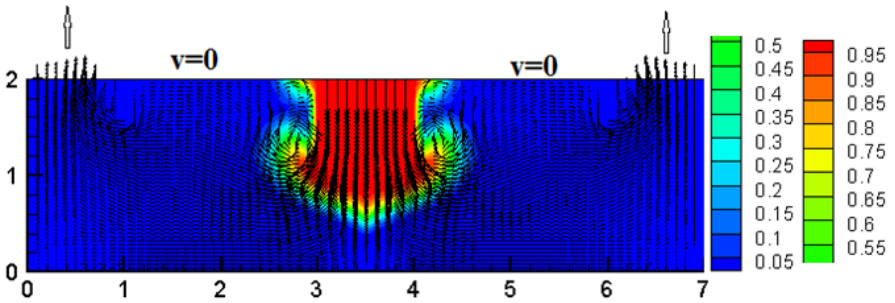


Fig.18

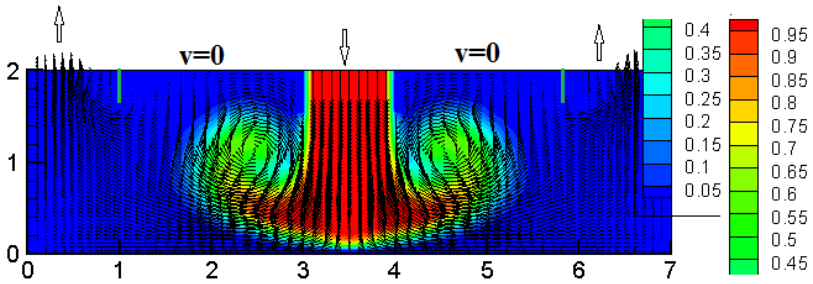


Fig.19

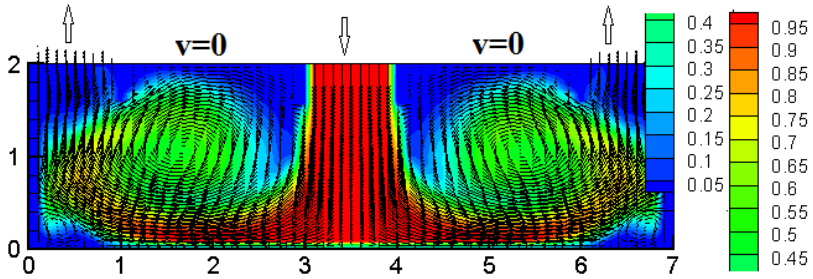


Fig.20

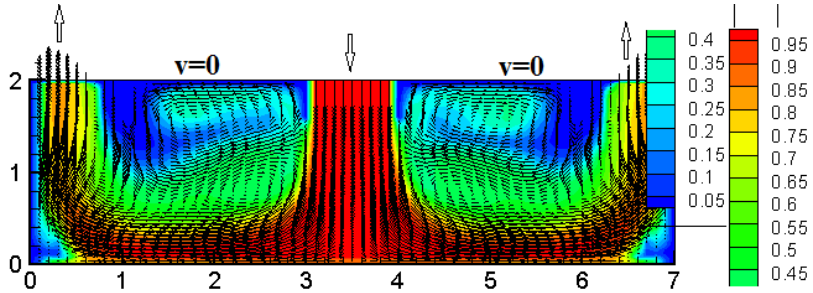


Fig.21

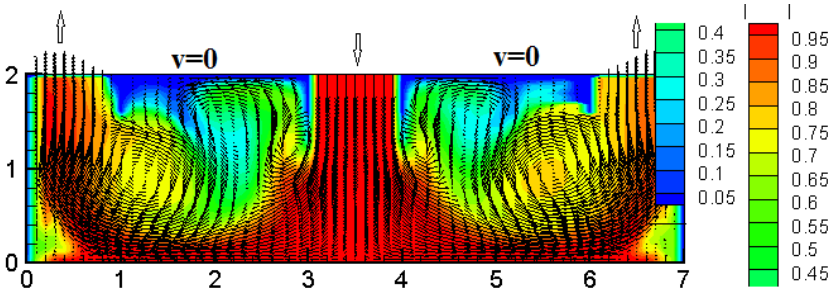


Fig.22

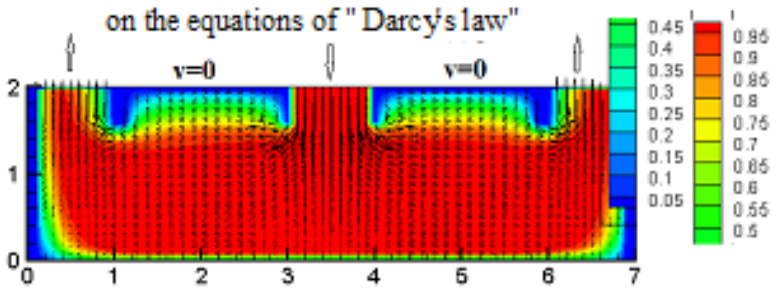


Fig.23

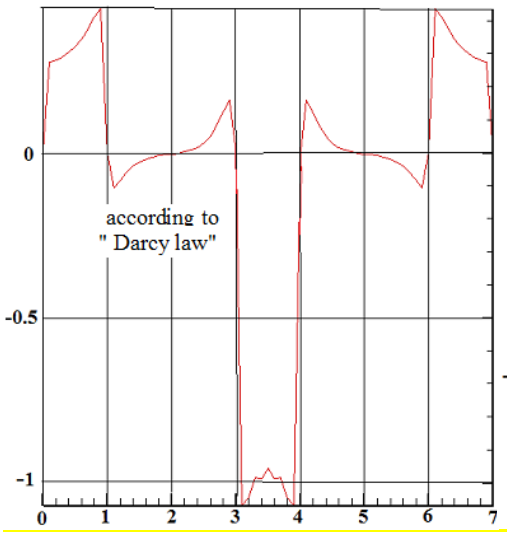


Fig.24

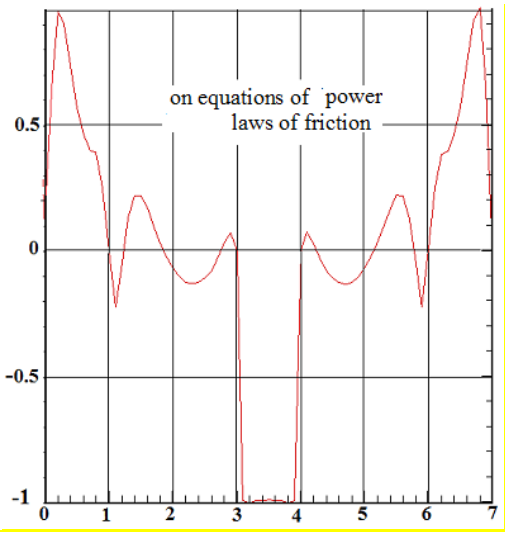


Fig.25

Fig.18-22 the pictures of water penetration into the soil with simultaneous oil displacement through 2 vertical pits $0<x'<1$ and 6<x'<7 are presented.Fig.23 the field of the velocity vector and the location of water at almost complete displacement of oil obtained by the equations of "Darcy's law"arepresented. Horizontal and vertical arrangement of velocity vectors $\mathbf{v}' = u'\mathbf{i} + v'\mathbf{j}$ fig.23 caused by neglect of the forces of inertia (acceleration) in the equations of "Darcy's law".The influence of the inertia forces to the filter demonstrated with paintings of fields of velocity vectors in Fig. 18-22.

Fig.24 and 25 shows a plot of the vertical velocity at the height of the wells

REFERENCES

[1] Jakupov K.B. COMPILATIVITY OF THE “DARCY LAW” IN THE THEORY OF FILTRATION // News of the National Academy of Sciences of Kazakhstan, Ser. Phys. Mat., 6 (316) 2017. P.115-131. ISSN 2518-1726 (Online) ISSN 1991-346X (Print)

[2] Jakupov K.B. GENEALOGY OF THE STOCK AND NAVIER EQUATIONS. POWERED RHEOLOGICAL LAWS AND EQUATIONS // News of the National Academy of Sciences of Kazakhstan, ser. Phys.-Math., 3 (313) 2017. C.64-81. ISSN 2518-1726 (Online) ISSN 1991-346X (Print)

[3] Basniev V.S., Kochina I.N., Maksimov V.M. Underground fluid mechanics. M .: Nedra, 1993. 416s.

[4] Ponomareva I.N., Mordvinov V.A. UNDERGROUND HYDROMECHANICS. -Perm 2009

[5] Polubarinova-KochinaP.Ya. Theory of groundwater movement. -M .: Publishing House "Science", 1977. P.664.

[6] H.Darcy. Les fontainespubliques de la ville de Dijon. Paris, 1856.

[7] Loitsyansky L.G. Fluid and gas mechanics. -M .: "Science", 1973.

[8] Aravin V.I. and Numerov S.N. Theory of motion of liquids and gases innon-deformable porous medium. M .: Gostekhizdat. 1953 P.616.

[9] Nied D.A., Bejan A. Convection in Porous Media. N. Y. etc: Springer, 1992. 408p. [ten].

[10] Numerov S.N. On the need to take into account the forces of inertia in the basic equations of the theory filtering // In Sat: Vopr. applied mathematics and geometric modeling. Leningrad Ing. -p. in-t, 1968, p.18-21.

[11] C.W.Hirt and B.D.Nichols. Volume of fluid (VOF) method boundaries // J.Comput.Phys., 1981, Vol.39, pp.201-225.

[12] Jakupov K.B. REPRESENTATIVITY OF THE METHOD OF FICTIVE AREAS IN HYDRODYNAMICS // News of the National Academy of Sciences of Kazakhstan, Phys. Mat, 6 (316) 2017. Pp.85-96. ISSN 2518-1726 (Online) ISSN 1991-346X (Print).

[13] Jakupov K.B. LIQUIDATION OF FALSIFICATIONS and MODERNIZATION OF THE BASES OF CONTINUOUS MECHANICS.-Almaty: “UlymOrdasy” Printing House, 2017.P.435. ISBN 978-601-280-859-9.

[14] Jakupov K.B. RHEOLOGICAL LAWS OF VISCOUS FLUID DYNAMICS // Proceedings of the National Academy of Sciences of the Republic of Kazakhstan, main body-mat, 1 (293), 2014, pp. 51-55. I SSN 1991-346X.

[15] Jakupov K.B. . Simulation of thermobarodiffusions with chemical reactions in liquids and gases // Proceedings of the National Academy of Sciences of Kazakhstan, series physical-mathematical., 6 (310), 2016, pp. 80-88. ISSN 2518-1726.ISSN 1991-346X.

DOI https://doi.org/10.32014/2018.2518-1726

К. Б. Жақып-тегі

ҚР БҒМ Математика және математикалық моделдеу институты, Алматы, Қазақстан

СҮЗГІНІҢ ТАБИҒАТТЫҚ ТЕҢДЕУЛЕРІ. «Дарси ЗАҢЫНЫҢ» ҚҰРЫҒАНЫ

Аннотация.Сүзгінің табиғаттық теңдеулерінің теориясы қорытлған.Жаңа теңдеу- лердің табиғаттығы мынада тұр: олар физиканың негізгі заңдарынан шыққан нақты сал- дары, грунттың тығыздығын және кеуектілігін, сүзбелі сұйықтықтың тұтқырлығы мен тығыздығын, дренажды, аурылық күштің әсерін тікелей есепке алғандығы. Сүзгі теория- сында пайдаланатын үзіліссіздік теңдеуідің жалғандығы көрсетілген. Сүзгінің жаңа тең- деулері тұтас ортаның динамикасының керенеулер арқылы жазылған теңдеулерінен шыға- рылған, олрадың құрылысына сұйықтықтың тұтқырлығы мен тығыздығы жәнегрунттың кеуектілігі еңгізілген.. Сүзгіні модельдеуге Ньютонның үйкеліс заңына сәйкес құрылған теңдеулерді пайдаланудың жәрәмсіздігі көрсетілген.Сүзгіні модельдеуге автордың дәрежелі үйкеліс заңдарына дәрежесі дақ сандарболған теңдеулерін пайдалану жәрәмді екені сандық түрде әшкереленген. Осыларды жұмсап скважинадағы сүзгі, ауырлық күш- пен болатын дренажды, жердің астындағы қоймадан мұнайды екі беттескен шурфтар арқылы сығып шығару сандық есептермен орындалған. Библ.18.

Тірек сөздер: сүзгі, қысым, жылдамдық, үдеу.

К.Б.Джакупов

Казахский Национальный Университет им.Аль-Фараби

УРАВНЕНИЯ ЕСТЕСТВЕННОЙ ФИЛЬТРАЦИИ. ФИАСКО "ЗАКОНА ДАРСИ"

Аннотация. Дана теориянатуральныхуравнений фильтраций. Натуральность новых уравнений фильтраций заключается в том, что они являются точными следствиями фунда- ментальных законовфизики, прямо учитывают плотность и пористость грунта, вязкость и плотность фильтрационной жидкости, дренаж, влияние силы тяжести и др. Установлена фальшивость традиционного уравнения неразрывности в теории фильтрации. Из уравне-ния динамики сплошной среды в напряжениях выводятся новые уравнения фильтрации, включающие плотность и вязкость жидкости и пористость грунта.Установленанеадекватность моделирования фильтрации уравнениями с законом трения Ньютона.Численно подтверждена эффективность моделированияфильтрации уравнениями Джакупова, осно- ванных на степенных законах трения с нечетными показателями степеней, с применением которых проведены расчеты фильтрации в скважине, дренажа под действием силы тяжести**,**вытеснениянефти водою из подземного ареала через две симметрично расположенные шурфы.

.**Ключевые слова:** фильтрация,давление, скорость, ускорение, уравнения.

About the author:

Dzhakupov Kenes Baikenovich - doctor of physico-mathematical Sciences, Professor, academician Office address: RSE Institute of mathematics and mathematical modeling KM MES RK, 050010, Pushkin street, 125, Almaty, Kazakhstan. E-mail: jakupovKB@mail.ru

МАЗМҰНЫ

СОДЕРЖАНИЕ

CONTENTS

Publication Ethics and Publication Malpractice in the journals of the National Academy of Sciences of the Republic of Kazakhstan

For information on Ethics in publishing and Ethical guidelines for journal publication see http://www.elsevier.com/publishingethics and http://www.elsevier.com/journal-authors/ethics.

Submission of an article to the National Academy of Sciences of the Republic of Kazakhstan implies that the described work has not been published previously (except in the form of an abstract or as part of a published lecture or academic thesis or as an electronic preprint, see http://www.elsevier.com/postingpolicy), that it is not under consideration for publication elsewhere, that its publication is approved by all authors and tacitly or explicitly by the responsible authorities where the work was carried out, and that, if accepted, it will not be published elsewhere in the same form, in English or in any other language, including electronically without the written consent of the copyright-holder. In particular, translations into English of papers already published in another language are not accepted.

No other forms of scientific misconduct are allowed, such as plagiarism, falsification, fraudulent data, incorrect interpretation of other works, incorrect citations, etc. The National Academy of Sciences of the Republic of Kazakhstan follows the Code of Conduct of the Committee on Publication Ethics (COPE), and follows the COPE Flowcharts for Resolving Cases of Suspected Misconduct (http://publicationethics.org/files/u2/New_Code.pdf). To verify originality, your article may be checked by the Cross Check originality detection service http://www.elsevier.com/editors/plagdetect.

The authors are obliged to participate in peer review process and be ready to provide corrections, clarifications, retractions and apologies when needed. All authors of a paper should have significantly contributed to the research.

The reviewers should provide objective judgments and should point out relevant published works which are not yet cited. Reviewed articles should be treated confidentially. The reviewers will be chosen in such a way that there is no conflict of interests with respect to the research, the authors and/or the research funders.

The editors have complete responsibility and authority to reject or accept a paper, and they will only accept a paper when reasonably certain. They will preserve anonymity of reviewers and promote publication of corrections, clarifications, retractions and apologies when needed. The acceptance of a paper automatically implies the copyright transfer to the National Academy of Sciences of the Republic of Kazakhstan.

The Editorial Board of the National Academy of Sciences of the Republic of Kazakhstan will monitor and safeguard publishing ethics.

Правила оформления статьи для публикации в журнале смотреть на сайтах:

www:nauka-nanrk.kz

http://www.physics-mathematics.kz

ISSN 2518-1726 (Online), ISSN 1991-346X (Print)

Редакторы *М. С. Ахметова, Т.А. Апендиев, Д.С. Аленов*
Верстка на компьютере *А.М. Кульгинбаевой*

Подписано в печать 05.12.2018.
Формат 60x881/8. Бумага офсетная. Печать – ризограф.
4,75 п.л. Тираж 300. Заказ 6.

Национальная академия наук РК
050010, Алматы, ул. Шевченко, 28, т. 272-13-18, 272-13-19