ISSN 2518-1726 (Online),
ISSN 1991-346X (Print)

ҚАЗАҚСТАН РЕСПУБЛИКАСЫ
ҰЛТТЫҚ ҒЫЛЫМ АКАДЕМИЯСЫНЫҢ

ӘЛЬ-ФАРАБИ АТЫНДАҒЫ
ҚАЗАҚ ҰЛТТЫҚ УНИВЕРСИТЕТІНІҢ

Х А Б А Р Л А Р Ы

ИЗВЕСТИЯ

НАЦИОНАЛЬНОЙ АКАДЕМИИ НАУК
РЕСПУБЛИКИ КАЗАХСТАН

КАЗАХСКИЙ НАЦИОНАЛЬНЫЙ
УНИВЕРСИТЕТ ИМЕНИ АЛЬ-ФАРАБИ

N E W S

OF THE NATIONAL ACADEMY OF SCIENCES
OF THE REPUBLIC OF KAZAKHSTAN

AL-FARABI KAZAKH
NATIONAL UNIVERSITY

ФИЗИКА-МАТЕМАТИКА СЕРИЯСЫ
♦
СЕРИЯ ФИЗИКО-МАТЕМАТИЧЕСКАЯ
♦
PHYSICO-MATHEMATICAL SERIES

5 (321)

ҚЫРКҮЙЕК – ҚАЗАН 2018 ж.
СЕНТЯБРЬ – ОКТЯБРЬ 2018 г.
SEPTEMBER – OCTOBER 2018

1963 ЖЫЛДЫҢ ҚАҢТАР АЙЫНАН ШЫҒА БАСТАҒАН
ИЗДАЕТСЯ С ЯНВАРЯ 1963 ГОДА
PUBLISHED SINCE JANUARY 1963

ЖЫЛЫНА 6 РЕТ ШЫҒАДЫ
ВЫХОДИТ 6 РАЗ В ГОД
PUBLISHED 6 TIMES A YEAR

АЛМАТЫ, ҚР ҰҒА   АЛМАТЫ, НАН РК   ALMATY, NAS RK

«ҚР ҰҒА Хабарлары. Физика-математикалық сериясы».

ISSN 2518-1726 (Online), ISSN 1991-346X (Print)

Меншіктенуші: «Қазақстан Республикасының Ұлттық ғылым академиясы» РҚБ (Алматы қ.)
Қазақстан республикасының Мәдениет пен ақпарат министрлігінің Ақпарат және мұрағат комитетінде 01.06.2006 ж. берілген **№5543-Ж** мерзімдік басылым тіркеуіне қойылу туралы куәлік

Мерзімділігі: жылына 6 рет.
Тиражы: 300 дана.

Редакцияның мекенжайы: 050010, Алматы қ., Шевченко көш., 28, 219 бөл., 220, тел.: 272-13-19, 272-13-18,
www:nauka-nanrk.kz / physics-mathematics.kz

Типографияның мекенжайы: «Аруна» ЖК, Алматы қ., Муратбаева көш., 75.

«Известия НАН РК. Серия физико-математическая».

ISSN 2518-1726 (Online), ISSN 1991-346X (Print)

Собственник: РОО «Национальная академия наук Республики Казахстан» (г. Алматы)
Свидетельство о постановке на учет периодического печатного издания в Комитете информации и архивов Министерства культуры и информации Республики Казахстан **№5543-Ж**, выданное 01.06.2006 г.

Периодичность: 6 раз в год.
Тираж: 300 экземпляров.

Адрес редакции: 050010, г. Алматы, ул. Шевченко, 28, ком. 219, 220, тел.: 272-13-19, 272-13-18,
www:nauka-nanrk.kz / physics-mathematics.kz

Адрес типографии: ИП «Аруна», г. Алматы, ул. Муратбаева, 75.

News of the National Academy of Sciences of the Republic of Kazakhstan. Physical-mathematical series.

ISSN 2518-1726 (Online), ISSN 1991-346X (Print)

Owner: RPA "National Academy of Sciences of the Republic of Kazakhstan" (Almaty)
The certificate of registration of a periodic printed publication in the Committee of information and archives of the Ministry of culture and information of the Republic of Kazakhstan N 5543-Ж, issued 01.06.2006

Periodicity: 6 times a year
Circulation: 300 copies

Editorial address: 28, Shevchenko str., of. 219, 220, Almaty, 050010, tel. 272-13-19, 272-13-18,
www:nauka-nanrk.kz / physics-mathematics.kz

Address of printing house: ST "Aruna", 75, Muratbayev str, Almaty

N E W S
OF THE NATIONAL ACADEMY OF SCIENCES OF THE REPUBLIC OF KAZAKHSTAN
PHYSICO-MATHEMATICAL SERIES
ISSN 1991-346X **https://doi.org/10.32014/2018.2518-1726.9**
Volume 5, Number 321 (2018), 68 – 74

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PROBLEM FROM THE THEORY OF BRIDGE EROSION

Abstract. In this paper, we represent the exact solution of a two phase Stefan problem. Radial heat polynomialsand integral error function are used for solving bridge problem. The recurrent expressions for the coefficients of these series are presented. The mathematical models describe the dynamics of contact opening and bridging.
Keywords: radial heat polynomials, Stefan problem.

Introduction

In considerence the heat transfer, the shape of the liquid bridge plays an important role. The overwhelming majority of researchers proceed from the fact that the visible part of the bridge has the shape of a cylinder whose axis is directed perpendicular to the plane of the electrodes [1]. In the general case, in the result of the action of surface tension and the pinch effect, the bridge takes the form of a certain surface of revolution about the z- axis. In this problem, we consider a symmetric model of the bridge, where the shape of the bridge is described by a surface $y(z,t) = z^{\nu/2}$. For a liquid bridge in we consider the generalized heat equation for and for solid contact we use the spherical Holm model [2].

Preliminaries

The fundamental solution for the equation

$$\frac{\partial \theta}{\partial t} = a^2 \left(\frac{\partial^2 \theta}{\partial x^2} + \frac{\nu}{x}\frac{\partial \theta}{\partial x}\right) \tag{1}$$

can be obtained by the solution of this equation with the initial condition containing delta-function using the Laplace transform in the form [3]

$$G(x,y,t) = \frac{C_\nu}{2t}(xy)^{-\beta} e^{-\frac{x^2+y^2}{4t}} I_\beta\left(\frac{xy}{2t}\right), \qquad \beta = \frac{\nu-1}{2}, \qquad C_\nu = 2^{-\beta}\Gamma(\beta+1) \tag{2}$$

If we consider the corresponding heat potentials for this solution

$$Q_{n,\nu}(x,t) = 2^{-\beta}\Gamma(\beta+1)^{-1}\int_0^\infty G(x,y,t)y^{2n+\nu}dy \tag{3}$$

and integrating by parts we obtain the explicit formula for the heat polynomials

$$Q_{n,\nu}(x,t) = \sum_{k=0}^{n} 2^{2k}\frac{n!\Gamma(\beta+1)}{k!(n-k)!\Gamma(\beta+1+n-k)}x^{2n-2k}t^k \tag{4}$$

It is more convenient for applications to multiply both sides of this formula by $\frac{\Gamma(\beta+1+n)}{\Gamma(\beta+1)}$.

Then $R_{n,\nu}(r,t)=\frac{\Gamma(\beta+1+n)}{\Gamma(\beta+1)}Q_{n,\nu}(x,t)=\sum_{k=0}^{n}2^{2k}\frac{n!\Gamma(\beta+1)}{k!(n-k)!\Gamma(\beta+1+n-k)}x^{2n-2k}t^k$ (5)

Mathematical model

The heat equations for each zone are

$$\frac{\partial\theta_1}{\partial t}=a_1^2\left(\frac{\partial^2\theta_1}{\partial x^2}+\frac{\upsilon}{x}\frac{\partial\theta_1}{\partial x}\right)\quad \alpha(t)<x<0 \tag{6}$$

$$\frac{\partial\theta_2}{\partial t}=a_1^2\left(\frac{\partial^2\theta_2}{\partial r^2}+\frac{2}{r}\frac{\partial\theta_2}{\partial r}\right)\quad r_0<r<\beta(t) \tag{7}$$

$$\frac{\partial\theta_3}{\partial t}=a_2^2\left(\frac{\partial^2\theta_3}{\partial r^2}+\frac{2}{r}\frac{\partial\theta_3}{\partial r}\right)\quad \beta(t)<r<\infty \tag{8}$$

with boundaryandinitialconditions:

$$\alpha(0)=0,\ \beta(0)=r_0,\ \theta_1(0,0)=\theta_2(r_0,0)=\theta_m,\ \theta_3(r,0)=f(r),\ f(r_0)=\theta_m \tag{9}$$

$$-\lambda\pi\alpha^{\nu}(t)\left.\frac{\partial\theta_1}{\partial x}\right|_{x=\alpha(t)}=Q(t) \tag{10}$$

$$\theta_1(0,t)=\theta_2(r_0,t) \tag{11}$$

$$\lambda_1\frac{\partial\theta_1(0,t)}{\partial x}=2\lambda_2\frac{\partial\theta_2(r_0,t)}{\partial r} \tag{12}$$

$$\theta_2(\beta(t),t)=\theta_m \tag{13a}$$

$$\theta_3(\beta(t),t)=\theta_m \tag{13b}$$

The Stefan's condition

$$-\lambda_1\left.\frac{\partial\theta_2}{\partial r}\right|_{r=\beta(t)}=-\lambda_2\left.\frac{\partial\theta_3}{\partial r}\right|_{r=\beta(t)}+L\gamma\frac{d\beta}{dt} \tag{14}$$

$$\theta_3(\infty,t)=0 \tag{15}$$

Method of solution

We represent solution of the problem (6)-(15) ii the form

$$\theta_1(x,t)=\sum_{n=0}^{\infty}A_n\sum_{k=0}^{n}\zeta_{n,k}x^{2n-2k}t^k \tag{16}$$

$$\theta_2(r,t)=\sum_{n=0}^{\infty}C_n\sum_{k=0}^{n}\zeta_{n,k,2}x^{2n-2k}t^k+\sum_{n=0}^{\infty}D_n\frac{(2a_1\sqrt{t})^{2n+1}}{r}\left(i^{2n+1}erfc\frac{-(r-r_0)}{2a_1\sqrt{t}}-i^{2n+1}erfc\frac{(r-r_0)}{2a_1\sqrt{t}}\right) \tag{17}$$

$$\theta_3(r,t)=\sum_{n=0}^{\infty}E_n\sum_{k=0}^{n}\zeta_{n,k,2}x^{2n-2k}t^k+\sum_{n=0}^{\infty}G_n\frac{(2a_2\sqrt{t})^{2n+1}}{r}\left(i^{2n+1}erfc\frac{-(r-r_0)}{2a_2\sqrt{t}}-i^{2n+1}erfc\frac{(r-r_0)}{2a_2\sqrt{t}}\right) \tag{18}$$

Where $\zeta_{n,k} = \dfrac{2^{2k} n!\Gamma\left(\dfrac{\nu-1}{2}+n+1\right)}{k!(n-k)!\Gamma\left(\dfrac{\nu-1}{2}+n+1-k\right)}$ and $\zeta_{n,k,2} = \dfrac{2^{2k} n!\Gamma\left(\dfrac{3}{2}+n\right)}{k!(n-k)!\Gamma\left(\dfrac{3}{2}+n-k\right)}$,

andcoefficients A_n, C_n, D_n, E_n, G_n have to be determined. Free boundary $\beta(t)$ represent in the form [4] $\beta(t) = \sum_{n=0}^{\infty} \beta_n t^{n/2}$.

Using the boundary condition (10) we get

$$\sum_{n=0}^{\infty} A_n \sum_{k=0}^{n} \zeta_{n,k} (n-k) \alpha(t)^{2n-2k-1} t^k = F(t) \quad (19)$$

where

$$F(t) = -\frac{\mathrm{Q}(t)}{2\lambda\pi\alpha^{\nu}(t)}$$

Taking into account that $\alpha(t)$ is given and using properties of raising the power series to a power[5]

$$\alpha(t)^{2n-2k-1} = \sum_{m=0}^{\infty} \beta(\alpha)_m t^m \quad (20)$$

Where coefficients $\beta_m(\alpha)$ determined by

$$\beta_0(\alpha) = \alpha_0^{2n-2k-1}$$

$$\beta(\alpha)_i = \frac{1}{i\alpha_0} \sum_{m=1}^{i} \left[2m(n-k) - i\right] \alpha_m \beta(\alpha)_{i-m} \qquad i \geq 1$$

Substituting the formula (20) into (19) we obtain

$$\sum_{n=0}^{\infty} A_n \sum_{k=0}^{n} \zeta_{n,k} (n-k) \sum_{m=0}^{\infty} \beta(\alpha)_m t^{m+k} = F(t) \quad (21)$$

F(t) is given, can be expanded in Maclaourin series thus to derive A_m, we take both sides of (21), l-times derivative at $t = 0$ we have

$$\sum_{n=0}^{l} A_{n+1} \hbar_{1,n} + \sum_{n=l}^{\infty} A_{n+1} \hbar_{2,n} = l! F_n \quad (22)$$

where

$$\hbar_{2,n} = \sum_{i=0}^{l} \zeta_{n+1,i} \beta(\alpha)_{l-i,i}\, l!(n-i+1)$$

$$\hbar_{1,n} = \sum_{i=0}^{n} \zeta_{n+1,i} \beta(\alpha)_{l-i,i}\, l!(n-i+1)$$

from (22) we can find A_n.

Satisfying the boundary conditions of conjugations of temperature (11) and heat flux (12) we get

$$\sum_{n=0}^{\infty} A_n \zeta_{n,n} t^m = \sum_{n=0}^{\infty} C_n \sum_{k=0}^{n} \zeta_{n,k,2} r_0^{2n-2k} t^k \quad (23)$$

$$\sum_{n=0}^{\infty} C_n \sum_{k=0}^{n} \zeta_{n,k,2} r_0^{2n-2k-1} t^k + \frac{2}{r_0} \sum_{n=0}^{\infty} D_n (2a_1\sqrt{t})^{2n} i^{2n} erfc 0 = 0 \quad (24)$$

from taking the m - times derivative of (23) and (24) we get

$$A_m \zeta_{\mathrm{m,m}} = \sum_{i=m}^{\infty} C_i \cdot \zeta_{\mathrm{i,m,2}} r_0^{2n-2m} \quad (23^*)$$

$$\sum_{i=0}^{\infty} C_n \zeta_{i+m+1,m,2} r_0^{2i+1}(i+1) = -\frac{1}{r_0} D_m (2a_1)^{2m} i^{2m} erfc0 \tag{25}$$

From expression (13a) when we put $r = \beta(t)$ we have

$$\sum_{n=0}^{\infty} C_n \sum_{k=0}^{n} \zeta_{n,k,2} \beta(\tau)^{2n-2k} \tau^{2k} + \sum_{n=0}^{\infty} \frac{1}{\beta(\tau)} (2a_1\tau)^{2n+1} (i^{2n+1} erfc(-\gamma(\tau)) - i^{2n+1} erfc(\gamma(\tau))) = \theta_m \tag{26}$$

where $\tau = \sqrt{t}$ and $\frac{\beta(\tau) - \beta_0}{2a_1\tau} = \frac{1}{2a_1} \sum_{n=0}^{\infty} \gamma_{n+1} \tau^n$

Multiplying both sides of (26) by $\beta(\tau)$ we get

$$\sum_{n=0}^{\infty} C_n \sum_{k=0}^{n} \zeta_{n,k,2} \sum_{m=0}^{\infty} \eta(\beta)_m \tau^{m+2k} + \sum_{n=0}^{\infty} D_n (2a_1\tau)^{2n+1} (i^{2n+1} erfc(-\gamma(\tau)) - i^{2n+1} erfc(\gamma(\tau))) = \theta_m \beta(\tau) \tag{27}$$

To comparing coefficients in (27) we applyLeibniz, Faa Di Bruno's formula and Bell polynomials. Using Leibniz we have

$$\left.\frac{\partial^l \left[(2a_1)^{2n+1} \tau^{2n+1} i^{2n+1} erfc(\gamma) \right]}{\partial \tau^l}\right|_{\tau=0} = \begin{cases} 0 \quad , \quad for \quad l < 2n+1 \\ \frac{(2a_1)^{2n+1} l!}{(l-2n-1)!} \left[i^{2n+1} erfc(\delta) \right]^{(l-2n-1)} \end{cases}$$

Using Faa Di Bruno's formula and Bell polynomials for a derivative of a composite function we have

$$\left.\frac{\partial^{l-2n-1}}{\partial \tau^{l-2n-1}} \left\{ i^{2n+1} erfc(\pm\delta) \right\}\right|_{\tau=0} = \sum_{m=1}^{l-2n-1} \left. \left(i^{2n+1} erfc(\pm\delta) \right)^{(m)} \right|_{\delta=0} B_{l-2n-1,m}$$

where

$$B_{l-2n-1,m} = \sum \frac{(l-2n-1)!}{j_1! j_2! \ldots j_{l-2n-m}!} \varsigma_1^{j_1} \varsigma_2^{j_2} \varsigma_3^{j_3} \ldots \varsigma_{l-2n-m}^{j_{l-2n-m}}$$

and $j_1, j_2, \ldots$ satisfy the following equations

$$j_1 + j_2 + \ldots + j_{l-2n-m} = m$$

$$j_1 + 2j_2 + \ldots + (l-2n-m) j_{l-2n-m} = l-2n-1$$

by taking both sides of (27) l-times derivative at $\tau = 0$ we get

$$\sum_{n=0}^{\left[\frac{l}{2}\right]-1} C_n \sum_{i=0}^{n} \zeta_{n,i,2} \eta(\beta)_{l-2i,i,2} + \sum_{n=\left[\frac{l}{2}\right]}^{\infty} C_n \sum_{i=0}^{\left[\frac{l}{2}\right]} \zeta_{n,i,2} \eta(\beta)_{l-2i,i,2} +$$

$$+ D_n \frac{2^{2n+1} l!}{(l-2n-1)!} \sum_{m=1}^{l-2n-1} \left(i^{2n+1-m} erfc(-\gamma_1) + (-1)^{2n+1-m} erfc(\gamma_1) \right) \beta_{l-2n-1,m} = \theta_m l! \beta_l \tag{28}$$

from (28), (23*) and (25) we can find C_n *and* D_n.

By the properties of Integral error functions [6] and condition (9) we get

$$\sum_{n=0}^{\infty} \left\{ E_n r^{2n} + \frac{G_n}{r} \frac{2}{(2n+1)!} r^{2n+1} \right\} = f(r)$$

Suggesting that the initial function can be expanded in $f(r) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} r^n$

we have

$$E_n + G_n \frac{2}{(2n+1)!} = \frac{f^{(2n)}(0)}{2n!} \quad (29)$$

From condition (13b) we have

$$\sum_{n=0}^{\infty} E_n \sum_{k=0}^{n} \zeta_{n,k,2} \beta(\tau)^{2n-2k} \tau^{2k} + \sum_{n=0}^{\infty} G_n \frac{(2a_1\tau)^{2n+1}}{\beta(\tau)} \left(i^{2n+1} erfc(-\xi(\tau)) - i^{2n+1} erfc(\xi(\tau)) \right) = \theta_m \quad (30)$$

where

$$\xi(\tau) = \frac{\beta(\tau)}{2a_2}$$

As previously by taking by both sides of (30), l – times derivatives at $\tau = 0$ by using Leibniz, Faa Di Bruno's formulas and Bell polynomials we have

$$\sum_{n=0}^{\left[\frac{l}{2}\right]-1} E_n \sum_{i=0}^{n} \zeta_{n,i,2} \eta(\beta)_{l-2i,i,2} + \sum_{n=\left[\frac{l}{2}\right]}^{\infty} E_n \sum_{i=0}^{\left[\frac{l}{2}\right]} \zeta_{n,i,2} \eta(\beta)_{l-2i,i,2} +$$

$$+G_n \frac{2^{2n+1} l!}{(l-2n-1)!} \sum_{m=1}^{l-2n-1} \left(i^{2n+1-m} erfc(-\xi_1) + (-1)^{2n+1-m} erfc(\xi_1) \right) \beta_{l-2n-1,m} = \theta_m l! \beta_l \quad (31)$$

From expression (29) and (31) we can find E_n and G_n

Satisfying Stefan's condition (14) and substituting $\sqrt{t} = \tau$ we have

$$-\lambda_1 \left\{ \sum_{n=0}^{\infty} C_n \sum_{k=0}^{n} \zeta_{n,k,2} \beta(\tau)^{2n-2k-1} \tau^{2k} - \frac{1}{\beta^2(\tau)} \sum_{n=0}^{\infty} D_n (2a_1\tau)^{2n+1} \left(i^{2n+1} erfc(-\gamma(\tau)) \right.\right.$$

$$\left.\left. - i^{2n+1} erfc(\gamma(\tau)) \right) - \frac{\lambda_1}{\beta(\tau)} \sum_{n=0}^{\infty} D_n (2a_1\tau)^{2n} \left(i^{2n} erfc(-\gamma(\tau)) + i^{2n} erfc(\gamma(\tau)) \right) \right\} =$$

$$= -\lambda_2 \left\{ \sum_{n=0}^{\infty} E_n \sum_{k=0}^{n} \zeta_{n,k,2} \beta(\tau)^{2n-2k-1} \tau^{2k} - \frac{1}{\beta^2(\tau)} \sum_{n=0}^{\infty} G_n (2a_2\tau)^{2n+1} \left(i^{2n+1} erfc(-\xi(\tau)) - \right.\right. \quad (32)$$

$$\left.\left. - i^{2n+1} erfc(\xi(\tau)) \right) + \frac{1}{\beta(\tau)} \sum_{n=0}^{\infty} D_n (2a_2\tau)^{2n} \left(i^{2n} erfc(-\xi(\tau)) + i^{2n} erfc(\xi(\tau)) \right) \right\} + L\gamma\beta'(\tau)$$

If multiply both sides of (32) by $\beta(\tau)$ and using conditions (13a), (13b) we have the following expression

$$-2\lambda_1 \sum_{n=0}^{\infty} C_n \sum_{k=0}^{n} \zeta_{n,k,2} \beta(\tau)^{2n-2k} \tau^{2k} - \lambda_1 \sum_{n=0}^{\infty} D_n (2a_1\tau)^{2n} \left(i^{2n} erfc(-\gamma(\tau)) + i^{2n} erfc(\gamma(\tau)) \right)$$

$$= -2\lambda_2 \sum_{n=0}^{\infty} E_n \sum_{k=0}^{n} \zeta_{n,k,2} \beta(\tau)^{2n-2k} \tau^{2k} - \lambda_2 \sum_{n=0}^{\infty} G_n (2a_2\tau)^{2n} \left(i^{2n} erfc(-\xi(\tau)) + i^{2n} erfc(\xi(\tau)) \right)$$

$$+ \left(\lambda_2 - \lambda_1 \right) \theta_m + L\gamma' \psi(\tau) \quad (33)$$

where

$$\psi(\tau) = \beta'(\tau)\beta(\tau) = \frac{1}{2} \frac{d}{d\tau} \beta^2(\tau)$$

$$\left(\beta(\tau)\right)^2=\sum_{n=0}^{\infty}\mu(\beta)_n\tau^n$$

$$\mu(\beta)=\beta_0^2 \qquad \mu(\beta)_m=\frac{1}{m\beta_0}\sum_{k=1}^{m}(3k-m)\beta_k\cdot\mu(\beta)_{m-k} \qquad m\geq 1$$

Previously by taking both sides of (33)l – times derivatives $\tau=0$ and for $l\geq 1$ we have

$$-2\lambda_1\left(\sum_{n=0}^{\left[\frac{l}{2}\right]-1}C_n\chi_{1,n}+\sum_{n=\left[\frac{l}{2}\right]}^{\infty}C_n\chi_{2,n}\right)-\lambda_1 D_n\frac{2^{2n}l!}{(l-2n)!}\sum_{m=1}^{l-2n}\left(i^{2n-m}erfc(-\gamma_1)+(-1)^m i^{2n-m}erfc(\gamma_1)\right)\beta_{l-2n,m}$$

$$=-2\lambda_2\left(\sum_{n=0}^{\left[\frac{l}{2}\right]-1}E_n\chi_{21n}+\sum_{n=\left[\frac{l}{2}\right]}^{\infty}E_n\chi_{2,n}\right)-\lambda_2 G_n\frac{2^{2n}l!}{(l-2n)!}\sum_{m=1}^{l-2n}\left(i^{2n-m}erfc(-\xi_1)+(-1)^m i^{2n-m}erfc(\xi_1)\right)\beta_{l-2n,m}$$

$$+\frac{L\gamma'}{2}l!\,\mu(\beta)_{l+1} \quad (34)$$

For $l=0$ we have

$$\theta_m-D_0\left(erfc(-\gamma_1)+erfc(\gamma_1)\right)-2\sum_{n=0}^{\infty}C_n\zeta_{n,0,2}r_0^{2n}=\frac{\lambda_2}{\lambda_1}\left(\theta_m-G_0\left(erfc(-\xi_1)+erfc(\xi_1)\right)-2\sum_{n=0}^{\infty}E_n\zeta_{n,0,2}r_0^{2n}\right)$$

where

$$\chi_{2,n}=\sum_{i=0}^{\left|\frac{l}{2}\right|}\zeta_{n,i,2}\upsilon(\beta)_{l-2i,i,2}$$

$$\chi_{1,n}=\sum_{i=0}^{n}\zeta_{n,i,2}\upsilon(\beta)_{l-2i,i,2}$$

From this recurrent formula we can express β_n.

Conclusion

To summarize, the coefficients A_n, C_n, D_n, E_n, G_n are determined from equations (22),(23*),(28),(29),(31) and (25), the moving boundary $\beta(t)$ obtained from equation (34). For the convergence of temperature functions $\theta_1, \theta_2, \theta_3$, it is possible to follow the idea proposed in [6].

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КӨПІР ЭРОЗИЯСЫНЫҢ ТЕОРИЯ ЕСЕБІ

Аннотация. Осы мақалада біз екі фазалық Стефан мәселесінің дәл шешімін ұсынамыз. Көпір мәселесін шешу үшінрадиалды жылу полиномы және интегралдық қателілфункциясы қолданылады. Осы қктарлардың коэффициенттері үшін қайталанатын өрнектер ұсынылған. Математиқалық модельдер байланыстың ашылу және көпіршікті динамикасын сипаттайды

Түйін сөздер: радиалды жылу полиномы,Стефанпроблемасы, интеграл

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ЗАДАЧА ИЗ ТЕОРИИ МОСТИКОВОЙ ЭРОЗИИ

Аннотация. В настоящей работе мы представляем точное решение двухфазной задачи Стефана. Для решения данной задачи использовали решение в виде радиальных тепловых полиномов и интегральной функции ошибок. Приводятся рекуррентные выражения для коэффициентов ряда. Математические модели описывают динамику размыкания металлических контактов.

Ключевые слова: радиальные тепловые полиномы, проблема Стефана.

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МАЗМҰНЫ

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ISSN 2518-1726 (Online), ISSN 1991-346X (Print)

Редакторы *М. С. Ахметова, Т.А. Апендиев, Д.С. Аленов*
Верстка на компьютере *А.М. Кульгинбаевой*

Подписано в печать 11.10.2018.
Формат 60х881/8. Бумага офсетная. Печать – ризограф.
4,9 п.л. Тираж 300. Заказ 5.

Национальная академия наук РК
050010, Алматы, ул. Шевченко, 28, т. 272-13-18, 272-13-19