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CONTENTS

INFORMATION AND COMMUNICATION TECHNOLOGIES

МАЗМҰНЫ

АҚПАРАТТЫҚ-КОММУНИКАЦИЯЛЫҚ ТЕХНОЛОГИЯЛАР

СОДЕРЖАНИЕ

ИНФОРМАЦИОННО-КОММУНИКАЦИОННЫЕ ТЕХНОЛОГИИ

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MODELING OF VIDRATIONAL PROCESSES IN WOLFRAM MATHEMATICA SYSTEM

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Abstract. The tower creates oscillations of the sleeve, during which the movement of the opposite edges can go in different directions, and the resulting compressive stresses alternate with tensile stresses. The constant alternation of one stress with another can cause fatigue of the material. The study of wave and vibration processes in the tower, taking into account its interaction with the surrounding layers, is of great importance in determining the strength of the structure. *Results.* As a result of these studies, the consideration of stationary, non-stationary, oscillatory and wave processes, mechanics of deformable solids, hydrodynamics, and geophysical sciences leads to good achievements. *Scientific novelty.* As a result of the research work carried out: - new stages of unsteady oscillations are theoretically investigated; - mathematical methods for studying various classes of spatial calculations are discussed.; - a model of the conditions of inclined oscillations in a drill string using the MAPLE system has been developed. *Practical value.* The results obtained lead to significant achievements in the field of mechanics of deformable solids, hydrodynamics and geophysics in the study

of stationary, non-stationary, oscillatory and wave processes. The topics discussed in the paper include modeling inclined vibrations of a drill string in the MAPLE system and theoretical study of new stages in the field of unstable vibrations, as well as the development of a new model of inclined vibrations of a drill string.

Keywords: model, drilling, wave theory, friction limits, numerical solutions, drilling column, wolfram mathematics

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ТЕРБЕЛІСТЕР ҮДЕРІСІН WOLFRAM MATHEMATICA ЖҮЙЕСІНДЕ МОДЕЛДЕУ

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Аннотация. Мақалада бұрғылау мұнарасы иын тербелістерді жасайды, бұл кезде қарсы орналасқан шеттерінің қозғалысы әр түрлі бағытта жүруі мүмкін, ал пайда болған сығылу кернеуі созылумен ауысады. Бір кернеуді басқалармен үнемі ауыстыру материалдың қажуының пайда болуын тудыруы мүмкін. Сол уақытта ұңғыма қабырғасы мен сазды ертіндінің жоғары тұтқырлығымен бұрғылау құбырының үйкелісі діріл бәсеңдеуіне себепші болады. Сондықтан оның қоршаған қатпарлармен әрекеттесуі есебімен мұнарадағы толқынды және тербелмелі үдерістерді зерттеу, құрылымның беріктігін қамтамасыз етуде өте маңызды. Жұмыста, динамикалық жүктемедегі мұнаралардың динамикалық қимылының заңдылығының табылуы – ұстап қалуды жою бойынша жұмыстар мен бұрғылау жұмыстарының сәтті жүргізілуіне қажетті жағдай жасау қарастырылған. *Нәтижелері*. Стационарлы, стационарлы емес, тербелмелі және толқынды процестердің қарастырылуы, деформацияланатын қатты дененің механикасы, гидродинамика, геофизика ғылымдарының бөлімдерінде жақсы жетістіктерге алып келеді. *Ғылыми жаңалығы*. Жүргізілген ғылыми зерттеу жұмыстарының нәтижесінде: - стационарлық

емес тербелістердің жаңа кезеңдері теориялық зерттелді; - кеңістіктік есептеулердің әртүрлі кластарын зерттеу үшін математикалық әдістері талқыланды; - MAPLE жүйесін қолдана отырып, бұрғылау бағанындағы көлбеу тербеліс жағдайларының моделі әзірленді. *Практикалық құндылық.* Алынған нәтижелер стационарлық, стационарлық емес, тербелмелі және толқындық процестерді зерттей отырып, деформацияланатын қатты заттар механикасы, гидродинамика және геофизика саласында айтарлықтай жетістіктерге әкеледі. Жұмыста қарастырылған MAPLE жүйесіндегі бұрғылау бағанының көлбеу тербелістерін модельдеуді және тұрақсыз тербелістер саласындағы жаңа кезеңдерді теориялық зерттеуді, сондай-ақ бұрғылау бағанындағы көлбеу тербеліс жағдайлары үшін жаңа модельді әзірлеуді қамтиды.

Түйін сөздер: модель, бұрғылау, толқындар теориясы, тежегіш шек, сандық шешімі, бұрғылау бағанасы, вольфрам математика

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МОДЕЛИРОВАНИЕ ВИБРАЦИОННОГО ПРОЦЕССА В СИСТЕМЕ WOLFRAM MATHEMATICA

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Аннотация. Вышка создает колебания рукова, при которых движение противоположных кромок может идти в разные стороны, а возникающие при этом сжимающие напряжения чередуются с растягивающими.Постоянное чередование одного напряжения с другим может вызвать усталость материала. В то же время трение бурильной трубы о высокой вязкости стенки скважины и глинистого раствора приводит к снижению вибрации. Поэтому исследование волновых и вибрационных процессов в башне с учетом ее взаимодействия с окружающими слоями имеет большое значение при определении прочности

конструкции.При работе выяснение законности динамического перемещения башен под действием динамической нагрузки необходимо для создания оптимальных условий для успешного выполнения работ по устранению ретенций и бурения. *Результаты.* В результате данных исследований к хорошим достижениям приводит рассмотрение стационарных, нестационарных, колебательных и волновых процессов, механики деформируемого твердого тела, гидродинамики, геофизических наук. *Научная новизна.* В результате проведенных научно-исследовательских работ: - теоретически исследованы новые этапы нестационарных колебаний; - обсуждены математические методы для изучения различных классов пространственных вычислений; - разработана модель условий наклонных колебаний в бурильной колонне с использованием системы MAPLE. *Практическая ценность*.Полученные результаты приводят к значительным достижениям в области механики деформируемых твердых тел, гидродинамики и геофизики при изучении стационарных, нестационарных,колебательных и волновых процессов. Рассмотренные в работе включают моделирование наклонных колебаний бурильной колонны в системе MAPLE и теоретическое исследование новых этапов в области неустойчивых колебаний, а также разработку новой модели условий наклонных колебаний бурильной колонны.

Ключевые слова: модель, бурение, теория волн, пределы трения, численные решения, буровая колонна, вольфрам математика

Introduction. The theoretical foundation of spring system dynamics is extensively discussed in classical monographs (Khajiyeva, et al, 2015). Analyzing the dynamic equation, Euler’s dynamic equation was examined by Academician L.S. Leibenzon, who first demonstrated that the positioning of hydraulic motors in the casing weakens the fixation of uncontrolled lateral displacement of drilling rigs (Bulatov, et al,1985). The stability of drilling rigs and the study of rotational drilling dynamics were researched by A.N. Dinnik. The strength calculations and stability of drilling rigs were discussed in his works. The theoretical understanding of wellbore deflection and the rig’s response was heavily influenced by flat stability models. To this day, most calculations for drilling rigs are based on this model. This work showed that static analysis only provides satisfactory results in specific cases. Wave phenomena and their effect on drilling rig dynamics, as well as the operational patterns of drilling pipes, have been addressed in established works.

The interaction between the drilling rig pipes and the wellbore walls, the adhesion forces, and the resistance forces in deflected wells have been studied in (Shashenko, 2017). In (Vasidzu,1987), the dynamic forces involved in the interaction of drilling pipes were explored through linear relations. The rigidity of the pipe material and the relation to the longitudinal progression coefficient are critical in defining areas where pipe sections may lack vibrational motion, causing formation zones. The dynamic aspects of drilling rig pipe motion, including internal and external friction, have been discussed in works (Simonov, et a ,1977) and (Barskij,1983).

The theoretical principles of drilling rig dynamics and its stability, along with methods of practical implementation, were explored in the works of I.L. Barsk (Barskij, 1985). His observations showed that the moment of rotation's effect on the rig depends not only on the magnitude of the rotation but on the change in the system's equilibrium, specifically when the rig transitions into a flutter motion. The energy applied to the rig transforms into oscillatory energy. The interaction with the wellbore walls reduces the amplitude of the rig's motion and leads to processional movement. The rig then impacts the wellbore walls, leading to the formation of a multi-angled bottom, causing stress and failure.

A critical dynamic task is the study of oscillations in the rig's core, particularly involving coordinative parameters such as centralizers, calibrators, and various other elements. The investigation of longitudinal and transverse vibrations in long interconnected pipe sections has been studied in works (Yasov,1991). Much of the work focuses on drilling apparatus control instruments and modern methods. The evaluation of physical processes during well construction and operation highlights the difficulties faced in drilling, where practical experience is highly valued. It is in these real-world experiences that the final technological decisions are made. Techniques to prevent deflection in the wellbore, based on practical drilling data, are outlined in (Aleksandrov, 1970).

The study of wave phenomena in drilling rigs is essential in solving issues related to mitigating stuck pipe events. This includes determining the optimal level of vertical alignment for the rig to avoid non-recoverable pipe displacements. Various strategies for mitigating sticking in wells due to the deflection or narrowing of the wellbore, as well as the adhesion forces of the rig pipes, have been presented inMardanova (Mardanova, 2002). In the zone of contact between the rig pipes and the wellbore walls, friction forces are governed by Coulomb's dry friction law. The resistance force in the contact area depends on the friction coefficient and the compressive force in the zone. Therefore, the selection of methods to prevent sticking involves defining the boundaries of the affected zone, which is often not clearly determined. In determining the upper boundary of sticking, methods based on axial load and the measured displacement of rig sections under static conditions have been suggested. However, methods for determining the lower boundary of the affected zone have yet to be developed. The influence of dynamic forces in the sticking zone and the breakdown of rock drilling methods have been explored in works led by S.S. Grigoryan (Grigoryan, 1993). A telescopic device developed under his leadership at the M.M.U. Mechanics Institute allows for the generation of shocks on the stuck rig, facilitating the removal of the rig from the sticking zone by applying dynamic impulses.The study of dynamic behavior in stuck drilling rigs and methods to address sticking have been elaborated in the works of Sh.M. Aytaliev (Ajtaliev, et al,1999). Various methods for simulating the interaction between the pipes and the wellbore, as well as methods for applying mechanical and non-mechanical forces to mitigate sticking, have been reviewed. The study of the dynamic behavior of drilling rigs under these conditions, using numerical-analytic methods, has been explored in this work.

Materials and basic methods.

During the research, it will be necessary to observe the variations of the drilling column deviations in the Oxz and Oyz planes at specific time intervals through individual visualizations. However, the complete and realistic visualization of the movement can only be observed in three-dimensional images. Therefore, the following figures present the dynamics of the drilling column's movement in three-dimensional space at certain moments in time.

Due to the variety of numerical methods for solving simple differential equations and systems of such equations, it is advisable to choose the method that provides relatively low error and minimizes computational costs to obtain the solution of the model describing the nonlinear motion of the drilling column.

To analyze the numerical methods and choose the most effective one, the following methods are considered:

1. The fourth-order explicit Runge-Kutta method;
2. Two methods based on stiffness switching;
3. Backward differentiation formula (BDF).

In the first stiffness switching method (Stiffness switching 1), two extrapolation numerical methods are used to solve the system of ordinary differential equations: the explicit method of "chechard" and the linear implicit Euler method. In the second stiffness switching method (Stiffness switching 2), the following two numerical methods are applied to obtain the solution of the system of equations: the eighth-order explicit Runge-Kutta method and the linear implicit Euler method (Zakir, et al, 2023).

To analyze the effectiveness of the methods mentioned above, they will be compared with the results of the classical fourth-order Runge-Kutta method (Tsitouras, 2019).

The effectiveness of the numerical method is characterized by the time step Δt used to obtain the solution, the number of arithmetic computations, as well as the error when compared to the fourth-order explicit Runge-Kutta method, and the computational time spent. The results of the comparative analysis are presented in Tables 1-2.

Table 1 – Comparison of Numerical Methods, t = 150s

Method	Number of Steps	Number of Operations	Error
Explicit Runge-Kutta	1020	4082	0
Stiffness switching 1	121	3464	4.61707×10^{-7}
Stiffness switching 2	161	2095	6.29397×10^{-8}
BDF	797	913	8.85473×10^{-6}
Implicit Runge-Kutta	5301	26504	6.35221×10^{-8}

Table 2 – Computational Time for Numerical Solutions

Method	Time Spent, s		
	$t = 50c$	$t = 200c$	$t = 350c$
Explicit Runge-Kutta	0.07825	0.171875	0.296875
Stiffness switching 1	0.03125	0.109375	0.203125
Stiffness switching 2	0.03125	0.078125	0.125
BDF	0.03125	0.078125	0.125
Implicit Runge-Kutta	0.328125	1.23438	2.14063

As seen from Tables 1-2, the stiffness-switching methods require a relatively small number of time steps to obtain a numerical solution, with a smaller number of arithmetic operations and a lower approximation error. These methods also require very little computational time, similar to the BDF method. Among the methods studied, although the explicit Runge-Kutta method of the fourth order has lower efficiency, it shows very high accuracy in numerical solutions.

When comparing the two stiffness-switching methods, the method with the eighth-order explicit Runge-Kutta and the explicit Euler method outperforms all other parameters, except for the larger number of time steps.

Therefore, using the stiffness-switching method with explicit Runge-Kutta and the explicit Euler method allows for obtaining a numerical solution with sufficient accuracy, requiring fewer integration steps and reducing the computational time for arithmetic operations. For this reason, this variation of stiffness-switching will be used for solving nonlinear models in further calculations.

The solution of the nonlinear system of ordinary differential equations will be done using the following two numerical methods with stiffness switching:

The eighth-order explicit Runge-Kutta method

The explicit linear Euler method

The use of these two numerical methods is based on the stiffness of the differential equations under study. Stiff differential equations (or systems of equations) refer to those equations that cannot be solved with explicit methods, i.e., using explicit methods in the given time steps would lead to a rapid increase in the number of computations or result in a significant growth of the error.

The mathematical model of the motion of pre-stressed drilling columns, which are subjected to a force varying according to the law N(z,t), with a length of l and moving at a frequency ω, is given as follows (Seitmuratov, et al, 2017).

$$\rho A\frac{\partial^2 u}{\partial t^2}-\rho I_y\frac{\partial^4 u}{\partial z^2\partial t^2}+EI_y\frac{\partial^4 u}{\partial z^4}-\frac{EA}{1-\nu}\left(\frac{\partial}{\partial z}\left(\frac{\partial u}{\partial z}\right)^3\right)-$$
$$-\frac{(5-6\nu)EA}{2(1-2\nu)}\left(2\frac{\partial v}{\partial z}\frac{\partial^2 v}{\partial z^2}\frac{\partial u}{\partial z}+\left(\frac{\partial v}{\partial z}\right)^2\frac{\partial^2 u}{\partial z^2}\right)+N(z,t)\frac{\partial^2 u}{\partial z^2}+$$
$$+\frac{A}{2}\left(\sigma_{xx}^0+\sigma_{zz}^0\right)\frac{\partial^2 u}{\partial z^2}+A\sigma_{xy}^0\frac{\partial^2 u}{\partial z^2}+\rho A\left(2\omega\frac{\partial v}{\partial t}-\omega^2 u\right)=0$$

$$\rho A\frac{\partial^2 v}{\partial t^2}-\rho I_x\frac{\partial^4 v}{\partial z^2\partial t^2}+EI_x\frac{\partial^4 v}{\partial z^4}--\frac{EA}{1-\nu}\left(\frac{\partial}{\partial z}\left(\frac{\partial v}{\partial z}\right)^3\right)-$$
$$-\frac{(5-6\nu)EA}{2(1-2\nu)}\left(2\frac{\partial u}{\partial z}\frac{\partial^2 u}{\partial z^2}\frac{\partial v}{\partial z}+\left(\frac{\partial u}{\partial z}\right)^2\frac{\partial^2 v}{\partial z^2}\right)+N(z,t)\frac{\partial^2 v}{\partial z^2}+ \tag{1}$$
$$+\frac{A}{2}\left(\sigma_{yy}^0+\sigma_{zz}^0\right)\frac{\partial^2 v}{\partial z^2}+A\sigma_{xy}^0\frac{\partial^2 v}{\partial z^2}-\rho A\left(2\omega\frac{\partial u}{\partial t}+\omega^2 v\right)=0$$

$u(z,t),v(z,t)-$ The transverse displacement components of the drilling column;

$\rho-$ Material density;

A– the cross-sectional area of the column;

E – Young’s modulus;

I_x,I_y – axial moments of inertia;

$\nu-$ Poisson’s ratio;

$\omega-$ Angular velocity;

$N(z,t)-$ Compressing force from above. In the calculations, we assume the compressive force to be constant.

σ_{ij}^0- Initial stresses.

Boundary conditions:

$$u(0,t)=0 \qquad u(l,t)=0$$
$$EI_y\frac{\partial^2 u(0,t)}{\partial z^2}=0 \quad EI_y\frac{\partial^2 u(l,t)}{\partial z^2}=0$$

$$v(0,t)=0 \qquad v(l,t)=0 \tag{2}$$
$$EI_x\frac{\partial^2 v(0,t)}{\partial z^2}=0 \quad EI_x\frac{\partial^2 v(l,t)}{\partial z^2}=0$$

The boundary conditions are considered as hinged support, meaning that both ends of the shaft are fixed.

Initial conditions:

$$\begin{aligned} &u(z,0)=0 \qquad v(z,0)=0 \\ &\frac{\partial u(z,0)}{\partial t}=C_1 \quad \frac{\partial v(z,0)}{\partial t}=C_2 \end{aligned} \tag{3}$$

C_1, C_2 – constants that describe the initial displacement velocity of the shaft in the Oxz and Oyz planes at the initial time moment.

The method of separation of variables by Bubnov-Galerkin was used to transform the partial differential equation into a system of ordinary differential equations.

$$\begin{aligned} &u(z,t)=\sum_{i=1}^{n} g_i(t)G_i(z)=\sum_{i=1}^{n} g_i(t)\sin\left(\frac{i\pi z}{l}\right),\ n=5 \\ &v(z,t)=\sum_{i=1}^{n} f_i(t)G_i(z)=\sum_{i=1}^{n} f_i(t)\sin\left(\frac{i\pi z}{l}\right),\ n=5 \end{aligned} \tag{4}$$

Results and discussion.

The implementation of the Bubnov-Galerkin method was carried out in the Wolfram Mathematica symbolic computation system, and the system of nonlinear ordinary differential equations was obtained, where the time components of the deviation are related to $f_i(t)$, $g_i(t)f_i(t)$, $g_i(t)$ (Hurshudyan, 2015).

$$\begin{aligned} g_i(t)=\int_0^l\Bigg[&\rho A\frac{\partial^2 u}{\partial t^2}-\rho I_y\frac{\partial^4 u}{\partial z^2\partial t^2}+EI_y\frac{\partial^4 u}{\partial z^4}-\frac{EA}{1-\nu}\left(\frac{\partial}{\partial z}\left(\frac{\partial u}{\partial z}\right)^3\right)- \\ &-\frac{(5-6\nu)EA}{2(1-2\nu)}\left(2\frac{\partial v}{\partial z}\frac{\partial^2 v}{\partial z^2}\frac{\partial u}{\partial z}+\left(\frac{\partial v}{\partial z}\right)^2\frac{\partial^2 u}{\partial z^2}\right)+N(z,t)\frac{\partial^2 u}{\partial z^2}+\frac{A}{2}\left(\sigma_{xx}^0+\sigma_{zz}^0\right)\frac{\partial^2 u}{\partial z^2}+ \\ &+A\sigma_{xy}^0\frac{\partial^2 u}{\partial z^2}+\rho A\left(2\omega\frac{\partial v}{\partial t}-\omega^2 u\right)\Bigg]\times\sin\frac{i\pi z}{l}dz \end{aligned}$$

$$\begin{aligned} f_i(t)=\int_0^l\Bigg[&\rho A\frac{\partial^2 v}{\partial t^2}-\rho I_x\frac{\partial^4 v}{\partial z^2\partial t^2}+EI_x\frac{\partial^4 v}{\partial z^4}--\frac{EA}{1-\nu}\left(\frac{\partial}{\partial z}\left(\frac{\partial v}{\partial z}\right)^3\right)- \\ &-\frac{(5-6\nu)EA}{2(1-2\nu)}\left(2\frac{\partial u}{\partial z}\frac{\partial^2 u}{\partial z^2}\frac{\partial v}{\partial z}+\left(\frac{\partial u}{\partial z}\right)^2\frac{\partial^2 v}{\partial z^2}\right)+N(z,t)\frac{\partial^2 v}{\partial z^2}+\frac{A}{2}\left(\sigma_{yy}^0+\sigma_{zz}^0\right)\frac{\partial^2 v}{\partial z^2}+ \\ &+A\sigma_{xy}^0\frac{\partial^2 v}{\partial z^2}-\rho A\left(2\omega\frac{\partial u}{\partial t}+\omega^2 v\right)\Bigg]\times\sin\frac{i\pi z}{l}dz \end{aligned} \tag{5}$$

When solving the given integrals using the Bubnov-Galerkin method for n=5, a system of 10 simple differential equations will be obtained.

The initial conditions for the differential equations will be as follows:

If the indices are odd: $i = 2k - 1, k = \overline{1,3}$:

$$g_i(0) = 0, \quad g_i'(0) = \frac{4C_1}{i\pi};$$

$$f_i(0) = 0, \quad f_i'(0) = \frac{4C_2}{i\pi} \tag{6}$$

If the indices are even: $i = 2k, k = \overline{1,3}$:

$$g_i(0) = 0, \quad g_i'(0) = 0;$$
$$f_i(0) = 0, \quad f_i'(0) = 0$$

The cross-sectional's area.:

$$A = \frac{\pi}{4}(D^2 - d^2) \approx 2.01 \cdot 10^{-2} m^2 \tag{7}$$

The axial moment of inertia:

$$I_x = I_y = \frac{\pi}{64}(D^4 - d^4) \approx 6.84 \cdot 10^{-5} m^4 \tag{8}$$

The results of the solution to the system of nonlinear differential equations, related to the time components of the deviation $f_i(t), g_i(t) f_i(t), g_i(t)$, are presented in Figures1-2

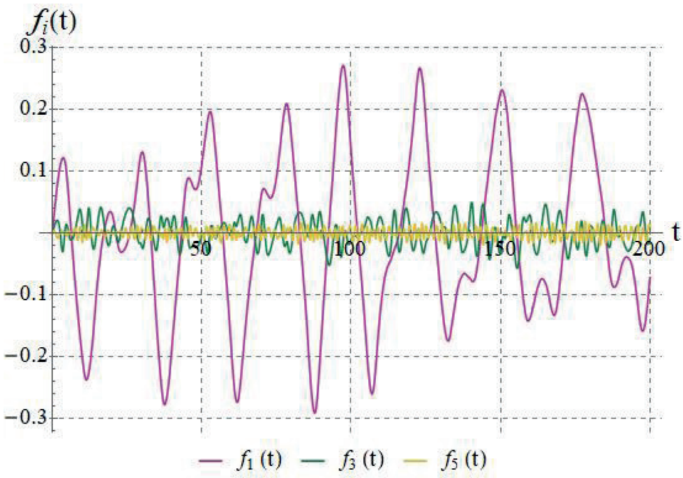


Figure 1 – The first three mods of the column's deviation.

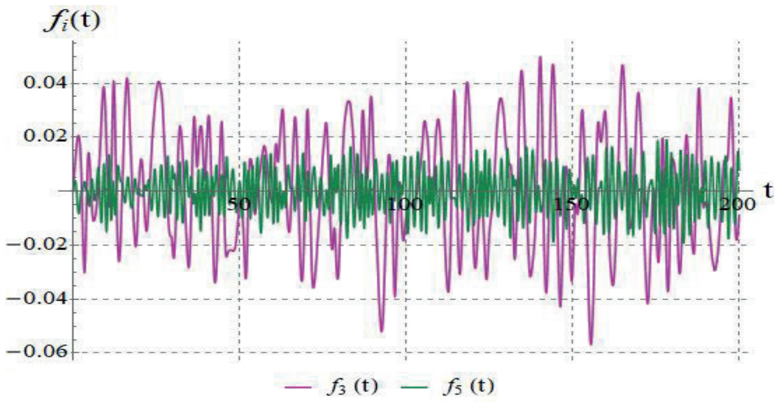


Figure 2 – The third and fifth modes of the column's deviation.

From Figure 1, we can clearly see that the first mode plays a significant role in the overall oscillation process due to its larger values compared to the other modes. It is worth noting that $f_i(t)f_i(t)$ the time components of the deviation, influence the deviation of the drilling column in the Oxz plane.

Similarly, from Figure 1, we can clearly see that the first mode (for the Oyz plane) plays a significant role in the overall oscillation process due to its larger values compared to the other modes.$g_i(t)g_i(t)$, the time components of the deviation, affect the deviation of the drilling column in the Oyz plane.

In the future, we will use the aforementioned time components $f_i(t)\text{and}g_i(t)$ $f_i(t)\text{and}g_i(t)$ to calculate the deviations of the drilling column in the corresponding Oxz and Oyz planes.

Figures 3-4 show the deviations of the drilling column in the Oxz and Oyz planes. The numerical calculations were made at the point $z = 0.49lz = 0.49l$ i.e., at the center of the column. All calculations were based on a column length of $l = 150ml = 150m$.

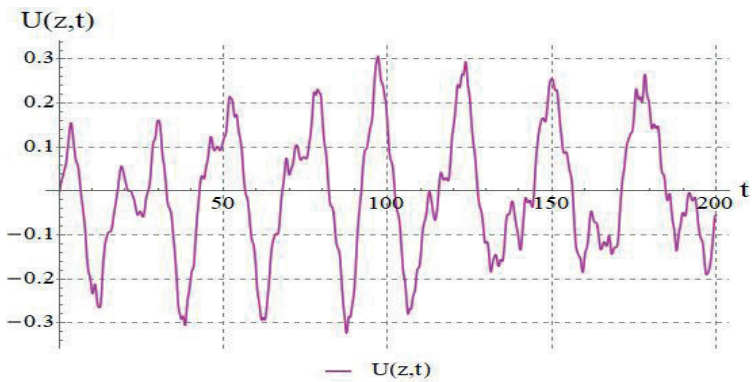


Figure 3 – Deviation of the drilling column (m).

From Figure 3, we can observe that the amplitude of the oscillations of the drilling column at the point $z = 049lz = 049l$ in the corresponding Oxz and Oyz planes is approximately around 0.3. These are the maximum values of the deviation for the given parameters.

For comparison and analysis, in Figures 4-5, the deviations of the column at the points$z = 0.33lz = 0.33l$ and$z = 0.66z = 0.66$along the length of the column are shown in the corresponding Oxz and Oyz planes.

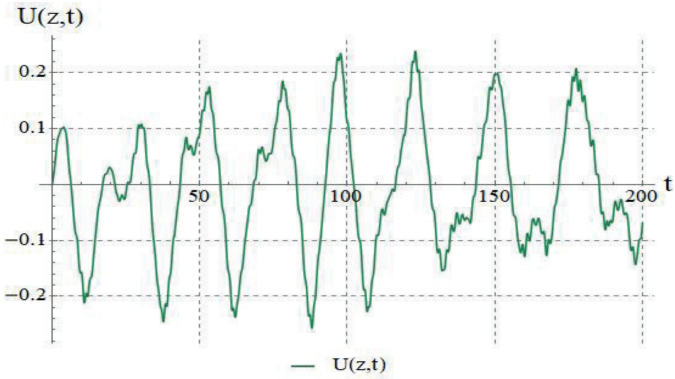


Figure 4 – Deviation of the drilling column at the point $z = 0.33lz = 0.33l$.

or the purpose of comparing the deviations at different parts of the drilling column, the column's length was conditionally divided into three parts. From Figures 4-5, we observe that the maximum deviation values of the drilling column are approximately around 0.2. As seen, this value is much smaller than the deviations at the center of the shaft shown in Figures 3-4. Thus, we conclude that the largest deviations in the motion of the drilling column occur in the middle section of the shaft.

In the following Figures 5-6, the dynamics of changes in the corresponding Oxz and Oyz planes for the entire length of the drilling column are presented for each fixed time moment..

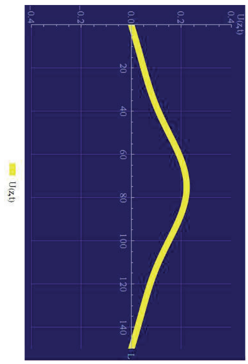
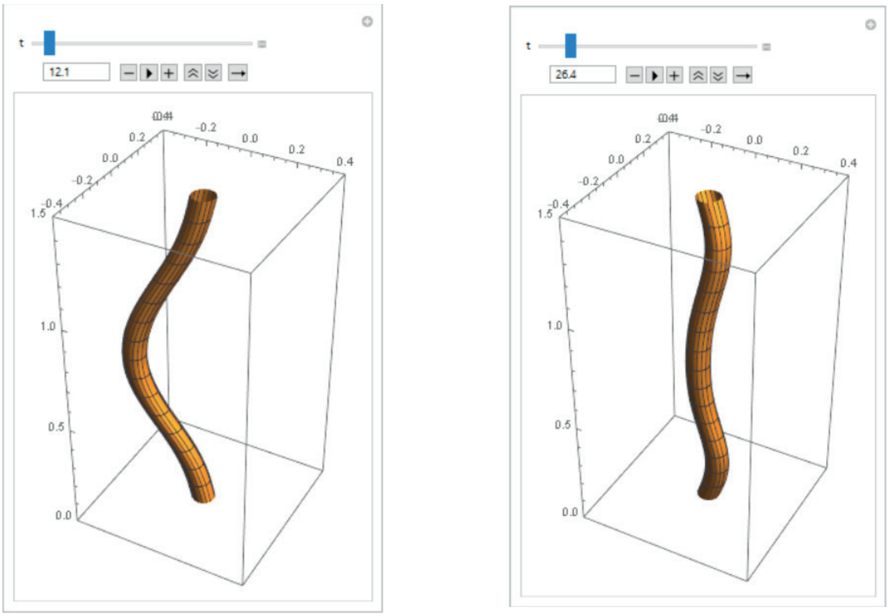


Figure 5 – View of the drilling column in the Oxz plane at the time t = 79s.

This was obtained using the ParametricPlot3d method. In the images, the calculations were carried out for a column length of l = 150m, but for convenience, this size was scaled. To set the dimensions of the considered area along the three axes, the PlotRange attribute of the ParametricPlot3d function was used. These dimensions were determined based on the maximum values of the deviation.

However, the disadvantage of these images is that performing individual calculations for each time moment leads to significant time expenditure. Therefore, for a specific time interval, the Wolfram Mathematica Manipulate function was used to perform the calculation just once. The Manipulate function is used to create dynamic images and is especially useful in understanding the solutions to differential equations. This function is used to visualize the dependency of the final values on the magnitude of the parameters. In our case, we use time as the changing parameter, thus obtaining a dynamic image of the drilling column for a specific time interval.

In the following Figures 1, we will examine the motion of the drilling column in space for the time interval $0 \leq t \leq 200s$.



a) $t = 12.1st = 12.1s$ б) $t = 26.4st = 26.4s$

Figure 6 – Motion of the drilling column in space for the time interval $0 \leq t \leq 200s$.

From Figure 6, we can observe the deformation of the pre-stressed drilling column in three-dimensional space for any limited time interval. The advantage of these images is that we can view the condition of the drilling column at any given time by simply pressing a button(Kister,1987)

In the next step, to give the images visual meaning, it is necessary to color the column in several colors to clearly highlight the "dangerous" values of the deviation.

Conclusion

Drilling operations are characterized by the use of powerful electric motors with large rotational speeds and automation of control and technological processes. All these factors exacerbate the issue of reliability concerning the equipment installed in the drilling wells.

There are various empirical equations for the drilling process that function under a range of favorable conditions, aimed at forecasting intensive control and efficiency indicators. However, these are based on deterministic influences, which are underpinned by the physical understanding of ongoing processes and, as a result, have a specific methodology. Using the methods developed, it is possible to make relatively simple changes within a wide range of process parameters, which are based on the impact of these changes on the system's performance.

When studying the vertical oscillations of the column in the fixing area, the moment of the supporting force acting on the column's torsion is taken into account. The effect of the support force on the columns in the fixing area depends on the lateral support and hydraulic pressure from massive fixings.

The friction parameters in the contact area change as the surrounding temperature rises. Therefore, design and implementation of drilling works, as well as addressing any adverse situations, should be scientifically studied and solved using the latest technological advancements.

The development of science and technology, the rise of scientific and technical progress, and the use of quality technologies that monitor the requirements of deformable environments and dynamics are crucial.

The development laws of applied problems and the study of deformable solid bodies in mechanics are being explored. For a complete analysis, the physical and mechanical properties of materials, their deformation characteristics over time, and the interconnection effects of temperature, electric, and magnetic fields on mechanical deformations are considered.

The results of the study contribute to the exploration of stationary, non-stationary, oscillatory, and wave processes, leading to significant achievements in the fields of mechanics of deformable solid bodies, hydrodynamics, and geophysics.

The scope of the work includes modeling the inclined oscillation of the drilling column in the MAPLE system and theoretical research on new stages in the non-stationary oscillation region, as well as developing a new model for inclined oscillation conditions in the drilling column. The effectiveness of mathematical methods in studying a wide class of spatial problems within known models is also explored.

In theoretical and experimental plans, among various classes of deformable environments, elastic and visco-elastic materials, as well as porous materials, are key areas of study. The interactions between phases (liquids and solids) can be relative, and their movement is interconnected. The degree of improvement in the connections between phases depends on binding characteristics, pore size, and other attributes of the skeleton.

The main problems of the dynamics of inclined oscillations have been transformed, and necessary data from elastic and visco-elastic theories are presented. In cases of small deformation, two-component visco-elastic environments have been considered.

The development laws of applied problems and the study of deformable solid bodies in mechanics are clarified. The basic problems of the dynamics of inclined oscillations in the drilling column have been transformed and modeled in the MAPLE system(Seitmuratov, et al,2011). The Maple system provides the ability to visualize several function graphs, allowing the distinction between them for display purposes.

The unique aspect of the graphs in the MAPLE system for inclined oscillations is the use of theplot3d function. Each point on these graphs, z_i corresponds to coordinates on the XY plane. Due to the computer screen's perspective, in practice, three-dimensional graphics serveas a projection of volume objects.

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