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РЕСПУБЛИКИ КАЗАХСТАН

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THE PROBLEM OF THE OSCILLATION OF THE ELASTIC LAYER BOUNDED BY RIGID BOUHDARIES

Abstract: In the case of harmonic oscillations of a cylindrical shell, the phase velocity is expressed in terms of the frequency of natural oscillations freely supported along the edges of the shell, and therefore, the study of waves in plane and circular elements has the most direct relation to the problem of determining its own forms and oscillation frequencies shells finite length. Below let us consider some problems of oscillation of an elastic layer bounded by rigid boundaries under the influence of a normal or rotational shear stress. The solutions of the problems under consideration are obtained by using integral transformations by the coordinate.

Key words: harmonic oscillations, cylindrical shells, phase velocity, frequency, eigenvibrations, Bessel function, wave, anisotropic, layer.

First we consider the problem for a half-space under the assumption that the half-space $z > 0$ is an anisotropic medium with the axis of symmetry of the mechanical properties (axis z), and the surface of which at the moment $t = 0$ impulse voltage applied $\xi_{z_0} = -f(r, t)$.

Because of the symmetry of the mechanical properties of the medium relative to the axis z of the unique nonzero component of the displacement vector $U_0(r, z, t)$, only the voltage ξ_{r_0} and ξ_{z_0} the ones determined by formulas

$$\begin{aligned}\xi_{r_0} &= C_{44} \left(\frac{\partial U_0}{\partial r} - \frac{U_0}{r} \right), \\ \xi_{z_0} &= C_{66} \frac{\partial U_0}{\partial z}\end{aligned}\tag{1}$$

The equation of motion reduces to one

$$\frac{\partial \xi_{r_0}}{\partial r} + \frac{\partial \xi_{r_0}}{\partial z} + \frac{2\xi_{r_0}}{r} = \rho \frac{\partial^2 U_0}{\partial t^2}\tag{2}$$

Substituting the expressions for ξ_{r_0} and ξ_{z_0} from relations (1) into equation (2), we bring it to the form:

$$\frac{\partial^2 U_0}{\partial r^2} + \frac{1}{r} \frac{\partial U_0}{\partial r} - \frac{U_0}{r^2} + \gamma^2 \frac{\partial^2 U_0}{\partial z^2} = \frac{1}{b^2} \frac{\partial^2 U_0}{\partial t^2}\tag{3}$$

where

$$b^2 = \frac{C_{44}}{\rho}; \quad \gamma^2 = \frac{C_{66}}{C_{44}}$$

If the half-space is isotropic, then $\gamma = 1$ and $b = \sqrt{\frac{\mu}{\rho}}$.

The boundary conditions for U_0 have the form:

$$\xi_{z=0} = -f(r, t) \text{ at } z = 0, \quad t \geq 0 \quad (4)$$

$$U_0 \rightarrow 0 \text{ at } z \rightarrow \infty \quad (5)$$

The initial conditions of the problem are zero, i.e.

$$U_0 = \frac{\partial U_0}{\partial t} = 0 \text{ at } t = 0 \quad (6)$$

The solution of equation (3) for the boundary (4) - (5) and the initial conditions (6) will be sought, by applying the Laplace transform t . Assuming that

$$U(r, z, p) = \int_0^{\infty} U_0(r, z, t) e^{-pt} dt, \quad \operatorname{Re} p > 0 \quad (7)$$

Obviously, for the function $U(r, z, p)$ we obtain equation

$$\frac{\partial^2 U}{\partial r^2} + \frac{1}{r} \frac{\partial U}{\partial r} - \left(\frac{1}{r^2} + \frac{p^2}{b^2} \right) U + \gamma^2 \frac{\partial^2 U}{\partial z^2} = 0 \quad (8)$$

Moreover, U must satisfy the boundary conditions:

$$\frac{\partial U}{\partial z} = -\frac{f_0(r, p)}{C_{66}} \text{ at } z = 0, \quad t > 0 \quad (9)$$

$$U_0 \rightarrow 0 \text{ at } z \rightarrow \infty \quad (10)$$

Where

$$f_0(r, p) = \int_0^{\infty} (r, t) e^{-pt} dt.$$

The general solution of equation (8) is sought by the method of separation of variables (the Fourier method) and has the form:

$$U(r, z, p) = \int_0^{\infty} \alpha \left[A(\alpha, p) e^{-\frac{z}{\gamma} \sqrt{\alpha^2 + \frac{p^2}{b^2}}} + B(\alpha, p) e^{\frac{z}{\gamma} \sqrt{\alpha^2 + \frac{p^2}{b^2}}} \right] J_1(\alpha r) d\alpha. \quad (11)$$

where $A(\alpha, p)$ and $B(\alpha, p)$ are determined from the boundary conditions (9) - (10) and from (10) it follows that

$$B(\alpha, p) = 0 \tag{12}$$

Using the boundary condition (9), to determine $A(\alpha, p)$ we obtain the integral equation:

$$\int_0^\infty \alpha A(\alpha, p) \sqrt{\alpha^2 + \frac{p^2}{b^2}} J_1(\alpha r) d\alpha = \frac{\gamma}{C_{66}} f_0(r, p) \tag{13}$$

Suppose

$$f_0(r, p) = \int_0^\infty \alpha f_1(\alpha, p) J_1(\alpha r) d\alpha \tag{14}$$

Then

$$A(\alpha, p) = \frac{f_1(\alpha, p)}{C_{66} \sqrt{\alpha^2 + \frac{p^2}{b^2}}} \tag{15}$$

Substituting expression (12) and (15) into formula (11), for $U(r, z, p)$, we obtain the following expression:

$$U(r, z, p) = \frac{\gamma}{C_{66}} \int_0^\infty \frac{\alpha f_1(\alpha, p)}{\sqrt{\alpha^2 + \frac{p^2}{b^2}}} J_1(\alpha r) e^{-\frac{z}{\gamma} \sqrt{\alpha^2 + \frac{p^2}{b^2}}} d\alpha \tag{16}$$

Let us consider a special case, when

$$f_0(r, p) = \frac{\varphi_0(p)}{r} \tag{17}$$

In the case (17), the function

$$f_1(\alpha, p) = \frac{\varphi_0(p)}{\alpha}$$

and (16)

$$U(r, z, p) = \frac{\gamma}{C_{66}} \int_0^\infty \frac{\varphi_0(p)}{\sqrt{\alpha^2 + \frac{p^2}{b^2}}} e^{-\frac{z}{\gamma} \sqrt{\alpha^2 + \frac{p^2}{b^2}}} J_1(\alpha r) d\alpha = \frac{1}{C_{66}} \varphi_0(p) I_{\frac{1}{2}} \left[\frac{p}{2b} \left(\sqrt{\frac{z^2}{\gamma^2} + r^2} - \frac{z}{\gamma} \right) \right] K_{\frac{1}{2}} \left[\frac{p}{2b} \left(\sqrt{\frac{z^2}{\gamma^2} + r^2} + \frac{z}{\gamma} \right) \right]$$

where $K_{\frac{1}{2}}, I_{\frac{1}{2}}$ is the Bessel function of the imaginary argument. Using the representations of the functions $I_{\frac{1}{2}}(\zeta)$ and $K_{\frac{1}{2}}(\zeta)$, for $U(r, z, p)$, we find that

$$U(r, z, p) = \frac{b \varphi_0(p)}{C_{66} r p} \left[e^{-\frac{pz}{\gamma a}} - e^{-\frac{p}{a} \sqrt{\frac{z^2}{\gamma^2} + r^2}} \right] \tag{18}$$

Turning expression (18) to p , for the sought quantity $U_0(r, z, t)$, we obtain expression

$$U_0(r, z, t) = \frac{b}{\gamma C_{66} r} \int_0^t f_1(t - \xi) \left[H\left(\xi - \frac{z}{\gamma b}\right) - H\left(\xi - \frac{1}{b} \sqrt{\frac{z^2}{\gamma^2} + r^2}\right) \right] d\xi \quad (19)$$

Where

$$f(r, t) = \frac{f_1(t)}{r}.$$

The resulting expression for $U_0(r, z, t)$ consists of two terms, the first term corresponding to a plane wave propagating in a half-space with a velocity γb and parallel to the plane $z = 0$, and the second term to a diffracted wave having the form of a semi-ellipsoid of revolution (hemispheres at $\gamma = 1$) and in contact with a plane wave on the axis of rotation at $z = b\gamma t$.

In addition, it follows from (19) that $U_0(r, z, t)$ decays in r as $1/r$.

If the acting function $f(r, t)$ is arbitrary, then we represent it in the form of a Schlemmich series:

$$f(r, t) = \frac{1}{r} \sum_{j=0}^{\infty} a_j(t) J_0(jr) = \frac{f_r(t)}{r} \quad (20)$$

Where

$$a_0(t) = \frac{1}{\pi} \int_0^{\pi} \left\{ f_r(0, t) + U \int_0^1 \frac{\partial f_r(\xi U, t)}{\sqrt{1 - \xi^2}} d\xi \right\} dU;$$

$$a_j(t) = \frac{2}{\pi} \int H \cos(jU) \left\{ \int_0^1 \frac{\partial f_r(\xi U, t)}{\sqrt{1 - \xi^2}} d\xi \right\} dU; j = 1, 2, \dots \quad (21)$$

For $f(r, t)$ the form (20), the function $f_1(\alpha, p)$ in formula (16) is equal to

$$f_1(\alpha, p) = \frac{1}{\alpha} \sum_{j=0}^{\infty} a_{j0}(p) H(\alpha - j) \quad (22)$$

Where

$$a_{j0}(p) = \int_0^{\infty} a_j(p) e^{-pt} dt. \quad (23)$$

Therefore,

$$U(r, z, p) = \frac{\gamma}{C_{66}} \sum_{j=0}^{\infty} a_{j0}(p) \int_0^{\infty} \frac{e^{-\frac{z}{\gamma} \sqrt{\alpha^2 + \frac{p^2}{b^2}}}}{\sqrt{\alpha^2 + \frac{p^2}{b^2}}} J_1(\alpha r) d\alpha \quad (24)$$

Turning (24) to p and applying the convolution theorem, we obtain

$$U_0(r, z, p) = \frac{\gamma}{2C_{66}} \sum_{j=0}^{\infty} \int_g^t a_j(t-\xi) T_j(r, z, \xi) d\xi \quad (25)$$

where

$$T_j(r, z, \xi) = b \int_j^{\infty} J_1(\alpha r) J_0 \left[\alpha \sqrt{b^2 t^2 - \frac{z^2}{\gamma^2}} \right] d\alpha \quad (26)$$

We generalize the problem for an anisotropic layer of thickness h . For $z = h$ there can be two types of boundary conditions:

- 1) $\tau_{z0} = -F(r, t)$ at $z = h$
- 2) $U_0 = 0$ at $z = h$

If $F(r, t) = 0$, then condition (27) means that surface $z = h$ is voltages -free.

First we consider the problem when the boundary condition (27) is given for $z = h$.

The general solution of the problem still has the form (11), and

$$A(\alpha, p) = B(\alpha, p) + \frac{\gamma}{C_{66}} f_1(\alpha, p)$$

$$B(\alpha, p) = \frac{\gamma}{2C_{66}} \frac{f_1(\alpha, p) e^{-\frac{h}{\gamma} \sqrt{\alpha^2 + \frac{p^2}{b^2}}} - f_2(\alpha, p)}{sh \left[\frac{h}{\gamma} \sqrt{\alpha^2 + \frac{p^2}{b^2}} \right]} \quad (29)$$

where $f_1(\alpha, p)$ is determined from equation (13), and $f_2(\alpha, p)$ is determined from equation

$$F_0(r, p) = \int_0^{\infty} \alpha f_2(\alpha, p) J_1(\alpha r) d\alpha \quad (30)$$

Suppose

$$F_0(r, p) = 0,$$

i.e. $f_2(\alpha, p) = 0$ and $f(r, t) = f_1(r, t)$.

Then

$$U(r, z, p) = \frac{1}{\gamma C_{66}} \int_0^{\infty} \frac{\varphi_0(p)}{\sqrt{\alpha^2 + \frac{p^2}{b^2}}} \times \left[\frac{e^{-\frac{z-h}{\gamma} \sqrt{\alpha^2 + \frac{p^2}{b^2}}} + e^{\frac{z-h}{\gamma} \sqrt{\alpha^2 + \frac{p^2}{b^2}}}}{e^{\frac{h}{\gamma} \sqrt{\alpha^2 + \frac{p^2}{b^2}}} - e^{-\frac{h}{\gamma} \sqrt{\alpha^2 + \frac{p^2}{b^2}}}} \right] J_1(\alpha r) d\alpha$$

or

$$U(r, z, p) = \frac{1}{\gamma C_{66}} \sum_{n=0}^{\infty} \int_0^{\infty} \frac{\varphi_0(p)}{\sqrt{\alpha^2 + \frac{p^2}{b^2}}} \times \left\{ e^{-\frac{z+2nh}{\gamma} \sqrt{\alpha^2 + \frac{p^2}{b^2}}} + e^{\frac{z-2(n+1)h}{\gamma} \sqrt{\alpha^2 + \frac{p^2}{b^2}}} \right\} J_1(\alpha r) d\alpha \quad (31)$$

Calculating the quadratures by α in (31) and then reversing by p , for $U_0(r, z, t)$ we obtain the

expression:

$$\begin{aligned}
 U_0(r, z, t) = & \frac{b}{r\gamma C_{66}} \sum_{n=0}^{n_1} \left\{ \int_0^t f_1(t-\xi) H \left[\xi - \frac{z+2nh}{\gamma b} \right] d\xi - \right. \\
 & \left. - \int_0^t f_1(t-\xi) H \left[\xi - \frac{1}{b} \sqrt{r^2 + \left(\frac{z+2nh}{\gamma} \right)^2} \right] d\xi \right\} + \\
 & + \frac{b}{r\gamma C_{66}} \sum_{n=0}^{n_2} \left\{ \int_0^t f_1(t-\xi) H \left[\xi + \frac{z-2(n+1)h}{\gamma b} \right] d\xi - \right. \\
 & \left. - \int_0^t f_1(t-\xi) H \left[\xi - \frac{1}{b} \sqrt{r^2 + \left(\frac{z-2(n+1)h}{\gamma} \right)^2} \right] d\xi \right\} \quad (32)
 \end{aligned}$$

where

$$n_1 = \left[\frac{z+2nh}{\gamma b t} \right], \quad n_2 = \left[\frac{-z+2(n+1)h}{\gamma b t} \right]$$

$[\xi]$ is the integer part of the number ξ .

Formula (32) contains all flat and diffracted incident and reflected waves.

Similarly, the problem for the layer under boundary condition (28) is solved and we have

$$\begin{aligned}
 U_0(r, z, t) = & \frac{b}{r\gamma C_{66}} \sum_{n=0}^{n_1} (-1)^n \left\{ \int_0^t f_1(t-\xi) H \left[\xi - \frac{z+2nh}{\gamma b} \right] d\xi - \right. \\
 & \left. - \int_0^t f_1(t-\xi) H \left[\xi - \frac{1}{b} \sqrt{r^2 + \left(\frac{z+2nh}{\gamma} \right)^2} \right] d\xi \right\} - \\
 & - \frac{b}{r\gamma C_{66}} \sum_{n=0}^{\infty} (-1)^n \left\{ \int_0^t f_1(t-\xi) H \left[\xi + \frac{z-2(n+1)h}{\gamma b} \right] d\xi - \right. \\
 & \left. - \int_0^t f_1(t-\xi) H \left[\xi - \frac{1}{b} \sqrt{r^2 + \left(\frac{z-2(n+1)h}{\gamma} \right)^2} \right] d\xi \right\} \quad (33)
 \end{aligned}$$

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[14] Seitmuratov, A. Zh B. M. Nurlanova, N. K. Medeubaev. BULLETIN of the Karaganda University 109-117 №3(81)/2017 Mathematics Series ISSN 2518-2729/ Equations of vibration of a two-dimensionally layered plate strictly based on the decision of various boundary-value

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ҚАТАҢ ШЕКАРАЛАРМЕН ШЕКТЕЛГЕН СЕРПІМДІ ҚАБАТ ТЕРБЕЛІСІ ЖАЙЛЫ ЕСЕП

Аннотация. Цилиндрлік қабықшалардың гармоникалық тербелісі жағдайында фазалық жылдамдық сол қабықшалардың шетіне еркін бекітілген өзіндік жиілік тендеуі арқылы өрнектеледі, сондықтан жалпақ және айналмалы элементтердің тербелісін зерттеу түпкілікті ұзындықтағы өзіндік пішіндері мен тербеліс жиілігіне тікелей қатысты. Берілген төмендегі есепте қалыпты немесе айналмалы керілу кернеуі жағдайында, қатаң шегарада шектелген серпімді қабат тербеліс теңдеулері қарастырылады. Қарастырылатын есепті шешу мәселелердің шешімдері координат бойынша интегралдық түрлендіру әдістерін қолдану арқылы алынды.

Түйін: гармоникалық тербеліс, цилиндрлік қабықшалар, фазалық жылдамдық, жиілік, өзіндік тербеліс, Бессель функциясы, толқын, анизотропты, қатпар

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ЗАДАЧА О КОЛЕБАНИИ УПРУГОГО СЛОЯ ОГРАНИЧЕННЫЕ ЖЕСТКИМИ ГРАНИЦАМИ

Аннотация В случае гармонических колебаний цилиндрической оболочки фазовая скорость выражается через частоту собственных колебаний свободно опертой по краям оболочки, и поэтому, исследование волн в плоских и круговых элементах имеет самое прямое отношение к проблеме определения собственных форм и частот колебаний оболочек конечной длины. Ниже рассматриваются некоторые задачи колебания упругого слоя ограниченные жесткими границами при воздействии на него нормального или вращательного касательного напряжения. Решения рассматриваемых задач получены с использованием интегральных преобразований по координате.

Ключевые слова: гармоническая колебания, цилиндрические оболочки, фазовая скорость, частота, собственная колебания, функция Бесселя, волна, анизотропный, слой.

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