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A MICROSCOPIC TWO-CLUSTER MODEL OF PROCESSES IN ^{6}Li

Abstract. Structure of bound and resonance states and dynamics of different processes in ^{6}Li are investigated within a microscopic two-cluster model. The model is an algebraic version of the resonating group method which correctly treats the Pauli principle and makes use of the oscillator basis to expand a wave function of a two-cluster system. Within this model the nucleus ^{6}Li is considered as two-cluster $\alpha + d$ system. Dynamics of the two-cluster system is totally governed by a semi-realistic nucleon-nucleon potential. We study interaction of ^{6}Li with electrons and photons. Form factors of elastic scattering of electron are determined for two nucleon-nucleon potentials. We also study the density distribution of protons and neutrons in ^{6}Li. We demonstrate that the model used correctly reproduces form factors for the ground state of ^{6}Li. Our results are compatible with results of other microscopic models. The capture reaction $\alpha + d = {}^6\text{Li} + \gamma$ are investigated in detail. The astrophysical S-factor of the reaction is obtained at low-energy region.

Keywords: cluster model, light nuclei, capture reaction, density distribution.

1. Introduction. In this paper we consider in more detail the structure of ^{6}Li. We will consider the processes initiated by the interaction of ^{6}Li with electrons and photons. Interaction of electrons with ^{6}Li will be represented by a form factor of the electrons elastic scattering from the ^{6}Li ground state. Interaction of photons with ^{6}Li will be represented by a cross section of the ^{6}Li photodisintegration $\gamma + {}^6\text{Li} = \alpha + d$, or by a cross section and the astrophysical S-factor of the radiative capture reaction $\alpha + d = {}^6\text{Li} + \gamma$. It is well known that the radiative capture reaction $\alpha + d = {}^6\text{Li} + \gamma$ is very important for the astrophysical applications.

To achieve these goals we apply a two-cluster microscopic model. This model is so-called the algebraic version of the resonating group method formulated in Refs. [1], [2]. The main peculiarity of the algebraic version is that it employs a full set of oscillator functions to describe the relative motion of the interacting clusters in bound and continuous spectrum states. In Ref. [3], which includes many important details of formulation and implementations of the algebraic version of the resonating group, this model was applied to study the lightest nuclei of the p-shell represented by the dominant two-cluster configuration. It was shown that this model reproduces the main features of the nucleus ^{5}He, ^{5}Li, ^{6}Li, ^{7}Li, ^{7}Be and ^{8}Be. In the present paper we concentrate our attention on some interesting features of ^{6}Li which have not been considered in Ref. [3].

2. Two-cluster model. The total wave function describing the state of two-cluster systems within the resonating group method can be represented as:

$$\Psi_J = \hat{A}\{[\,\varphi_1(A_1)\,\varphi_2(A_2)]_s\psi^J_{LS}(\,\vec{q})\}, \qquad (1)$$

where $\varphi_1(A_1)$ is a wave function describing the internal motion of A_1 nucleons which comprise the first cluster, $\varphi_2(A_2)$ is a wave function describing the internal motion of A_2 nucleons which comprise the second cluster. Both functions depend on the spatial, spin and isospin coordinates of individual nucleons.

And both functions are antisymmetric with respect to a permutation of any pair of nucleons. It is assumed within the resonating group method that these functions are known. There are some simple methods how to construct such functions. Contrary to the functions $\varphi_1(A_1)$ and $\varphi_2(A_2)$, the wave function $\psi_{LS}^J(\vec{q})$ which describes the relative motion of two clusters is unknown and has to be determined by solving dynamical equations of the resonating group method. This function is a function of the Jacobi vector $\vec{q}$, which determines the distance between clusters.

Within the algebraic version of the resonating group method, the wave function $\psi_{LS}^J(q)$ is decomposed into an infinite series of the three-dimension harmonic oscillator wave functions $\psi_n(q, r_0)$:

$$\psi_{LS}^J(q) = \sum_{n=n_0}^{\infty} \mathrm{C}_{nL}\, \psi_{nL}(q, r_0), \tag{2}$$

where C_{nL} is the expansion coefficient, q is the modulus of the vector $\vec{q}$. The explicit form of the oscillator functions $\psi_n(q, r_0)$ can be found in Ref. [3]. As oscillator functions form a complete set of orthonormal functions, then any wave function of a two-cluster system can be expanded over these functions. In the framework of the algebraic version of the resonating group method, the wave function (1) of two-cluster system, by taking into account Eq. (2), can be represented as a generalized Fourier series

$$\Psi_J = \sum_{n=n_0}^{\infty} \mathrm{C}_{nL}\, \Psi_{nL}, \tag{3}$$

it also yields the dynamical equations for the expansion coefficients as a set of the linear algebraic equations

$$\sum_{m=n_0}^{\infty} [\langle \bar{n}L \,|\hat{\mathrm{H}}|\, \bar{m}L\rangle - \mathrm{E}\cdot\delta_{n,m}]\, \bar{\mathrm{C}}_{mL} = 0, \tag{4}$$

where $\hat{\mathrm{H}}$ is the many-particle Hamiltonian of the nucleus, E is the total energy of the nuclear system, and

$$\psi_{nL} = A\{\varphi_1(A_1)\, \varphi_2(A_2)\psi_{nL}(q, b)Y_{LM}(\vec{\boldsymbol{q}})\} \tag{5}$$

is the many-particle, cluster oscillator function.

To construct matrix elements of Hamiltonian and other operators of physical importance we employ the technique of the generating functions [4]. The generating functions of two-cluster system is the Slater determinant constructed from single-particle Brink orbitals. A single-particle Brink orbital is connected with Gaussian functions:

$$\phi(\boldsymbol{r}_i, \boldsymbol{R}_j) = \exp\left\{-\left(\boldsymbol{r}_i - \boldsymbol{R}_j\right)^2 / (2b^2)\right\}, \tag{6}$$

where $\boldsymbol{r}_i$ is a coordinate of the ith nucleon (i = 1, 2, …, A) and $\boldsymbol{R}_j$ (j = 1, 2) is a generator parameter representing center of mass of jth cluster. After elimination of a wave function of the center of mass motion, we obtain the translationally invariant generating function of A nucleons system. We denote this function as $\boldsymbol{\Phi(R)}$, where $\boldsymbol{R}$ is the generator coordinate associated with relative position of clusters in the space and equals

$$\boldsymbol{R} = \sqrt{\frac{A_1 A_2}{A_1 + A_2}}\,[\boldsymbol{R}_1 - \boldsymbol{R}_2]. \tag{7}$$

By using properties of the determinant functions, we can easy calculate the kernel of the Hamiltonian (or the generating matrix elements of the Hamiltonian)

$$\langle \boldsymbol{R} \,|\hat{H}|\, \widetilde{\boldsymbol{R}}\rangle = \langle \boldsymbol{\Phi}(\boldsymbol{R}) \,|\hat{\mathrm{H}}|\, \boldsymbol{\Phi}(\widetilde{\boldsymbol{R}})\rangle, \tag{8}$$

the norm kernel

$$\langle \boldsymbol{R}|\widetilde{\boldsymbol{R}}\rangle = \langle \boldsymbol{\Phi}(\boldsymbol{R})|\boldsymbol{\Phi}(\widetilde{\boldsymbol{R}})\rangle \tag{9}$$

and the generating matrix elements of other operators.

The important feature of the generating function $\boldsymbol{\Phi}(\boldsymbol{R})$, as was shown in Ref. [4], is that it generates an infinite set of cluster oscillator functions (5)

$$\boldsymbol{\Phi}(\boldsymbol{R}) = \sum_{n=0}^{\infty} \sum_{L,M} a_{nL} R^{2n+L} Y_{LM}(\widehat{\boldsymbol{R}}) \psi_{nL}\,. \tag{10}$$

To extract the necessary cluster oscillator function ψ_{nL} from the generating function we use the following procedure

$$\psi_{nL} = \frac{1}{a_{nL}} \frac{1}{(2n+L)!} \left(\frac{d}{dR}\right)^{2n+L} \int d\widehat{\boldsymbol{R}} Y^*_{LM}(\widehat{\boldsymbol{R}}) \boldsymbol{\Phi}(\boldsymbol{R})|_{R=0}. \quad (11)$$

Thus we need to integrate over the unit vector $\widehat{\boldsymbol{R}}$ with the weight $Y^*_{LM}(\widehat{\boldsymbol{R}})$ and then differentiate $2n + L$ times with respect to R. After differentiation we have to put $R = 0$. Such a projecting procedure can be applied directly to the matrix elements of the kernel of the Hamiltonian

$$\langle nL \,|\hat{\mathrm{H}}|\, mL\rangle = \frac{1}{a_{nL}} \frac{1}{(2n+L)!} \left(\frac{d}{dR}\right)^{2n+L} \int d\widehat{\boldsymbol{R}} Y^*_{LM}(\widehat{\boldsymbol{R}}) \quad (12)$$

$$\times \frac{1}{a_{mL}} \frac{1}{(2m+L)!} \left(\frac{d}{d\tilde{R}}\right)^{2m+L} \int d\widehat{\tilde{\boldsymbol{R}}} Y^*_{LM}\left(\widehat{\tilde{\boldsymbol{R}}}\right) \langle \boldsymbol{R} \,|\hat{H}|\, \tilde{\boldsymbol{R}}\rangle|_{R=\tilde{R}=0}\,.$$

We use this procedure to construct matrix elements of proton and neutron form factors and operator of electromagnetic transitions, which we need to calculating cross sections of the capture reactions or photodisintegration reactions.

3. Definition of form factor and density distribution. The proton and neutron form factors are determined as the matrix elements

$$F_p(q) = \langle \psi_{EJ^\pi} | \widehat{F_p} | \psi_{EJ^\pi} \rangle, \quad (13)$$

$$F_n(q) = \langle \psi_{EJ^\pi} | \widehat{F_n} | \psi_{EJ^\pi} \rangle, \quad (14)$$

calculated with a wave function ψ_{EJ^π} describing a bound states of a nucleus. Here the operators $\widehat{F_p}$ and $\widehat{F_n}$ are

$$\widehat{F_p} = \frac{1}{2} \sum_{i=1}^{A} (1 + \widehat{\tau_{iz}}) \exp\{i(\mathrm{q}r_i)\}\,, \quad (15)$$

$$\widehat{F_n} = \frac{1}{2} \sum_{i=1}^{A} (1 - \widehat{\tau_{iz}}) \exp\{i(\mathrm{q}r_i)\}\,, \quad (16)$$

and the operators $\frac{1}{2}(1 + \widehat{\tau_{iz}})$ and $\frac{1}{2}(1 - \widehat{\tau_{iz}})$ are projection operators on proton and neutron state, respectively.

The proton and neutron density distribution are determined as

$$D_p(r) = \left\langle \psi_{EJ^\pi} \middle| \tfrac{1}{2} \textstyle\sum_{i=1}^{A} (1 + \widehat{\tau_{iz}})\, \delta(r - r_i) \middle| \psi_{EJ^\pi} \right\rangle, \quad (17)$$

$$D_n(r) = \left\langle \psi_{EJ^\pi} \middle| \tfrac{1}{2} \textstyle\sum_{i=1}^{A} (1 - \widehat{\tau_{iz}})\, \delta(r - r_i) \middle| \psi_{EJ^\pi} \right\rangle, \quad (18)$$

To calculate $F_p(q)$, $F_n(q)$, $D_p(r)$ and $D_n(r)$, one needs to perform in Eqs. (13), (14), (17) and (18) integrations over all spatial, spin and isospin coordinates.

For numerical calculations of properties of the ^{6}Li nucleus, we selected two nucleon-nucleon potentials: the Minnesota potential (MP) [5] and the modified Hasegawa-Nagata [6, 7] potential (MHNP). This is done in order to study the dependence of calculated quantities of ^{6}Li on the shape of selected nucleon-nucleon potentials.

Here, we are not going to discuss the wave function of the ground state of ^{6}Li as it was discussed in Ref. [3]. And we will not present wave functions or phase shifts of the elastic scattering of deuterons from an alpha particle as they were presented in Ref. [3]. It is necessary to note that phase shifts of the elastic α + d scattering and the parameters of resonance state are in a good agreement with the experimental data.

4. Form factor and density distribution. Within the two-cluster model used, the total isospin T = 0 for ^{6}Li, thus the neutron form factor coincides with the proton form factor. This can be deduced from the definitions of the proton and neutron form factors Eqs. (13) and (14). And this is why the proton, neutron and mass density distributions are also identical. Thus we display only the proton form factor and proton density distribution. The proton density distributions D(r) obtained with MP and MHNP are display in figure 1. There is a substantial difference in the proton density distributions for these potential at small distances r. The proton density distribution in the ground 1^+ state of ^{6}Li calculated with MP and MHNP. The proton form factor F(q) for elastic scattering of electrons from the 1^+ ground state of ^{6}Li are shown in figure 2. As we can see, the first minimum of the form factor are in the same point of the transferred momentum q for both potentials. As it is well known the range of small values of q associated with the root-mean-square radius of ^{6}Li, which has approximately the same value for MP and MHNP [8, 9].

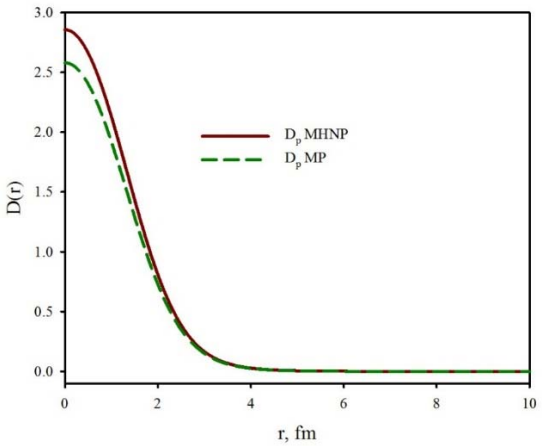

Figure 1 – The proton density distribution in the ground 1^+ state of ^{6}Li calculated with the MP and MHNP

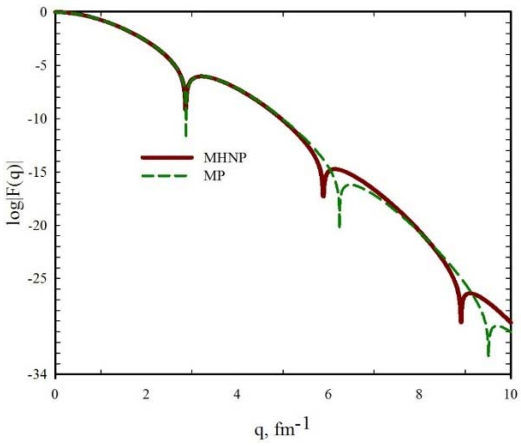

Figure 2 – The proton form factor F(q) from the ^{6}Li ground state. Results are obtained with MP and MHNP

5. The capture reaction α + d = ^{6}Li + γ. The cross section and astrophysical S factor of the reaction α + d = ^{6}Li + γ are calculated by employing the quadrupole transitions only. It has been repeatedly shown that the quadrupole transitions from continuous spectrum states to the ground state of ^{6}Li dominates in the low energy region. Having calculated wave functions of the ground state of ^{6}Li and continuous spectrum states in the channel α + d, we then easily calculated the quadrupole transitions and cross section of the capture reaction. The explicit formula for calculating the cross section of the capture reaction can be found, for example, in Ref. [10, 11]. It is worthwhile noticing that the quadrupole transitions connect the ground 1^+ state with three states of the continuous spectrum, namely, $J^\pi = 3^+$, $J^\pi = 2^+$ and $J^\pi = 1^+$. Within the present model, the total spin S is a good quantum number and S=1. Thus, the total orbital momentum L is also a good quantum number. For the ground state L = 0, and for the scattering states $J^\pi = 3^+$, $J^\pi = 2^+$ and $J^\pi = 1^+$ the total orbital momentum L = 2. The spin-orbit interaction splits the state with the total orbital momentum L = 2 on three states with the total angular momentum $J^\pi = 3^+$, $J^\pi = 2^+$ and $J^\pi = 1^+$. As the total isospin of ^{6}Li T=0, then the dipole transitions are forbidden in the long-wave approximation. It is well-known, that the dipole transition operator in the long-wave approximation contains only the isovector part, thus the dipole transitions are allowed for the nuclei with total isospin T ≥ ½.

In table 1 we show the energy and width of the $J^\pi = 3^+$, $J^\pi = 2^+$ and $J^\pi = 1^+$. These resonance states, as we can see later, play an important role in the capture reaction. It was shown in Ref. [3] that these resonance states have a great impact on the cross section of the elastic α + d scattering. The energy E of the resonance states are measured from the α + d threshold. Results of our calculation with the MP and the MHNP are compared with the experimental data [12-14]. There is a fairly good agreement between the theoretical and experimental data. As we can see, the MHNP provides more correct description of the 3^+, 2^+ and 1^+ resonance state, than the MP.

Table 1 - Experimental and theoretical values of the parameters of resonance states of ^{6}Li

Potential	L; J^π	E (MeV)	Γ (MeV)
MHNP	2; 1^+	4.100	2.357
	2; 2^+	3.063	1.013
	2; 3^+	0.763	0.019
MP	2; 1^+	5.174	6.33
	2; 2^+	4.288	3.005
	2; 3^+	0.848	0.028
Experimental data [12, 13]	2; 1^+	4.176±0.050	1.5±0.2
	2; 2^+	2.838 ±0.022	1.30 ± 1.00
	2; 3^+	0.712 ±0.002	0.024 ± 0.002

As was indicated above, all resonances in the ^{6}Li nucleus mentioned in Table 1 have the total orbital momentum L = 2, which indicates the presence of a centrifugal barrier in these states. The effective barrier in the α + d channels is complemented also by the Coulomb barrier. The spin-orbital forces increase the effective attraction between clusters in the 3^+ state and thus generates a very narrow resonance at low energies. At this energy range, the width of the barrier is large which difficult to penetrate. This stipulates the appearance of a very narrow (long-lived) resonance state. In the 1^+ state, the spin-orbital forces, contrary to the 3^+ state, reduce the interaction of the alpha particle and the deuteron and push the resonance state to a relatively larger energy, where the effective barrier is small. The spin-orbit forces have small effect on the 2^+state, as the consequence the resonance width is less than in the 1^+ state, but larger than in the 3^+ state.

In figures 3 and 4 we display the partial and the total astrophysical S factors of the reaction α + d = ^{6}Li + γ as a function of the energy E of relative motion of clusters alpha-particle and deuteron. One can immediately notice a peak below the energy E = 1 MeV. This peak is associated with the narrow 3^+ resonance state. The contribution to the partial and total S factors of wider 2^+ and 1^+ resonance states is not so prominent. By comparing Fig. 3 and Fig. 4, we see explicitly how the astrophysical S factor of the capture reaction α + d = ^{6}Li + γ depends on the shape of the nucleon-nucleon potentials used in our calculations. It is important to note, that the transition from the 3^+ continuous spectrum states to the ground state dominates at the low energy range 0 ≤ E < 1 MeV. At the higher energies, the transition from the 2^+ continuous spectrum states to the ground state is more prominent than the other transitions.

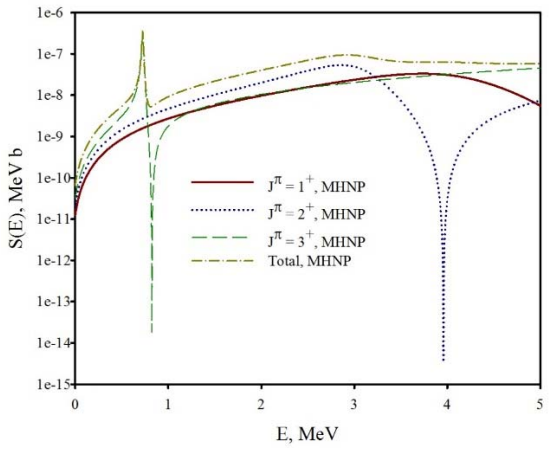

Figure 3 – The astrophysical S-factors generated by the quadrupole transitions. Results are obtained with the modified Hasegawa-Nagata potential

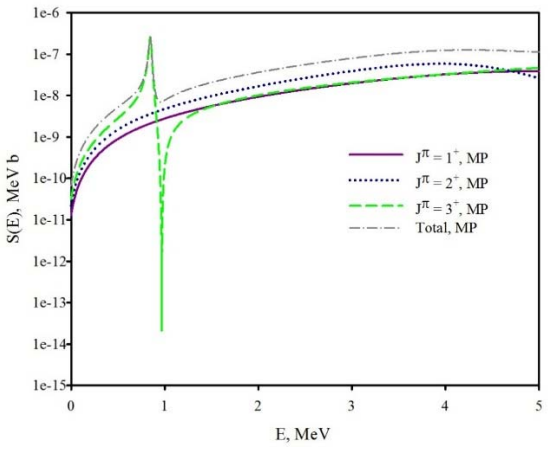

Figure 4 – The astrophysical S-factors of reaction α + d = ^{6}Li + γ generated by the quadrupole transitions. Results are obtained with the potential of Minnesota

The astrophysical S factor of the capture reaction α + d = ^{6}Li + γ obtained by the theoretical or experimental methods usually is approximated by the three terms expression:

$$S(E) = S_0 + S_1E + S_2E^2 \,. \qquad (19)$$

For this aim we selected the energy interval $0 \leq E \leq 0.2$ MeV. In Table 2 we display parameters of the three terms fit to the calculated astrophysical S-factor of the capture reactions α + d = ^{6}Li + γ. In the table we show partial astrophysical *S*-factors $S\,(E; J_i^\pi \Rightarrow J_f^\pi)$ and the total *S*-factor

$$S_T(E) = \sum_{J_i^\pi} S\big(E;\, J_i^\pi \Rightarrow J_f^\pi\big), \tag{20}$$

In our calculations this sum involves three partial astrophysical *S*-factors.

Table 2 - Parameters of three-term fitting of the calculated astrophysical *S*-factor of the reaction α + d = ^{6}Li + γ.

Potential	Transition	S_0	S_1	S_2
MHNP	$3^+ \Rightarrow 1^+$	3.092×10^{-11}	10.923×10^{-10}	7.455×10^{-9}
	$2^+ \Rightarrow 1^+$	2.035×10^{-11}	7.878×10^{-10}	4.370×10^{-9}
	$1^+ \Rightarrow 1^+$	1.214×10^{-11}	4.716×10^{-10}	2.593×10^{-9}
	Total	6.341×10^{-11}	23.517×10^{-10}	14.418×10^{-9}

The total astrophysical S factor obtained in our model is in a fairly good agreement with the available experimental data and is also compatible with results of other theoretical two- and three-cluster models.

6. Conclusions. We have applied a two-cluster microscopic model to study structure of the nucleus ^{6}Li. This nucleus was represented as a two-cluster configuration α + d, which determines the main properties of ^{6}Li at the low energy region. Two well-known nucleon-nucleon potentials were employed to study properties of ^{6}Li and their dependence on the shape of the potentials. The present model correctly reproduced the main features of bound and scattering states in ^{6}Li.

We have calculated the form factor of the elastic scattering of electron from the ground state of ^{6}Li. It was shown that at region of the small values of the transferred momentum q the form factor F(q) slightly depends on the shape of the nucleon-nucleon potentials, however for large values of the momentum q the shape of the potentials has the large impact on the form factor. As for the proton density distribution D(r), the shape of the nucleon-nucleon potentials has noticeable impact at the region of small distances r.

We have also studied the dynamics of the capture reaction α + d = ^{6}Li + γ at small energy region which is of a great importance for the different astrophysical applications. We have calculated the partial and total astrophysical S factor S(E) of the reaction. We have demonstrated how the low-lying resonance states 3^+, 2^+ and 1^+ contribute to astrophysical S factor of the capture reaction. It was shown that our model provides a satisfactory description of the S factor in the range of energy $0 \leq E \leq 1$ MeV.

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^{6}LI-ДЕГІ МИКРОСКОПИЯЛЫҚ ЕКІКЛАСТЕРЛІК ҮДЕРІС ҮЛГІСІ

Аннотация. Жұмыста байланысқан және резонансты күйлердің құрылымы, сонымен бірге ^{6}Li-дегі әртүрлі үдеріс динамикасы микроскопиялық екікластерлік үлгі аясында зерттелген. Бұл тәсілдің негізі abinitio және екікластерлік жүйенің толқындық функцияларын осцилляторлық негізде жіктеу үшін қолданатын резонансты топ әдісінің алгебралық нұсқасы (РТӘАН). ^{6}Li мәселесін қарастыру бастапқы нуклеосинтез кезеңінде ^{6}Li шығуының негізгі реакциясы саналатын радиациялық қарпу реакциясы тұрғысынан туындайды. Осы мәселеге екі көзқарасты қолдану өте қажет, өйткені ^{6}Li радиациялық қарпу

реакциясында алты өзара әрекеттесетін нуклондар бар, олар сәйкесінше алты дене қатысатын күрделі есепті тудырады. РТӘАН құралдары бұл мәселені екі дене есебіне: альфа-бөлшек α және сутегі изотопы дейтерий *d*-ге азайта алады. Паули принципін және антисимметризацияны ескермей, өзара әрекеттесетін кластерлердің толқындық функциясын гармоникалық осциллятор функциясының базистер жиынтығы бойынша жіктеуге мүмкіндік береді. Бұл өз кезегінде есептеуді едәуір жеңілдетуге және жұмыс соңында тәжірибелік мәліметтер негізінде сәйкес нәтижелерге қол жеткізуге мүмкіндік берді.

Зерттеліп жатқан жүйе динамикасы толығымен нуклон-нуклондық потенциалымен анықталды. Нәтижесінде ядролық әсерлесуді сипаттау үшін екі нуклон-нуклондық потенциал қолданылды: модификацияланған Хасегава-Нагата потенциалы (МХНП) және Миннесота потенциалы. Модификацияланған Хасегава-Нагата потенциалы жұп нуклон-нуклондық әсерлесу үшін алыс арақашықтықтарда тартылысты және жақын арақашықтықтарда тебілісті жақсы шығарды. Бұл потенциал көптеген жағдайда, атап айтқанда, жеңіл ядролардың энергияның кең диапазонында шашырау мәліметтерін сипаттауда, электромагниттік қасиеттердің және жеңіл ядроларының қатысуы арқылы өтетін үдерістерді сипаттау үшін қолданды. Миннесота потенциалы спин-орбиталық компоненті бар үш гаусс функциясымен анықталады.

Жұмыстың негізгі мақсаты – эксперименталды түрде резонансты күйлері бақыланған, астрофизикалық 5 МэВ энергияға жақын төменгі энергиялы интервалда ^{6}Li жеңіл ядро реакцияларындағы ядролық күштердің әсері мен табиғатын зерттеу. Жасалған жұмыстың мақсаты осындай резонанстық күйді теориялық есептеу.

Кластерлік үлгінің артықшылығы, зерттеліп жатқан бөлшектер, олардың құрылымы кейбір жағдайларда назардан тыс қалуы мүмкін нуклондардың ядролық уақытының конгломерациялары өлшемі түрінде ұсынылды. Сонымен қатар, зерттелген ядролар бірқатар қызықты сипаттамаларға ие болды. Радиациялық қарпу реакциясындағы ^{6}Li ядросында ұзақ толқындардың жақындауына байланысты дипольдік ауысуы мүмкін болмады, сондықтан тек квадруполдық ауысуы үшін зерттеулер жүргізілді.

Шредингер теңдеулерін шешуде қолданылатын шекаралық шарттарды енгізіп, осцилляторлық функциялардың базистері бойынша кластерлік қозғалыспен салыстырмалы түрдегі толқындық функцияларды алгебралық формада жіктеуге күрделі интегралды-дифференциалдық есептеулерді келтіріп, жеңіл ядролардың байланысқан және квазибайланысқан күйлерін есептеу үшін резонансты топтар әдісінің алгебралық нұсқасы қолданылды. Оның әртүрлі нуклон-нуклондық потенциалын қолдану арқылы астрофизикалық S-факторлары мен шашырау фазалары туралы мәліметтер алынды, олар кейіннен өңделді

Резонансты топтар әдісінің алгебралық нұсқасын қолдану арқылы алынған S-факторлар түріндегі теориялық нәтижелер тәжірибелік мәліметтермен салыстырылды, содан кейін әдіс пен эксперимент нәтижелері жақсы сәйкесті.

^{6}Li ядросын екікластерлік жүйе ретінде қарастыру кезінде туындайтын негізгі мәселенің бірі – ядроның электрондар мен фотондар әрекеттесуін зерттеу. Нәтижесінде екі нуклон-нуклондық потенциал үшін серпімді электронды шашыраудың форм-факторлары анықталды. Сондай-ақ, ядроның құрылымын зерттеу үшін ^{6}Li-дегі протондар мен нейтрондардың тығыздығының таралуын зерттеу қажет болды.

Резонанстық топтар әдісінің алгебралық нұсқасын қолдана отырып, жүргізілген есептеулер нәтижесінде электронның серпімді шашырау форма коэффициенті ^{6}Li ядросының негізгі күйі үшін есептелді. Осының салдарынан, берілген q импульстің аз шамасы аймағында F (q) форм-фактор нуклон-нуклондық потенциалынан әлсіз тәуелді екендігі, алайда q импульсінің үлкен мәнінде потенциалдың формасы форм-факторына көбірек әсер ете бастады. Протон тығыздығының D (r) таралуы нуклон-нуклондық потенциалдарының r арақашықтық аз болғанда аймақта айтарлықтай әсер ететінін көрсетті. Алынған ақпарат зерттеліп жатқан ядроның ішкі құрылымын қарастырып қана қоймай, сонымен бірге зерттеу деректерінде қолданылатын нуклон-нуклондық потенциалының табиғаты мен сипаттамалары туралы ақпарат берді.

Түйін сөздер: кластерлік үлгі, жеңіл ядролар, қарпу реакциясы, таралу тығыздығы.

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МИКРОСКОПИЧЕСКАЯ ДВУКЛАСТЕРНАЯ МОДЕЛЬ ПРОЦЕССОВ В ^{6}Li

Аннотация. Структура связанных и резонансных состояний, а также динамика различных процессов в ^{6}Li были исследованы в данной работе в рамках микроскопической двухкластерной модели. Основой данного подхода являются ab initio и алгебраическая версия метода резонирующих групп (АВМРГ), что использует осцилляторную основу для разложения волновой функции двухкластерной системы. Рассмотрение проблемы ^{6}Li происходит с позиции реакции радиационного захвата, которая является основной реакцией происхождения ^{6}Li в период первичного нуклеосинтеза. Использование этих двух подходов к данной задаче крайне необходимы, так как в реакции радиационного захвата ^{6}Li присутствует

шесть взаимодействующих между собой нуклонов, что создаёт весьма сложную задачу с участием шести тел. Инструменты АВМРГ позволяют сократить данную задачу до рассмотрения двух тел: альфа частицы α и изотопа водорода дейтерия *d*. Не пренебрегая принципом запрета Паули и антисимметризацией, данный подход позволяет разложить волновую функцию взаимодействующих кластеров по набору базисов функций гармонического осциллятора. Это в свою очередь позволило в значительной степени упростить расчеты и получить результаты, которые по окончанию работы были сопоставлены с экспериментальными данными.

Динамика же исследуемой системы полностью определялась полуреалистичным нуклон-нуклонным потенциалом. Вследствии этого для описания ядерного взаимодействия использовались два нуклон-нуклонных потенциала: модифицированный потенциал Хасегавы-Нагаты (МПХН) и потенциал Миннесоты. Модифицированный потенциал Хасегавы-Нагаты для парного нуклон-нуклонного взаимодействия хорошо воспроизводил притяжение на больших расстояниях и отталкивание на малых расстояниях. Применялся данный потенциал во многих случаях, в частности для описания данных по рассеянию легких ядер в широком диапазоне энергий, электромагнитных свойств и процессов с участием легких ядер. Потенциал Миннесоты в свою очередь определяется тремя гауссовыми функциями с присутствием спин-орбитальной компоненты.

Основой задачи данной работы было исследование характера и влияния ядерных сил в реакциях легких ядер ^{6}Li, при которых в интервале низких энергий близкому к астрофизическим энергиям до 5 МэВ, экспериментально наблюдались резонансные состояния. Целью же проделанной работы был теоретический расчет такого резонансного состояния.

Преимуществом кластерной модели являлось то, что исследуемые частицы представлялись в виде довольно устойчивых по меркам ядерного времени нуклонных конгломераций, внутренней структурой которых в некоторых случаях можно было пренебречь. К тому же исследуемые ядра обладали рядом интересных характеристик. Так ядро ^{6}Li в реакции радиационного захвата из-за длинноволнового приближения не могло иметь дипольный переход, в связи с чем исследования проводились лишь для квадрупольного перехода.

Алгебраическая версия метода резонирующих групп использовалась для расчетов, связанных и квазисвязанных состояний в легких ядрах, приводя сложные интегрально-дифференциальные вычисления к алгебраической форме методом разложения волновой функции относительного движения кластеров по базисам осцилляторных функций и, вводя граничные условия, по которым будет решаться уравнение Шредингера. Его решение с использованием различных нуклон-нуклонных потенциалов дало данные по астрофизическим S-факторам и фазам рассеяния, которые в последствии были обработаны.

Теоретические результаты в виде S-факторов, полученных путем расчетов с использованием алгебраической версии метода резонирующих групп сравнивались с экспериментальными данными, после чего было получено хорошее соответствие результатов метода с экспериментом.

Одной из основных задач возникающей при рассмотрении ядра ^{6}Li как двух-кластерной системы является изучение взаимодействия данного ядра с электронами и фотонами. Вследствие этого были определены форм-факторы упругого рассеяния электрона для двух нуклон-нуклонных потенциалов. Также для исследования структуры ядра нужно было изучить распределение плотности протонов и нейтронов в ^{6}Li.

В результате проводимых расчетов с использованием алгебраической версии метода резонирующих групп форм-фактор упругого рассеяния электрона вычислен из основного состояния ядра ^{6}Li. Вследствие этого было показано, что в области малых значений переданного импульса q форм-фактор F(q) слабо зависел от формы нуклон-нуклонных потенциалов, однако при больших значениях импульса q форма потенциалов стала иметь большее влияние на форм-фактор. Распределение плотности протонов D(r) показало, что форма нуклон-нуклонных потенциалов оказывает заметное влияние в области, когда расстояние r было мало.

Полученная информация позволила не только рассмотреть внутреннюю структуру исследуемого ядра, но также дала сведения о природе и характеристиках, используемых в данных исследования нуклон-нуклонных потенциалах.

Ключевые слова: кластерная модель, легкие ядра, реакция захвата, плотность распределения.

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