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INVESTIGATION OF VIRTUAL STATE OF ^{8}Be+n SYSTEM USING THE COMPLEX SCALING METHOD

Abstract. Nuclear states observed around threshold energies provide us with interesting problems associated with the nuclear cluster structure. Most of them are also interesting astrophysically from the viewpoint of nucleosynthesis.

The first excited $1/2^+$ state in ^{9}Be has been observed as a sharp peak above the ^{8}Be+n threshold energy in the photo-disintegration cross section of $\gamma+^{9}\text{Be}->\alpha+\alpha+n$. Since the size of the peak has a strong influence on the reaction rate of the ^{9}Be synthesis, new experimental data have been investigated. Recently, we performed calculations using an α+α+n three-cluster model together with the complex scaling method (CSM), which well reproduces the recently-observed photo-disintegration cross section. The results indicate that the virtual-state character of the $1/2^+$ state plays an important role in formation of the peak structure appearing in the cross section observed above the ^{8}Be+n threshold. From these results, we discuss that the first excited $1/2^+$ state in ^{9}Be is a ^{8}Be+*n* virtual state but not resonant one. However, the virtual state cannot be directly obtained as an isolated pole solution in the CSM, because the scaling angle in the CSM cannot be increased over the position of the virtual state pole on the negative imaginary axis of the complex momentum plane.

In our previous work, the cross section form of the photodisintegration and the phase shift behavior of a virtual state assuming a simple two-body model are discussed. In the CSM, the virtual state cannot be obtained as an isolated solution, but the continuum solutions are considered to include the effect of the virtual state. There is no previous study that the CSM can be applied successfully to virtual state. Applying the CSM to the simple schematic potential model, we have shown that the sharp peak of the photodisintegration cross section calculated just above the threshold which does not correspond to a usual Breit-Wigner type pole. A new approach for the CSM to describe the virtual state was proposed, and we discussed the pole position of the virtual state using the continuum level density (CLD), the scattering phase shift, and scattering length calculated in the CSM. The next problem is how to distinguish a virtual state from a resonant state and how to see the difference in the observed photo-disintegration cross sections.

The purpose of this work is to investigate the reliability of the virtual state solutions in the CSM as comparing with the solutions of the Jost function method. To investigate the structure of the virtual state, we calculate the energy eigenvalues, phase shifts and photodisintegration cross section of the two-body model with a two-range Gaussian potential by changing the strength of the attractive potential.

Keywords: Complex scaling method, continuum level density, phase shift.

Introduction. It is a long-standing problem to determine its resonance energy and width of the first excited $1/2^+$ state of ^{9}Be, which is closely connected with the problem to clarify whether it is a resonant state or not. Recently, we studied the $1/2^+$ unbound state of ^{9}Be and the photodisintegration cross section

applying the complex scaling method (CSM) [1-2] to the *α*+*α*+*n* three-cluster model [3]. The results indicate that there is no sharp resonant state corresponding to the distinguished peak observed just above the ^{8}Be+n threshold in the photodisintegration cross section of ^{9}Be. However, the recent experimental cross section data [4, 5] can be well explained by the *α*+*α*+*n* calculation. From these results, we concluded that the first excited $1/2^{+}$ state in ^{9}Be is a ^{8}Be+n virtual state but not resonant one.

The purpose of this work is to investigate the reliability of the virtual state solutions in the CSM as comparing with the solutions of the Jost function method. To investigate the structure of the virtual state, we calculate the energy eigenvalues, phase shifts and photodisintegration cross section of the two-body model with a two-range Gaussian potential by changing the strength of the attractive potential.

Complex scaled two-body model. To understand the characterization of the virtual state [7-9], we investigate the simple two-body model corresponding to the ^{8}Be+n structure in ^{9}Be. In this model, both clusters are assumed to have no-spin and the relative motion between clusters is described by the following Schrödinger equation:

$$H\Psi^{\nu}_{J^{\pi}} = E_{\nu}\Psi^{\nu}_{J^{\pi}} . \tag{1}$$

Where J^{π} is the spin and parity, and ν is the state index. The Hamiltonian is given as

$$H = -\frac{\hbar^2}{2\mu}\nabla^2 + V(r) . \tag{2}$$

Where we assume a simple Gaussian potential

$$V(r) = -V_1 \exp(-ar^2) , \tag{3}$$

where a = 0.16 fm^{-2}. For simplicity, we put $\frac{\hbar^2}{\mu} = 1$ (MeV fm^2).

In this study we use the CSM, in the CSM the relative coordinate r is transformed as

$$U(\theta): \quad r \to re^{i\theta} , \tag{4}$$

Where $U(\theta)$ is a complex scaling operator given by a scaling parameter θ. Applying this trans formation to Eq.(1), we obtain the complex-scaled Schrödinger equation:

$$H^{\theta}\Psi^{\nu}_{J^{\pi}}(\theta) = E^{\theta}_{\nu}\Psi^{\nu}_{J^{\pi}}(\theta) . \tag{5}$$

The complex-scaled Hamiltonian H^{θ} and wave function $\Psi^{\nu}_{J^{\pi}}(\theta)$ are defined as $U(\theta)HU(\theta)^{-1}$ and $U(\theta)\Psi^{\nu}_{J^{\pi}}$, respectively, and see Ref. [10] for detail.

Applying the L^2 basis function method, the wave function is expanded as

$$\Psi^{\nu}_{J^{\pi}}(\theta) = \sum_{n=1}^{N} c_n^{J^{\pi}\nu}(\theta)\phi_n(r) , \tag{6}$$

Where $\{\phi_n(r)\}$ is an appropriate basis function set. The expansion coefficients $c_n^{J^{\pi}\nu}(\theta)$ and the complex energy eigenvalues E^{θ}_{ν} are obtained by solving the complex-eigenvalue problem.

The scattering phase shifts $\delta^{J^{\pi}}(E)$ for the Hamiltonian Eq.(2) can be calculated using the solutions of the complex scaled Schrödinger Eq.(5) with and without interaction.

From Eq. (12) in Ref. [11-14], we have

$$\delta^{J^\pi}(E) = N_b\pi + \sum_{r=1}^{N_r^\theta}\left\{-\cot^{-1}\left(\frac{E-E_r^{res}}{\Gamma_r/2}\right)\right\} + \sum_{c=1}^{N_c^\theta}\left\{-\cot^{-1}\left(\frac{E-\varepsilon_c^r}{\varepsilon_c^i}\right)\right\} - \sum_{k=1}^{N}\left\{-\cot^{-1}\left(\frac{E-\varepsilon_k^{0r}}{\varepsilon_k^{0i}}\right)\right\}, \quad (7)$$

where the eigenvalues E_ν^θ are classified into N_b bound states, N_r^θ resonant states and N_c^θ continuum states for a given θ; $N = N_b + N_r^\theta + N_c^\theta$. The N eigenvalues of the free Hamiltonian given by the only kinetic energy operator are expressed as $\varepsilon_k^{0r} - i\varepsilon_k^{0i}$ $(k = 1,\ldots,M)$.

The photodisintegration cross section due to the electric dipole transition from the ground state $J_{gs}^\pi = 1^-$ to the continuum $J_f^\pi = 0^+$ states is expressed as

$$\sigma_{E1}(E_\gamma) = \frac{16\pi^2}{9}\cdot\left(\frac{E_\gamma}{\hbar c}\right)\frac{dB(E1,E_\gamma)}{dE_\gamma}, \quad (8)$$

where the transition strength is calculated by using the solutions of the CSM

$$\frac{dB(E1,E_\gamma)}{dE_\gamma} = -\frac{1}{\pi}\frac{1}{2J_{gs}+1}\mathrm{Im}\left[\sum_{\nu=1}^{N}\left\langle\tilde{\Psi}_{J_{gs}^\pi}^{gs}(\theta)\right\| \hat{O}^{\theta+}(E1) \left\|\Psi_{J_f^\pi}^{\nu}(\theta)\right\rangle \times\frac{1}{E-E_\nu^\theta}\left\langle\tilde{\Psi}_{J_f^\pi}^{\nu}(\theta)\right\| \hat{O}^{\theta}(E1) \left\|\Psi_{J_{gs}^\pi}^{gs}(\theta)\right\rangle\right]. \quad (9)$$

Numerical results. To understand the characterization of a virtual state, we investigate a simple two-body potential model for the ^{8}Be+n system in the CSM. The potential strength V_1 in Eq.(3) is taken to reproduce one bound state of s-and p-waves. The energy levels of $J^\pi = 0^+$ and 1^- obtained by solving the complex-scaled Schrödinger equation which are presented schematically in figure 1. The potential strength V_1 is taken to reproduce one bound 0^+state of s-waves. In this model, this 0^+ solution is assumed as the Pauli forbidden state, because this model simulates ^{8}Be(0^+)+n. Hence the 1^- solution describes the ground state, and there is no 0^+ bound state.

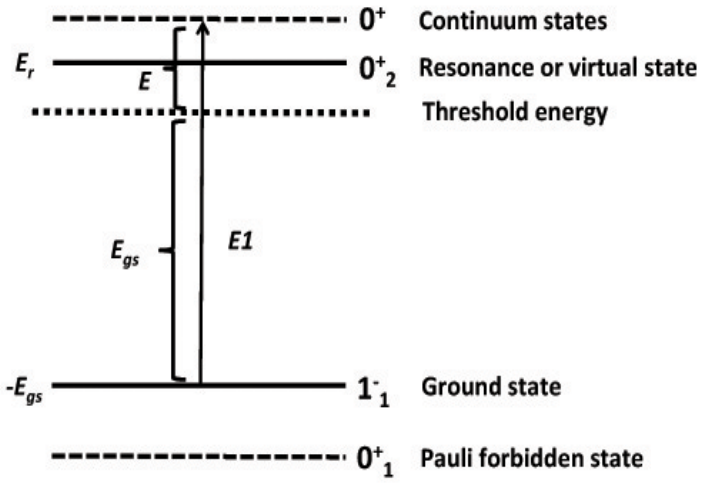

Figure 1 - Schematic diagram of the energy spectrum

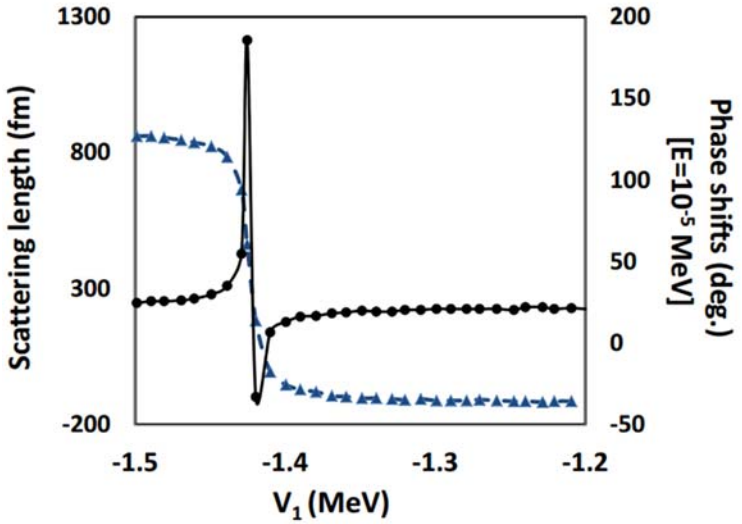

Figure 2 - The scattering length and phase shifts

In this work, we first apply different method ideally the Jost function method [8] for analyzing the virtual state to the simple schematic potential and two-body model. The Jost function method is applicable not only to study resonant states but also bound and scattering states. Using the Jost function method, poles of the *S*-matrix for a virtual state can be obtained without the complex rotation in the same way as bound states. The Jost function method is applied to calculate scattering phase shifts and scattering length at different strengths between $V_1 = -1.5$ MeV and -1.2 MeV. In figure 2, we show the calculated scattering length and scattering phase shifts applying the Jost function method. The blue with filled triangle and black with filled circle lines show the phase shifts and scattering length, respectively. From the scattering length we find that an increase at the strength $V_1 = -1.43$ MeV and sudden decrease at the strength $V_1 = -1.42$ MeV. It can be seen from figure 2, the scattering length shows an extreme sensitivity to the choice of the potential strength V_1. This result indicates the existence of the virtual state at the strength $V_1 = -1.42$ MeV which is consistent with our previous results in Ref. [6].

Next we consider the different case when there exists the resonance state to see how the resonance state contributes to the photodisintegration cross section. It is needed to perform calculation using both types of potentials (with and without a repulsion term). As a purely attractive potential we choose the potential Eq.(3) and with a repulsive term of the potential we employ as following

$$V(r) = -V_1 \exp(-ar^2) + V_2 \exp(-\gamma r^2), \tag{10}$$

Where $\gamma = 0.04$ fm^{-2}. We adjusted the strength of the potential for second term of Eq.(10) so that the resonant state generates when the V_2 switch on.

To investigate the structure of the obtained state, we calculate the energy eigenvalues of the system by changing the strength of the repulsive potential V_2 in Eq.(10), which is shown in figure 3. In the present calculation, when the strength of the repulsive potential $V_2 = 0.7$ MeV, the resonance pole suddenly appears just below the threshold. This resonance pole with a narrow decay width moves smoothly to the bound state region as the repulsive potential becomes zero, and we finally obtain the bound state with the region of $V_2 = 0.6$ MeV. On the other hand, we consider the pole trajectory in the opposite case of the repulsive potential with $V_2 = 0.7$ MeV. If the resonance exists, the pole with a narrow decay width should appear above the threshold as the analytical continuation from the resonance pole. However, we found that no resonances appear above the threshold for $V_2 > 0.7$ MeV of the repulsive potential.

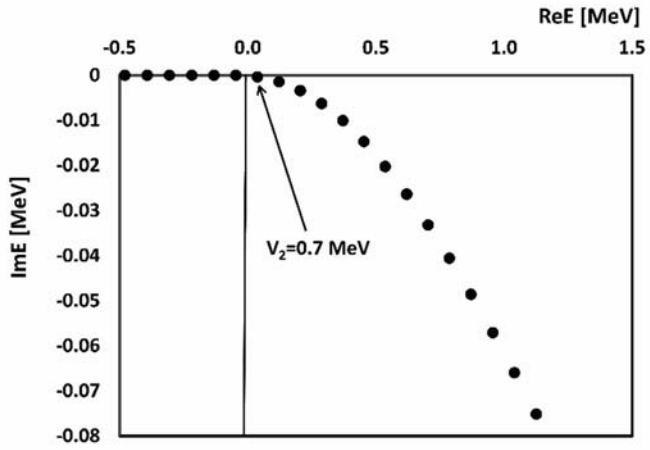

Figure 3 - Pole trajectory of the state in a complex energy plane by changing the repulsive potential

We consider the different case when there exists the resonance state to see how the resonance state contributes to the phase shifts. By using the repulsive potential with $V_2 = 0.7$ MeV, the resonance state is obtained with the energy and decay width being 0.038 and 6.58×10^{-4} MeV, respectively.

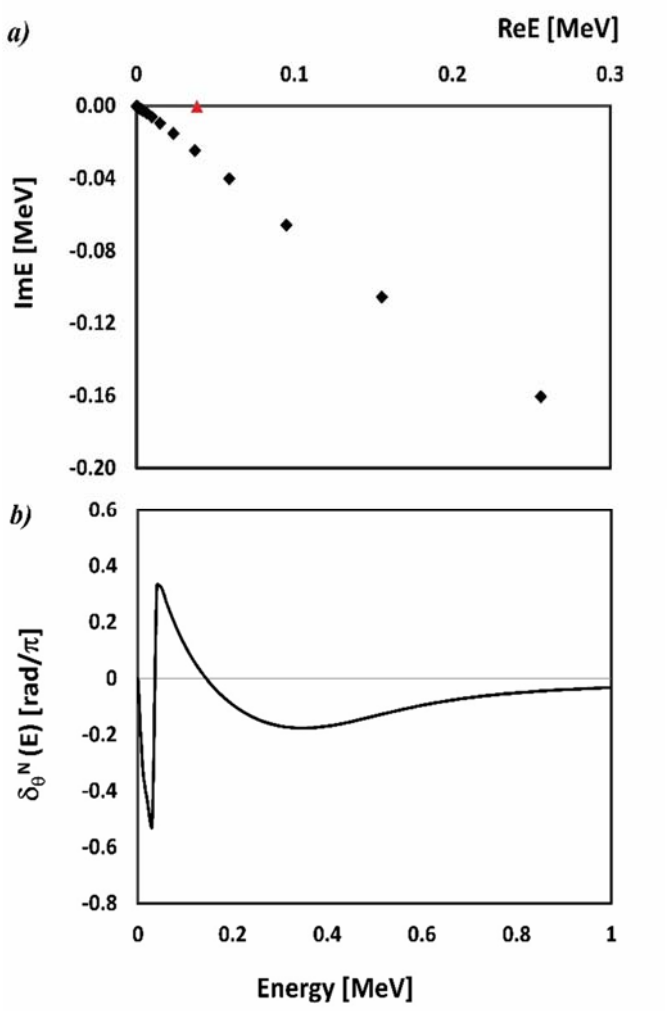

Figure 4 - Upper panel: Energy eigenvalues on the complex energy plane at V_1=-1.42 MeV and V_2=0.7 MeV. Bottom panel: Phase shifts at V_1=-1.42 MeV and V_2=0.7 MeV

In figure 4*a*) we show the distribution of eigenvalues in the complex energy plane calculated at the strengths $V_1 = -1.42$ MeV and $V_2 = 0.7$ MeV in Eq.(10). It can be seen from figure 4*a*), the segregated pole is obtained near the zero energy imaginary part which means a narrow resonance state is calculated and it is presented as a red triangle and continuum states are displayed by black diamonds. We can calculate the scattering phase shift using the eigenvalue solutions of $H(\theta)$ and $H(\theta)$ in the CSM [10], where $H_0(\theta)$ is the free-Hamiltonian without potentials. In figure 4*b*) the scattering phase shift is given which is calculated at the strengths $V_1 = -1.42$ MeV and $V_2 = 0.7$ MeV in Eq.(10). It is found that the effect of the narrow resonance state to the calculated phase shift is remarkable.

We investigate $E1$ transition from the 1^- ground state to the excited 0^+ unbound state including the 0_2^+ state. From the $E1$ transition we can calculate photodisintegration cross section using Eq.(9). In the present calculation, when the strength of the repulsive potential $V_2 = 0.7$ MeV, the resonance pole appears just below the threshold. In figure 5, we show the photodisintegration cross section, which shows a sharp peak at the resonance energy. It can be seen from the calculated photodisintegration cross section, we obtain the peak with a usual Breit-Wigner type pole and the peak shape of the calculated photodisintegration cross section is much different from figure 4 in Ref.[6] for the virtual state. This trend is much different from that figure 4 in Ref. [6]. Comparing with Fig. 4 in Ref.[6], we can understand how the shape of the photodisintegration cross section deviates from the Breit-Wigner type form for virtual state. As was discussed by Fano [15], deviation from the Breit-Wigner form can be investigated by calculating the interference between resonance and continuum terms.

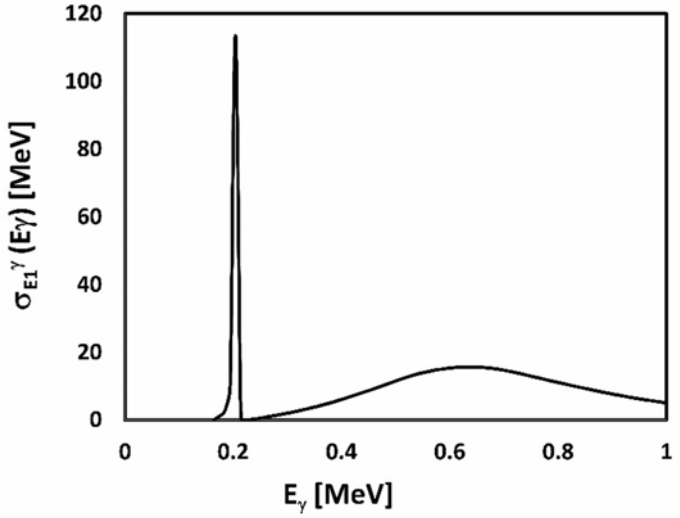

Figure 5 - Photodisintegration cross section at V_1=-1.42 MeV and V_2=0.7 MeV

Discussion and summary. We investigate the character of the virtual state using the CSM and the Jost function method. In both methods, the virtual state is obtained $V_1 = -1.42$ MeV. The pole trajectory of the resonance state is calculated at different repulsive potential strengths.

In the present calculation, we do not investigate the decomposed photodisintegration cross section in detail. It would be important to understand the structure of the resonance states and the virtual states, and the detailed analysis will be performed in a forth coming paper.

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^{8}Be + n ЖҮЙЕСІН КЕШЕНДІ МАСШТАБТАУ ӘДІСІ АРҚЫЛЫ ВИРТУАЛДЫ КҮЙІН ЗЕРТТЕУ

Аннотация. Табалдырық энергиялардың айналасында байқалған ядролық күйлер бізге ядролық кластер құрылымымен байланысты қызықты сұрақтар мен мәселелерді көтереді. Олардың көпшілігі нуклеосинтез тұрғысынан қарастырғанда астрофизикалық қызығушылық тудырады.

^{9}Be-дегі алғашқы қозған $1/2^{+}$ күйі γ + ^{9}Be-> α + α + n-дің фото-ыдырау қимасындағы ^{8}Be + n реакциясының табалдырық энергиясынан жоғары айқын шың ретінде байқалды. Шыңның өлшемі ^{9}Be синтезінің реакция жылдамдығына қатты әсер ететіндіктен, жаңа тәжірибелік мәліметтер зерттелді. Жақында біз үш өлшемді α + α + n моделін қолдана отырып, кешенді масштабтау әдісі (КМӘ) қолдандық, ол жақында байқалған фото-ыдырау қимасына жақсы сәйкес келеді. Нәтижелер $1/2^{+}$ күйінің виртуалды күйінің сипаттамасы ^{8}Be + n шегінен жоғары көлденең қимада пайда болатын шың құрылымын қалыптастыруда маңызды рөл атқаратынын көрсетеді. Осы нәтижелерге сүйене отырып, ^{9} Be-дегі алғашқы қозған $1/2^{+}$ күйі ^{8}Be + *n* виртуалды күйі екенін талқылаймыз, бірақ ол резонанс тудырмайды. Алайда виртуалды күйді кешенді масштабтау әдісінде (КМӘ) оқшауланған полюстің шешімі ретінде алу мүмкін емес, өйткені кешенді масштабтау әдісіндегі масштабтау бұрышы күрделі импульстік жазықтықтың виртуалды осіне қатысты виртуалды күй полюсінің орнына көбейтілмейді.

Біздің алдыңғы жұмысымызда фотодисинтеграцияның көлденең қимасы және виртуалды күйдің фазалық беталысы қарапайым екі денелі модельдің болжамы негізінде қарастырылды. Кешенді масштабтау әдісінде (КМӘ) виртуалды күйді оқшауланған шешім ретінде алу мүмкін емес, бірақ континуумдық шешімдер виртуалды күйдің әсерін қамтиды деп есептеледі. Кешенді масштабтау әдісін (КМӘ) виртуалды күйге сәтті қолдануға болатындығы туралы зерттеулер кездеспейді. Схемалық потенциалдың қарапайым моделіне кешенді масштабтау әдісін қолдана отырып, біз шекті деңгейден жоғары есептелген фотодиинтеграция қимасының шыңы Брейт-Вигнер типті полюсте сәйкес келмейтінін көрсеттік.

Виртуалды күйді сипаттау үшін кешенді масштабтау әдісінде жаңа тәсіл ұсынылды, біз виртуалды күйдің полюсті жағдайын үзіліссіз деңгейдің тығыздығын (ҮДТ), шашырау фазалық ығысуын және кешенді масштабтау әдісінде (КМӘ) есептелген шашырау ұзындығын қолдана отырып талқыладық. Келесі мәселе виртуалды күйді резонанстық күйден қалай ажыратуға және байқалған фото-ыдыраудың көлденең қималарындағы айырмашылықты көру жолдары болып саналады.

Жұмыстың мақсаты – Йост функциясы әдісінің шешімімен салыстырғанда кешенді масштабтау әдісіндегі (КМӘ) виртуалды күй шешімінің сенімділігін зерттеу. Виртуалды күйдің құрылымын зерттеу үшін алдымен тартылыс потенциалының күшін өзгерту арқылы екі диапазонды Гаусс потенциалы бар екі денелі моделіндегі энергияның меншікті мәндерін, фазалық жылжуын және фотодисинтеграция көлденең қимасын есептейміз.

Түйін сөздер: кешенді масштабтау әдісі, үзіліссіз деңгейінің тығыздығы, фазалық ығысу.

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ИССЛЕДОВАНИЕ ВИРТУАЛЬНОГО СОСТОЯНИЯ СИСТЕМЫ ^{8}Be + n С ИСПОЛЬЗОВАНИЕМ МЕТОДА КОМПЛЕКСНОГО МАСШТАБИРОВАНИЯ

Аннотация. Ядерные состояния, наблюдаемые вокруг пороговых энергий, предоставляют нам весьма интересные вопросы и задачи, связанные со структурой ядерного кластера. Большинство из них также имеет астрофизический интерес с точки зрения нуклеосинтеза.

Первое возбужденное состояние $1/2^+$ в ^{9}Be наблюдалось в виде резкого пика выше пороговой энергии ^{8}Be + n в сечении фоторасщепления γ + ^{9}Be-> α + α + n. Новые экспериментальные данные были исследованы, поскольку размер пика сильно влияет на скорость реакции синтеза ^{9}Be. Недавно мы провели расчеты с использованием трехкластерной модели α + α + n вместе с методом комплексного масштабирования (МКМ), который хорошо воспроизводит недавно наблюдаемое сечение фоторасщепления. Результаты показывают, что характер виртуального состояния $1/2^+$ играет важную роль в формировании структуры пика, появляющейся в поперечном сечении, наблюдаемом выше порога 8Be + n. Исходя из этих результатов, мы обсуждаем, что первое возбужденное состояние $1/2^+$ в ^{9}Be является виртуальным состоянием ^{8}Be + n, но не является резонансным. Однако виртуальное состояние не может быть непосредственно получено как решение с изолированным полюсом в МКМ, потому что угол масштабирования в методе комплексного масштабирования (МКМ) не может быть увеличен по положению полюса виртуального состояния на отрицательной мнимой оси плоскости комплексного импульса.

В нашей предыдущей работе было рассмотрено сечение фоторасщепления и фазовое поведение виртуального состояния в предположении простой модели двух тел. В методе комплексного масштабирования (МКМ) виртуальное состояние не может быть получено как изолированное решение, но считается, что континуальные решения включают в себя влияние виртуального состояния. Нет предыдущих исследований, что метод комплексного масштабирования (МКМ) может быть успешно применен к виртуальному состоянию. Применяя метод комплексного масштабирования (МКМ) к простой модели схематического потенциала, мы показали, что резкий пик сечения фоторасщепления, рассчитанный чуть выше порога, который не соответствует обычному полюсу типа Брейта-Вигнера.

Был предложен новый подход для метода комплексного масштабирования (МКМ) для описания виртуального состояния, и мы обсудили положение полюса виртуального состояния, используя плотность уровня континуума (ПУК), фазовый сдвиг рассеяния и длину рассеяния, вычисленные методом комплексного масштабирования (МКМ). Следующая проблема состоит в том, как отличить виртуальное состояние от резонансного и как увидеть разницу в наблюдаемых сечениях фото-распада.

Целью данной работы является исследование надежности решений виртуального состояния в методе комплексного масштабирования (МКМ), сравнивая их с решениями метода функции Йоста. Для того чтобы исследовать структуру виртуального состояния, мы в самом начале вычисляем собственные значения энергии, затем определяем фазовые сдвиги, а также путем изменения напряженности потенциала сил притяжения находим поперечное сечение фоторасщепления модели двух тел с двух диапазонным гауссовым потенциалом.

Ключевые слова: метод комплексного масштабирования, плотность уровня континуума, фазовый сдвиг.

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