

NEWS

OF THE NATIONAL ACADEMY OF SCIENCES OF THE REPUBLIC OF KAZAKHSTAN
PHYSICO-MATHEMATICAL SERIES

ISSN 1991-346X

<https://doi.org/10.32014/2020.2518-1726.102>

Volume 6, Number 334 (2020), 91 – 98

MPHTI: 41.29.21; 41.29.25

UDK 524 D.

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**MASS DISTRIBUTION OF DARK MATTER HALO
AND SCALE EVOLUTION OF EARLY TYPE GALAXIES**

Abstract. In this paper we use two suites of ultra-high resolution N-body simulations Phoenix and Aquarius Projects to study the assembly history of sub-halos and its dependence on host halo mass. We found that more massive haloes have more progenitors, which is in contrast with former works because they counted dynamical progenitors repeatedly. Less massive halos have larger fraction of dynamical progenitors than more massive ones. The typical accretion time depends strongly on host halo mass. Progenitors of galactic halos are accreted at higher redshift than that of cluster halos. Once these progenitors orbit their primary systems, they rapidly lose their original mass but not their identifiers. Most of the progenitors are able to survive to present day. At given redshift, the survival fraction of accreted sub-halos is independent of host halo mass, while sub-halos in high mass halos lost more mass.

In the second part, we use a semi-analytical galaxy formation model compiled on a Millennium Simulation to study the size evolution of massive early-type galaxies from redshift $z = 2$ to present days. We find that the model we used is able to well reproduce the amplitude and slope of size-mass relation, as well as its evolution. The amplitude of this relation reflects the typical compactness of dark matter halos at the time when most stars are formed. This link between size and star formation epoch is propagated in through galaxy combinations. Minor combinations are increasingly important with increasing present day stellar mass for galaxies more massive than $10^{11.4}M_{\odot}$. At lower masses, major combinations are more important. In situ star formation contributes more to the size growth than it does to stellar mass growth. Similar to former works, we find that minor combinations dominate the subsequent growth both in stellar mass and in size for early formed early-type galaxies.

Key words: dark matter halo, semi-analytical galaxy formation model.

1. Introduction. The un-evolution sub-dark matter halo mass function (USMF) describes the mass distribution of all precursor dark matter halos involved in the construction of the dark halo and its substructures throughout the dark halo formation. The mass distribution of dark matter halos. Some of these dark bodies that fell into the main dark halo disappeared, and some survived as a sub-dark matter halo. Their mass distribution plus the influence of evolution is the current sub-dark matter halo mass distribution. In order to understand the dependence of the sub-dark matter halo mass function on the quality of the main dark halo, we need to know more about it from the USMF. First, whether the USMF of the galaxy cluster darkness and the galaxy dark halo is the same; second, whether the current sub-dark is dependent on the quality of the main dark halo is caused the evolution is different at the beginning. In paper [1] were studied the distribution of the current dark halo in 2009. They found that the USMF does not depend on the quality of the main dark halo. This result is a bit strange, because the energy spectrum of the standard cosmological model is not scale-independent, so it is difficult to understand why the main dark halo of different scales actually have the same USMF.

We use the dim sum of the tree to track all the predecessors. First we need to build the main branches of the dark matter halo and the trees. Our method is to start with the main dark halo of the redshift $z = 0$, find the predecessor of the highest quality of its last time point, and then find the predecessor of the

highest quality at this moment. Repeat this step until the darkness of the predecessor is too low cannot be resolved (there are 32 particles in the numerical simulation we used), and the precursors of the largest mass at each moment constitute the main branch of the combined tree. Then, if a dark halo eventually enters the radius of the main branch (R_{200}), it is defined as the darkness of the predecessor. The darkness of the predecessor is very important when accretion is a sub-dark halo, and many previous work gives a variety of definitions. For example: in the paper [2] used the moment when the dark halo entered the FOF group of the main branch; in the paper [3] used this dark matter halo to reach the moment of its greatest quality in history. In this work, we define the maximum wraparound velocity (V_{max}) of a sub-dark matter halo, and it is also an independent dark halo, defined as its accretion time. Correspondingly, its mass is now defined as the accretion mass. We use this definition because of Gao et al. [4] found that the mass of a satellite galaxy closely related to the peak of the maximum surrounding velocity of the sub-dark matter halo.

The combined tree of dark matter halos is generally very complicated, we need to consider two special cases, I will explain it in detail next. 1) Ejected halo: a dark halo has been briefly appeared within the radius of the main branch, but it is eventually popped out when the redshift $z = 0$, and it is disappeared in the outside of the virial radius of the main dark halo. Previous work found that the dark halo that was ejected within the virial radius of 1~3 times around the main dark halo, and the probability of being popped was related to their own quality, which is the main cause of the deviation of the aggregation. Because the main dark halo is less affected by the current sub-dark halo distribution, we did not include this part of the dark halo in the predecessor dark halo sample. 2) Through the dynamical progenitor: that is, the former branch through multiple times of the main branch in dark matter halo. After the dark halos of these predecessors entered the radius of the main branch, one or several redshifts appeared outside of the radius of main branch, but eventually it is disappeared within the main branch or within the main dark halo's virial radius when the redshift $z = 0$.

The definition of the dark halo used in our predecessor is completely different from the early observations [5]. They use the combined tree of the FOF group to find the dark halo of the predecessor, and the dark halo which we are looking for the main combination into the R_{200} of the main dark halo. Because there are some dark halos in the FOF group are only connected by a thin particle "bridge" and do not belong to the same system, our method can remove these false "sub-dark halos". In addition, the way we track the darkness of our predecessors is completely different. When tracking the predecessor's dark halo, they only considered the dark halo of the main branch into the main combination, ignoring the dark halo of the predecessors, which first combined into other dark halos and followed other dark halos into the main branches. This method can give us less statistics of a large number of the dark halo's predecessors. In the work of Li and Mo [6] counted all the dark halo's predecessors, including dark matter halos which first entered other dark halos. But these dark matter halos, which first enter other dark halos, may have been broken up before entering the main dark halo, so they ultimately did not contribute to the sub-structure of the main dark halo, and their quality has been incorporated into their host dark halo at the time. When the host darkness enters the main branch, it is counted as the quality of their host dark halo, so they repeat the statistics of the quality of these dark halos. In our work, there is no statistics of dark halo which are disappears in the darkness of other predecessors. The two important differences between Li Yun's work and our work are that they count as a predecessor of dark halo every time they meet.

2. Mass distribution of dark matter halo. In figure 1, we compare the ratio of Phoenix dark halo and Aquarius dark halo through the predecessor dark halo to all front of main dark halos. The figure shows the median values of 7 Phoenix dark halos and 6 Aquarius dark halos. Through the darkness of the predecessor dark halo, the R_{200} that repeatedly enters the main branch is repeated, and the predecessor dark halo that eventually disappears in the main dark halo. The results for Phoenix are indicated by solid red lines and the results of Aquarius are indicated by solid black lines. The horizontal axis is the quality of the dark halo of the predecessor normalized by the quality of the main dark halo. Note that at this time we are counting the predecessor dark halo which passed through the front, not the number of crossing times of the predecessor dark halo. It can be seen from the figure that when the quality of the current dark halo is greater than 1/1000 of their main dark halo quality, whether it is Phoenix dark halo or Aquarius dark halo, the proportion of predecessor dark halo through the front is small. But the proportion of dark halo lower than this quality is very large. Phoenix's galaxy group has about 35%, and Aquarius's galaxy group has about 50% of its predecessor dark halo has crossed the main branches for many times.

In figure 2, we present the USMF for the integration of the Phoenix numerical simulation and the Aquarius numerical simulation. The red solid and black solid lines represent the median of the USMF for each of the seven Phoenix galaxy cluster halos and the six Aquarius galaxy dark halos, respectively. The quality of the main dark halo limits the quality of its sub-dark halo, and the dark halo of the galaxy cluster is even greater than the mass of the galaxy dark halo. In order to remove the dominant main dark halo dependence to the quality, we use the quality of the predecessor dark halo normalized by the main dark halo mass M_{sub} / M_{halo} . The figure 2 gives the mass function of the integral, which makes the difference between the different samples more obvious. As can be seen from the figure 2, the USMF is obviously dependent on the quality of the main dark halo, and the Phoenix main dark halo of the galaxy cluster level contains an average of 30% substructures more than the Aquarius main dark halo of the galaxy cluster level.

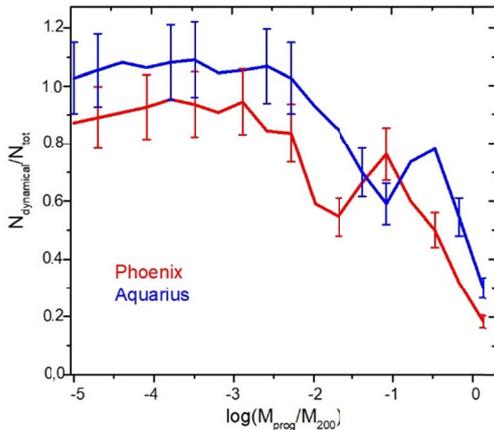


Figure 1 - The median ratio of the seven Phoenix and the six Aquarius to the dark halo of the predecessor to its quality

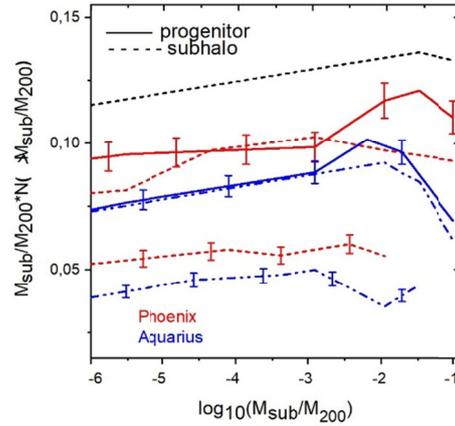


Figure 2 - The median of the seven Phoenix dark halos, and the median of six Aquarius dark halos

Below, we study the survival rate and survival mass ratio of the two redshifts $z = 2$ and $z = 4$, which are accreted by the precursors. The triangle line in figure 3 represents the survival rate, and the number of survivors compared to the number of surviving dark halo, which was accreted by the redshift. The square line indicates the survival quality ratio, and the mass of the dark halo which is surviving to the present, compared to the mass when they were just accreted. Red and black indicate the result of Phoenix dark halo and Aquarius dark halo, respectively. It can be seen that whether it is in Phoenix dark halo or in Aquarius dark halo, at the redshift $z = 2$, the accreted predecessor of the dark halo, which mass is larger than the ($\log(M_{prog} / M_{200, z=0}) > 10^{-5}$), about 90% have survived to the present.

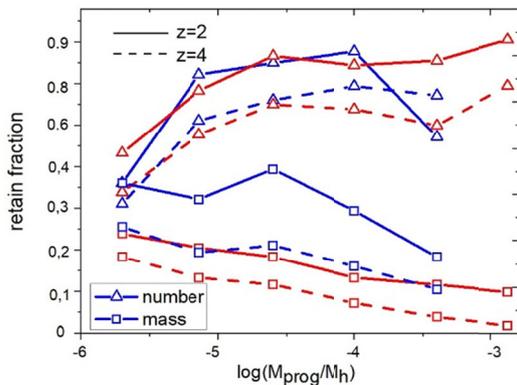


Figure 3 - Phoenix dark halo and Aquarius dark halo in the redshift $z = 4$ and $z = 2$ accreted of all precursors

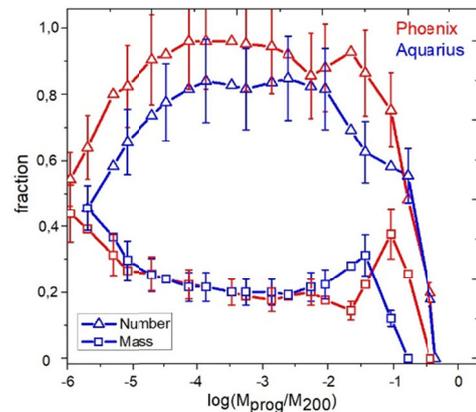


Figure 4 - Survival and survival mass ratios for all precursors

In figure 4, we show the relationship between the survival rate and the survival mass ratio of all the precursors, which accreted at all times in the history of the whole accretion, and the quality of the predecessors. Red and black line give the median values of Phoenix Dark Halo and Aquarius Dark Halo, respectively. The error bars represent the full range of diffusion for each of the seven Phoenix dark halos and six Aquarius dark halos. Triangles and squares represent survival and survival mass ratios, respectively. It can be seen from the figure 4 the survival rate of the dark halo of the predecessor is more dependent on its quality, and the massive predecessor dark halo is more easier to be destroyed, because the greater the mass of the dark halo, the stronger the dynamic friction. The dynamic friction and the tidal stripping force work together to make the massive dark halo predecessor's survival rate lower.

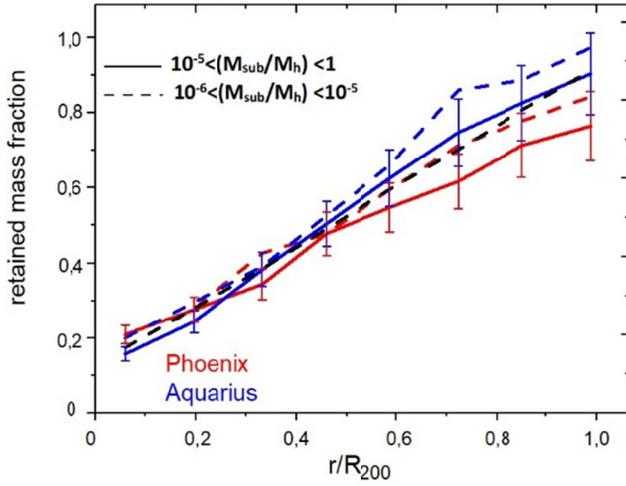


Figure 5 - The median value of the existing sub-dark matter halo in the Phoenix dark and Aquarius dark halo

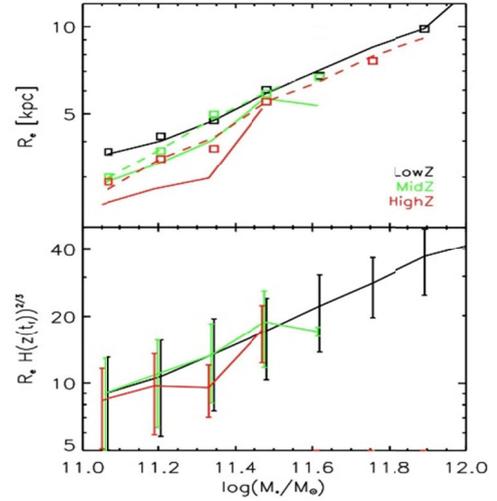


Figure 6 - Scale-quality relationship of early-type galaxies at different redshifts

In figure 5, we show the median value of the survival mass ratio of Phoenix dark halo and Aquarius neutron dark halo and their relationship r / r_{200} in the main dark halo. To compare the dependence of this relationship on the sub-dark halo mass function, we divided the sub-dark halo samples into two parts according to the mass: $10^{-6} < M_{sub} / M_h < 10^{-5}$ and $10^{-5} < M_{sub} / M_h$. The error bars represent the full dispersion of Phoenix Dark Halo and Aquarius Dark Halo. It can be seen that the survival mass ratio of the sub-dark matter halo has a strong correlation with their distance from the center, and this correlation does not depend on the mass of the sub-dark halo.

In figure 6 gives the median distribution of the scale-quality relationships for the three samples (samples represented by red, green, and black solid lines). It can be seen that the slope of the scale-quality relationship is almost independent of the redshift, and the amplitude has increased by about 1.2 times from the redshift $z \sim 1.6$ to the present. The square in the upper graph in figure 6 represents the scale-mass distribution of the galaxies of each sample in the redshift $z = 0$. Unlike the dashed lines, they only contain the early galaxies of the satellite galaxies when the redshift is $z = 0$. About 27% of the MidZ galaxies, about 21% of the HighZ galaxies are satellite galaxies in nowadays. The higher the dark matter of the same mass, the higher the redshift and the denser, the evolution of the dimension follows to this formula $R_{200}(z) \propto H(z)^{-2/3}$.

3. Formation of early galaxies. In figure 7, we examine the contribution of different physical processes to the mass of early-type galaxies. We follow the main branches of the combined trees of each galaxy, and record the in-situ star formation, the galaxy main combination, the minor combination, starburst, and this contribution of these four physical processes to the mass growth during the growth of the galaxy. Divided by the total stellar mass, and all sample galaxies take the median by mass interval. The solid line indicates the result of the LowZ galaxies, and the dashed line indicates the results of the

MidZ galaxies. In the process of in-situ star formation, we distinguish between stars formed by starbursts caused by galaxies (indicated by black) and stars formed statically on galaxies (indicated by green). We distinguish the main combination (indicated by blue) or minor combination (indicated by red) according to the mass ratio of the two galaxies.

In figure 8, we present the scale-growth process of four galaxies with redshift $z = 0$ in the model. We record the changes in the mass and radius of the galaxies caused by different physical mechanisms at each moment, and link the changes on the scale-quality map with lines of different colors. The change in quality is monotonous, but the scale is usually increasing or decreasing. At low quality (high redshift time), the scale and quality of our sample galaxies grow mainly through the formation of static stars (black lines) on the galaxies, and the quality formed during this period is only comparable to the final mass of the galaxies. Very small, but the most scaled growth is very effective. After the redshift $z = 2$ (vertical dotted line), the evolution of the galaxies is dominated by the combination. Consistent with the statistical average, the main combination (blue) dominates the growth of relatively small-mass galaxies, and the minor combination dominates the growth of large-mass galaxies.

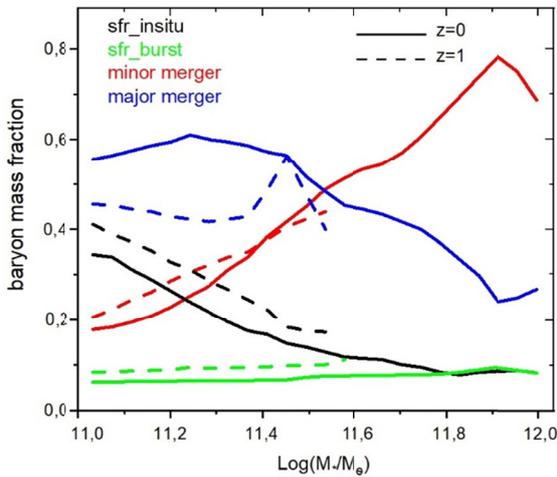


Figure 7 - The contribution of different physical mechanisms to stellar mass growth

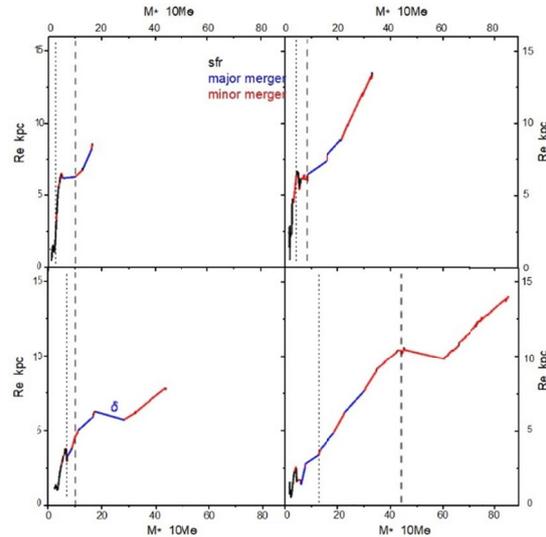


Figure 8 - The ratio of the three main physics to the galaxy-scale growth of the main combination

As shown in figure 9, there are several very important features in the sample which we are tracking. For example: there is no significant difference in the scale of growth between main and minor combination in terms of growth unit mass. Part of the reason is because we used in the calculation of the galaxies' scale changes, contains the mass ratio. Before the combination, the radius of the two galaxies were: 5.9 kpc and 1.7 kpc , and the corresponding masses were: $12.2 \times 10^{10} M_{\square}$ and $8.8 \times 10^{10} M_{\square}$. After the combination, the projected radius of the newly formed galaxies is 5.1 kpc .

Figure 10 shows the average scale and mass change from the (HighZ (red solid line) and MidZ (blue solid line)) high redshift early galaxies to the low redshifts. We will compare the evolution of the galaxies after the minor combination process, which will cause the shape of the galaxies to change. We only consider the scale and quality of the nucleus to remove the influence of the disk of the galaxies. We found that although the MidZ galaxies are not early galaxies when the redshift $z = 2$, their scale and quality are pursuing HighZ's early galaxies. As can be seen in figure 16, the HighZ early-type galaxies are redshifted from $z = 2$ to redshifted $z = 0$. The quality has increased by 2 times and the scale has increased by 1.9 times. The MidZ galaxies' mass increased by 1.7 times from the redshift $z = 1$ to the redshift $z = 0$, and the scale increased by 1.5 times.

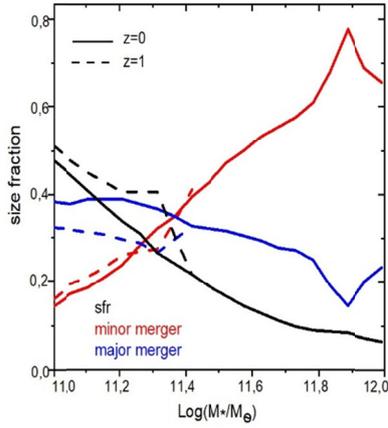


Figure 9 - Four different masses of redshift $z = 0$ and $z = 0$ of the early galaxy's mass and scale growth process

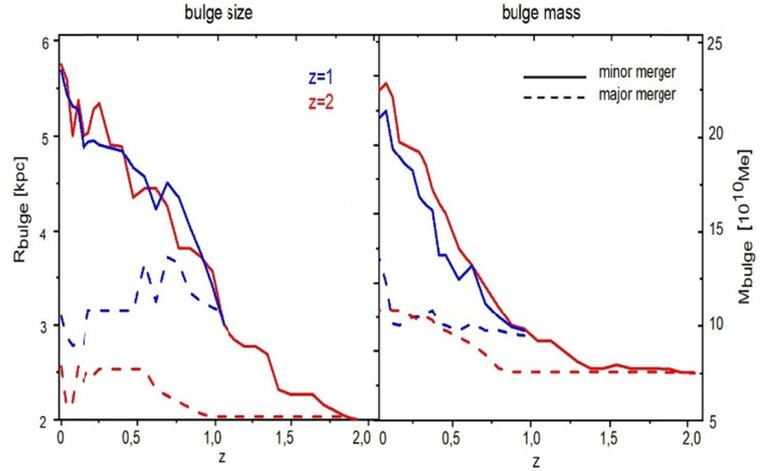


Figure 10 - The mass of the galaxy and the median value of the effective radius of the projection with the redshift

4. Conclusion. We use the latest half-system galaxies formation model to study the scale evolution of large-scale early galaxies. We found that the half-mass radius of the early-type galaxies with redshift $z = 0$ in the simulation is 1.8 times larger than the radius of the early-type galaxies with the redshift $z = 2$ selected by the same method. This discovery is consistent with many recent observations.

When we choose galaxies in the same way in different redshifts, we are not choosing the same galaxies. In low redshifts, new galaxies are constantly becoming early galaxies. The number of early galaxies has increased by 100 times from redshift $z = 1$ to $z = 2$. Therefore, most of the current early-type galaxies are not early-type galaxies in high redshifts, and the mass range of early-type galaxies are also changed. Therefore, there are two main reasons for the changes in the scale of early-type galaxies.

Convergence, stellar accretion, and in situ star formation preserved this dependence on formation time, to a greater or lesser extent, in the evolution of early-type galaxies during low redshift periods. Compared with star formation, the combination dominates the quality and scale evolution of early low-shifted galaxies, and the minor combination is most important for the evolution of early-type galaxies with $M_{star} > 3 \times 10^{11} M_{\odot}$. The starburst caused by the combination only contributed 5% in quality growth, indicating that starbursts occurred less. Previous studies have shown that the red-shifted $z = 1$ after the combination is a lack of gas, consistent with our conclusions. In-situ star formation contributes more to scale growth than to mass growth. But for large-scale galaxies, in-situ star formation contributes less to the scale or mass than to the combination process. We found that early-type galaxies formed by high redshifts mainly grew by sub-combination in the subsequent evolution. Their nuclear mass and nuclear sphere radius increased by 2 and 1.9 times from redshift $z = 1$ to redshift $z = 0$, respectively. When we studied the growth of individual galaxies, we found that a single galaxies combination or star formation may also shrink the scale of the galaxies.

The model we used did not incorporate the physical process of gas interaction energy dissipation when combined, which resulted in our results being too large for observations with high redshift gas [10].

Most early-type galaxies formed by high redshifts have evolved to today's central galaxies (of course, most of the current central galaxies are not early-type galaxies at high redshifts). Then it is very interesting to study the high redshift early galaxies in the low redshift state, or the low redshift galaxies in the high redshift mass form distribution. These studies are relatively easy to implement in a half-cutting system. We have conducted some research and look forward to publishing follow-up articles. In addition, the observations emphasize the “binomiality” of galaxies evolution (Huang et al., 2013). The observations show that high-redshift galaxies and low-redshift galaxies are consistent with the mass density profile within 1 kpc from the center. The density profile within the internal 1 kpc is also independent of galaxy mass. That is to say, the early-type galaxies with high redshifts only change externally during the subsequent evolution. This question is also very interesting, but after some tests we made it, we regretted that we could not study this issue using the current model.

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ҚАРАҒЫ МАТЕРИЯ ГАЛОСЫНЫҢ МАССАЛЫҚ ТАРАЛУЫ ЖӘНЕ ЕРТЕ ТИПТЕГІ ГАЛАКТИКАЛАРДЫҢ ТҮЗІЛУ ЭВОЛЮЦИЯСЫ

Аннотация. Жұмыста суб-галолардың жинақталу тарихын және оның гало массасына тәуелділігін зерттеу үшін Phoenix пен Aquarius Projects-тің жоғары ультра ажыратымдылығының N-денелік модельдеуін қолданамыз. Массалық галоньң негізін қалаушыны анықтадық, бұрынғы жұмыстармен салыстырғанда, олар динамикалық негізін қалаған галоларды бірнеше рет есептеген. Аз массивті галоға қарағанда массасы аз галоларға динамикалық үйкелістің үлкен әсері бар. Әдеттегі жинақтау уақыты гало массасына тәуелді. Галактикалық қараңғы материя галолардың негізін құраушы қызыл ығысу кезінде кластерлік галактикалық галолаларға қарағанда жоғары болады. Бұл бастау өздерінің алғашқы жүйелерінің айналасында болғаннан кейін, идентификаторларын емес, бастапқы массасын тез жоғалтады. Аталық галолардың көпшілігі бүгінге дейін өмір сүріп келді. Берілген қызыл ығысу кезінде аккрецияланған суб-галостардың тіршілік ету үлесі негізін қалаған гало массасына тәуелді емес, ал үлкен галолардағы субгалолар көп массасын жоғалтады.

Екінші бөлімде біз $z = 2$ қызыл ығысуынан бастап бүгінге дейінгі массивтік ерте типті галактикалардың көлемдік эволюциясын зерттеу үшін Мыңжылдық Модельдеуі бойынша құрастырылған галактиканың пайда болуының жартылай аналитикалық моделін қолданамыз. Біз қолданған модель амплитудасы мен көлбеу мөлшерін, сондай-ақ оның эволюциясын жақсы шығаруға қабілетті екенін анықтаймыз. Бұл қатынастың амплитудасы көптеген жұлдыздар пайда болғандағы қараңғы материя галосының типтік ықшамдылығын көрсетеді. Бұл өлшем мен жұлдыздардың пайда болу дәуірі арасындағы байланыс галактиканың тіркесімдері арқылы таралады. Галактикалар үшін қазіргі жұлдыз массасының өсуіне байланысты кішігірім комбинациялар $10^{11.4} M_{\odot}$ -ден үлкен маңызға ие. Төменгі массада негізгі комбинациялар маңызды. Жұлдыз өлшемінің ұлғаюына қарағанда жұлдыз массасының көбеюіне көбірек ықпал етеді. Бұрынғы жұмыстарға ұқсас, кішігірім комбинациялар жұлдыздық масса бойынша да, ерте пайда болған ерте типтегі галактикаларда көлем бойынша да басым болады.

Түйін сөздер: қараңғы материя гало, галактиканың жартылай аналитикалық түзілуі.

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МАССОВОЕ РАСПРЕДЕЛЕНИЕ ГАЛО ТЕМНОЙ МАТЕРИИ И МАСШТАБНАЯ ЭВОЛЮЦИЯ ГАЛАКТИК РАННИХ ТИПОВ

Аннотация. В статье мы используем два набора моделирования N-тела сверхвысокого разрешения Phoenix и Aquarius Projects для изучения истории сборки суб-гало и ее зависимости от массы основного гало. Мы обнаружили, что более массивные гало имеют больше предшественников, что контрастирует с предыдущими работами, потому что они неоднократно считали динамические предшественники. Менее массивные гало имеют большую долю динамических предшественников, чем более массивные. Типичное время аккреции сильно зависит от массы основного гало. Предшественники галактических гало аккрецируются на более высоком красном смещении, чем у гало скоплений. Как только эти предшественники вращаются вокруг своих первичных систем, они быстро теряют свою первоначальную массу, но не свои идентификаторы. Большинство предшественников доживают до наших дней. При данном красном смещении доля выживания суб-гало не зависит от массы основного гало, в то время как суб-гало, ϕ в основном гало с большой массой теряют больше массы.

Во второй части мы используем полуаналитическую модель образования галактик, скомпилированную с помощью Millennium Simulation, для изучения эволюции размеров массивных галактик ранних типов от красного смещения $z = 2$ до наших дней. Мы обнаружили, что использованная нами модель способна хорошо воспроизвести амплитуду и наклон зависимости размер-масса, а также ее эволюцию. Амплитуда этого соотношения отражает типичную компактность гало темной материи в то время, когда образуется большинство звезд. Эта связь между размером и эпохой звездообразования распространяется через комбинации галактик. Второстепенные комбинации становятся все более важными с увеличением современной звездной массы для галактик с массивом более $10^{11.4} M_{\odot}$. При более низких массах более важны основные комбинации. Звездо-

образование больше способствует увеличению размеров, чем росту звездной массы. Подобно предыдущим работам, мы обнаруживаем, что незначительные комбинации доминируют в последующем росте как звездной массы, так и размера для рано сформировавшихся галактик ранних типов.

Ключевые слова: гало темной материи, полуаналитическое образование галактик.

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