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THE METHOD OF NUMERICAL SOLUTION OF NONLINEAR VOLTERRA INTEGRAL EQUATIONS OF THE FIRST KIND

Abstract. When considering systems of differential equations with very general boundary conditions, exact solution methods encounter great difficulties, which become insurmountable in the study of nonlinear problems. In this case it is necessary to apply to certain numerical methods. It is important to note that the use of numerical methods often allows you to abandon the simplified interpretation of the mathematical model of the process. The problems of numerical solution of nonlinear Volterra integral equations of the first kind with a differentiable kernel, which degenerates at the initial point of the diagonal, are studied in the paper. This equation is reduced to the Volterra integral equation of the third kind and a numerical method is developed on the basis of that regularized equation. The convergence of the numerical solution to the exact solution of the Volterra integral equation of the first kind is proved, an estimate of the permissible error and a recursive formula of the computational process are obtained.

Keywords: nonlinear integral equation, system of nonlinear algebraic equations, error vectors, the Volterra equation, small parameter, numerical methods.

Introduction

The problem of solving integral equations arises as an auxiliary problem for solving boundary value problems for partial differential equations and as an independent one in the study of the operation of nuclear reactors, in solving the so-called inverse problems of geophysics, in processing the results of observations, and so on. We confine ourselves to the case of nonlinear Volterra integral equations of the first kind.

Questions about the numerical solution of linear integrated equations of Vol'terraof the first sort are explored in two case, when source K(x,t) on diagonal doesn’t return zero in any points of a section and the source on diagonal is identical zero, a derivative on x on diagonal doesn’t return zero in any points of the section [4-7]. In this research we considered the case non-liner integrated equations Vol’terra of the first sort with allocated source, which can return zero in some points of the section of the diagonal.

Formulation of the problem

Consider the nonlinear Volterra integral equation of the first kind:

$$\int_0^x N_0\big(x,t,u(t)\big)dt = g(x), \qquad (1)$$

where $N_0\big(x,t,u(t)\big) = K(x,t)u(t) + N\big(x,t,u(t)\big)$ and known functions $K(x,t), N\big(x,t,u(t)\big)$, $g(x)$ obey conditions:

a) $g(x) \in C^2[0,b], K(x,t) \in C^{2,1}(D), D = \{(x,t)/0 \le t \le x \le b\}$,
$g^{(i)}(0) = 0, i = 0,1, k(x) = K(x,x), k(0) = 0, 0 < k(x)\ \forall x \in (0,b]$, $k(x)$ – nondecreasing function;

b) $G(x) \ge d_1, G(x) = L(x,x) + C_1 g(x), L(x,t) = C_2 K(x,t) + K_x(x,t), 0 < d_1, C_1, C_2 = const$;

c) $\boldsymbol{N(x,t,u) \in C^{1,0,1}(D \times R^1), M_0(x,t,u) \in C^{0,0,1}(D \times R^1), M_0(x,x,u) = 0, M_0(x,t,u(t)) = C_2 N(x,t,u(t)) + N_x(x,t,u(t))}$, **for** $\boldsymbol{x > t, (x,s), (t,s) \in D, (x,s,u), (x,s,w), (t,s,w), (t,s,u) \in D \times R^1}$ **the following inequality holds true**

$\boldsymbol{|M_0(x,s,u) - M_0(x,s,w) - M_0(t,s,w) + M_0(t,s,u)| \le L_N(x-t)|u-w|}$,

$0 < L_N = const.$

The research and solution of the basic equation

We get Volterra integral equation of the third kind from equation (1) after applying $D + C_1 T + C_2 I$, where I is an identical operator, *D* is an operator of differentiation with respect to x, *T* is Volterra operator kind of $(Tv)(x) = \int_0^x u(t)v(t)dt$, [1]:

$$k(x)u(x) + \int_0^x G(t)u(t)dt = \int_0^x M(x,t,u(t))dt + C_1 \int_0^x u(t)dt \times$$

$$\times \int_t^x K(s,t)u(s)ds + C_1 \int_0^x \int_t^x N(s,t,u(t))u(s)ds\,dt + f(x), (1_0)$$

where $M(x,t,u(t)) = -M_0(x,t,u(t)) + (L(t,t) - L(x,t))u(t), \qquad f(x) = C_2 g(x) + g'(x).$

Consider regularized equation with a small parameter

$$(\varepsilon + k(x))u_\varepsilon(x) + \int_0^x G(t)u_\varepsilon(t)dt = \int_0^x M(x,t,u_\varepsilon(t))dt + C_1 \int_0^x u_\varepsilon(t)dt \times$$

$$\times \int_t^x K(\tau,t)\, u_\varepsilon(\tau)d\tau + C_1 \int_0^x \int_t^x N(\tau,t,u_\varepsilon(t))u_\varepsilon(\tau)\, d\tau\, dt + \varepsilon u(0) + f(x), \qquad (2)$$

Transform equation (2) to the following form

$$u_\varepsilon(x) = -\frac{1}{\varepsilon + k(x)} \int_0^x exp\left(-\int_t^x \frac{G(\tau)}{\varepsilon + k(\tau)} d\tau\right) \frac{G(t)}{\varepsilon + k(t)} \left\{ \int_0^t M(t,\tau,u_\varepsilon(\tau))d\tau - \right.$$

$$- \int_0^x M(x,\tau,u_\varepsilon(\tau))d\tau - C_1 \left[\int_0^t u_\varepsilon(\tau)d\tau \int_s^t K(\nu,\tau)\, u_\varepsilon(\nu)d\nu + \right.$$

$$+ \int_0^x u_\varepsilon(\tau)d\tau \int_s^x K(\nu,\tau)\, u_\varepsilon(\nu)d\nu - \int_0^t \int_\tau^t N(\nu,\tau,u_\varepsilon(\tau))u_\varepsilon(\nu)d\nu\, d\tau +$$

$$\left. \left. + \int_0^x \int_\tau^x N(\nu,\tau,u_\varepsilon(\tau))u_\varepsilon(\nu)d\nu\, d\tau \right] + f(t) - f(x) \right\} dt + \frac{1}{\varepsilon + k(x)} \times$$

$$\times exp\left(-\int_0^x \frac{G(\tau)}{\varepsilon + k(\tau)} d\tau\right) \left\{ \int_0^x M(x,t,u_\varepsilon(t))\, dt + C_1 \int_0^x u_\varepsilon(t)dt \times \right.$$

$$\left. \times \int_t^x K(\tau,t)\, u_\varepsilon(\tau)d\tau + \int_0^x \int_t^x N(\tau,t,u_\varepsilon(t))u_\varepsilon(\tau)d\tau\, dt \right] + \varepsilon u_{0,h} + f(x) \Big\}. \qquad (3)$$

Introduce a uniform grid $\omega_h = \{x_i = ih, i = 0..n, b = nh\}$ on the $[0,b]$ segment, *n* – natural number and C_h – space of grid functions $u_i = u(x_i)$ with the following norm

$$\|u_i\|_{C_h} = \max_{0 \le i \le n} |u_i|.$$

Using the right Riemann sum and replacing $u(0)$ to $u_{0h} = f_1/(hG_1 + k_1)$, we obtain the next system of nonlinear algebraic equations from equation (3):

$$u_{\varepsilon,i} = -\frac{1}{\varepsilon + k_i} h \sum_{j=1}^{i-1} exp\left(-h \sum_{s=j+1}^{i} \frac{G_s}{\varepsilon + k_s}\right) \frac{G_j}{\varepsilon + k_j} \times$$

$$\times \left\{ h \sum_{s=1}^{j} [M_{j,s}(u_{\varepsilon,s}) - M_{i,s}(u_{\varepsilon,s})] - h \sum_{s=j+1}^{i-1} M_{i,s}(u_{\varepsilon,s}) - C_1 h \sum_{s=1}^{j-1} u_{\varepsilon,s} \times \right.$$

$$\times h \sum_{m=j+1}^{i} K_{m,s} u_{\varepsilon,m} - C_1 h \sum_{s=j+1}^{i-1} u_{\varepsilon,s}\, h \sum_{m=s+1}^{i} K_{m,s} u_{\varepsilon,m} - C_1 h \sum_{s=1}^{j-1} h \times$$

$$\left. \times \sum_{m=j+1}^{i} N_{m,s}(u_{\varepsilon,s})\, u_{\varepsilon,m} - C_1 h \sum_{s=j}^{i-1} h \sum_{m=s+1}^{i} N_{m,s}(u_{\varepsilon,s}) u_{\varepsilon,m} + f_j - f_i \right\} +$$

$$+ \frac{1}{\varepsilon + k_i} exp\left(-h \sum_{s=1}^{i} \frac{G_s}{\varepsilon + k_s}\right) \left\{ h \sum_{j=1}^{i-1} M_{i,j}(u_{\varepsilon,j}) + C_1 h \sum_{j=1}^{i-1} u_{\varepsilon,j}\, h \sum_{s=j+1}^{i} K_{s,j} u_{\varepsilon,s} + \right.$$

$$\left. + C_1 h \sum_{j=1}^{i-1} h \sum_{s=j+1}^{i} N_{s,j}(u_{\varepsilon,j})\, u_{\varepsilon,s} + \varepsilon u_{0,h} + f_i \right\}, i = 1..n, \qquad (4)$$

where $M_{i,j}(u_{\varepsilon,j}) = M(x_i, x_j, u(x_j)), f_i = f(x_i), x_j = jh, j = 1..i, i = 1..n.$

Introduce the notations

$$q = \frac{d_2 b T_0}{d_1}(L_2 + C_2 L_1 + L_N)\left(2\frac{h}{\varepsilon} + e^{-1}\right) + \frac{2C_1 T_0 M b r d_2 h}{d_1 \varepsilon} +$$

$$+ C_1 b \left(\frac{2T_0 d_2 h}{d_1 \varepsilon} + \frac{1}{e d_1}\right)(M_N + K_N r), M = \max_D |K(x,t)|, |u_\varepsilon(x)| \le r.$$

$$L_1 = \max_D |K_x(x,t)|, L_2 = \max_D |K_{xx}(x,t)|, T_0 = \max_{x\in[0,b]} |G(x)|,$$

$$d_2 = sup\left(\sum_{j=1}^{i-1} exp\left(-h \sum_{s=j+1}^{i} \frac{G_s}{\varepsilon + k_s}\right)\left(h \sum_{s=j+1}^{i} \frac{G_s}{\varepsilon + k_s}\right)\right),$$

$$M_N = \max_{D\times R^1} |N(x,t,u)|, K_N = \max_{D\times R^1} |N_u(x,t,u)|,$$

Theorem. If the conditions *a)–c)*, $q < 1$ and $\varepsilon = O(h^\alpha)$ for all $0 < \alpha < 1/2$ are satisfied, then the solution of the system of equations (4) converges uniformly to the exact solution u_i of equation (1) when $h \to 0$, thus we have the estimate

$$\|u_{\varepsilon,i} - u_i\| \le N_1 h^\alpha + N_2 h^{1-\alpha} + N_3 h, 0 < N_i = const, i = 1,2,3.$$

Proof. Adding the quantity $\varepsilon u(x)$ to both sides of equation (1_0), we reduce it to the form (3), where $u_\varepsilon(x)$ and εu_0 are respectively instead of $u(x)$ and $\varepsilon u(x)$. Putting $x = x_i, i = 1..n$ in the obtained equation, we use the formula of the right Riemann sum and consider the difference of the resulting system of algebraic equations with the remainder term and the system of equations (4). Then, using the error vector $\eta^h_{\varepsilon,i} = u_\varepsilon(x_i) - u(x_i), i = 1..n$, we obtain

$$\eta_{\varepsilon,i}^{h} = -\frac{1}{\varepsilon+k_i} h \sum_{j=1}^{i-1} exp\left(-h \sum_{s=j+1}^{i} \frac{G_s}{\varepsilon+k_s}\right) \frac{G_j}{\varepsilon+k_j} \left\{ h \sum_{s=1}^{j-1} [M_{j,s}(u_{\varepsilon,s}) - \right.$$

$$-M_{j,s}(u_s) - M_{i,s}(u_{\varepsilon,s}) + M_{i,s}(u_s)] - h \sum_{s=j+1}^{i-1} [M_{i,s}(u_{\varepsilon,s}) - M_{i,s}(u_s)] -$$

$$-C_1 h \sum_{s=1}^{j-1} u_s\, h \sum_{m=j+1}^{i} K_{m,s}\eta_{\varepsilon,m}^{h} - C_1 h \sum_{s=j+1}^{i-1} u_s\, h \sum_{m=s+1}^{i} K_{m,s}\, \eta_{\varepsilon,m}^{h} -$$

$$- - C_1 h \sum_{s=1}^{j-1} \eta_{\varepsilon,s}^{h}\, h \sum_{m=j+1}^{i} K_{m,s}\, u_m - C_1 h \sum_{s=j+1}^{i-1} \eta_{\varepsilon,s}^{h}\, h \sum_{m=s+1}^{i} K_{m,s} u_m -$$

$$-C_1 h \sum_{s=1}^{j-1} h \sum_{m=j+1}^{i} N_{m,s}(u_s)\, \eta_{\varepsilon,m}^{h} - C_1 h \sum_{s=j+1}^{i-1} h \sum_{m=s+1}^{i} N_{m,s}(u_s)\, \eta_{\varepsilon,m}^{h} -$$

$$-C_1 h \sum_{s=1}^{j-1} h \sum_{m=j+1}^{i} [N_{m,s}(u_{\varepsilon,s}) - N_{m,s}(u_s)] u_m -$$

$$\left. -C_1 h \sum_{s=j+1}^{i-1} h \sum_{m=s+1}^{i} [N_{m,s}(u_{\varepsilon,s}) - N_{m,s}(u_s)] u_m + \varepsilon(u_j - u_i) \right\} +$$

$$+\frac{1}{\varepsilon+k_i} exp\left(-h \sum_{s=1}^{i} \frac{G_s}{\varepsilon+k_s}\right) \left\{ h \sum_{j=1}^{i-1} [M_{i,j}\,(u_{\varepsilon,j}) - M_{i,j}(u_j)] + \right.$$

$$+C_1 h \sum_{j=1}^{i-1} \eta_{\varepsilon,j}^{h} \sum_{s=j+1}^{i} K_{s,j} u_s + C_1 h \sum_{j=1}^{i-1} u_j \times h \sum_{s=j+1}^{i} K_{s,j} \eta_{\varepsilon,s}^{h} +$$

$$+C_1 h \sum_{j=1}^{i-1} h \sum_{s=j+1}^{i} [N_{s,j}(u_{\varepsilon,j}) - N_{s,j}(u_j)]\, u_s +$$

$$\left. +C_1 h \sum_{j=1}^{i-1} h \sum_{s=j+1}^{i} N_{s,j}(u_j) \eta_{\varepsilon,s}^{h} + \varepsilon u_{0,h} - \varepsilon u_i \right\} - R_i, i = 1..n, \qquad (5)$$

where R_i is a remainder term. We have the following estimates from (5):

1) $$\left| -\frac{1}{\varepsilon+k_i} h \sum_{j=1}^{i-1} exp\left(-h \sum_{s=j+1}^{i} \frac{G_s}{\varepsilon+k_s}\right) \frac{G_j}{\varepsilon+k_j} \left\{ h \sum_{s=1}^{j-1} [M_{j,s}(u_{\varepsilon,s}) - \right. \right.$$

$$\left. -M_{j,s}(u_s) - M_{i,s}(u_{\varepsilon,s}) + M_{i,s}(u_s)] - h \sum_{s=j+1}^{i-1} [M_{i,s}(u_{\varepsilon,s}) - M_{i,s}(u_s)] \right| \le$$

$$\le \frac{2d_2 b T_0 h}{d_1 \varepsilon} (L_2 + C_2 L_1 + L_N) \left\| \eta_{\varepsilon,i}^{h} \right\|_{C_h},$$

where $L_1 = max_D |K_x(x,t)|, L_2 = max_D |K_{xx}(x,t)|, T_0 = max_{x\in[0,b]} |G(x)|$,

$$d_2 = sup\left(\sum_{j=1}^{i-1} exp\left(-h \sum_{s=j+1}^{i} \frac{G_s}{\varepsilon+k_s}\right) \left(h \sum_{s=j+1}^{i} \frac{G_s}{\varepsilon+k_s}\right) \right);$$

2) $$\left|\frac{C_1}{\varepsilon+k_i}h\sum_{j=1}^{i-1}exp\left(-h\sum_{s=j+1}^{i}\frac{G_s}{\varepsilon+k_s}\right)\frac{G_j}{\varepsilon+k_j}\left\{h\sum_{s=1}^{j-1}h\sum_{m=j+1}^{i}N_{m,s}(u_s)\,\eta_{\varepsilon,m}^h+\right.\right.$$

$$\left.+h\sum_{s=j+1}^{i-1}h\sum_{m=s+1}^{i}N_{m,s}(u_s)\,\eta_{\varepsilon,m}^h\right|\le\frac{2C_1T_0M_Nd_2bh}{d_1\varepsilon}\left\|\eta_{\varepsilon,i}^h\right\|_{C_h},$$

$where\ M_N=\max\limits_{D\times R^1}|N(x,t,u)|;$

3) $$\left|\frac{C_1}{\varepsilon+k_i}h\sum_{j=1}^{i-1}exp\left(-h\sum_{s=j+1}^{i}\frac{G_s}{\varepsilon+k_s}\right)\frac{G_j}{\varepsilon+k_j}\left\{h\sum_{s=1}^{j-1}h\sum_{m=j+1}^{i}[N_{m,s}(u_{\varepsilon,s})-\right.\right.$$

$$\left.\left.-N_{m,s}(u_s)]u_m+h\sum_{s=j+1}^{i-1}h\sum_{m=s+1}^{i}[N_{m,s}(u_{\varepsilon,s})-N_{m,s}(u_s)]u_m\right\}\right|\le$$

$$\le\frac{2C_1T_0K_Nrbd_2h}{d_1\varepsilon}\left\|\eta_{\varepsilon,i}^h\right\|_{C_h},K_N=\max\limits_{D\times R^1}|N_u(x,t,u)|;$$

4) $$\left|\frac{C_1h}{\varepsilon+k_i}\sum_{j=1}^{i-1}exp\left(-h\sum_{s=j+1}^{i}\frac{G_s}{\varepsilon+k_s}\right)\frac{G_j}{\varepsilon+k_j}\left\{h\sum_{s=1}^{j-1}u_s\,h\sum_{m=j+1}^{i}K_{m,s}\eta_{\varepsilon,m}^h+\right.\right.$$

$$\left.\left.+h\sum_{s=j+1}^{i-1}u_s\,h\sum_{m=s+1}^{i}K_{m,s}\eta_{\varepsilon,m}^h\right\}\right|\le\frac{C_1T_0Mh}{\varepsilon+p_i}\sum_{j=1}^{i-1}exp\left(-h\sum_{k=j+1}^{i}\frac{G_k}{\varepsilon+k_s}\right)\frac{1}{\varepsilon+k_j}\times$$

$$\times\left\{h\sum_{s=1}^{j}|u_s|\,h\sum_{m=j+1}^{i}\left|\eta_{\varepsilon,m}^h\right|+h\sum_{s=j+1}^{i-1}|u_s|\,h\sum_{m=j+1}^{i}\left|\eta_{\varepsilon,m}^h\right|\right\}\le$$

$$\le\frac{C_1T_0Mbrd_2h}{d_1\varepsilon}\left\|\eta_{\varepsilon,i}^h\right\|_{C_h};$$

where $M=\max\limits_{D}|K(x,t)|,|u(x)|\le r;$

5) $$\left|\frac{C_1h}{\varepsilon+\ k_i}\sum_{j=1}^{i-1}exp\left(-h\sum_{s=j+1}^{i}\frac{G_s}{\varepsilon+k_s}\right)\frac{G_j}{\varepsilon+k_j}\left\{h\sum_{s=1}^{j-1}\eta_{\varepsilon,s}^h\,h\sum_{m=j+1}^{i}K_{m,s}\,u_{\varepsilon,m}+\right.\right.$$

$$\left.\left.+h\sum_{s=j+1}^{i-1}\eta_{\varepsilon,s}^h\,h\sum_{m=s+1}^{i}K_{m,s}u_{\varepsilon,m}\right\}\right|\le\frac{C_1T_0Mh}{\varepsilon+k_i}\sum_{j=1}^{i-1}exp\left(-h\sum_{s=j+1}^{i}\frac{G_s}{\varepsilon+k_s}\right)\frac{1}{\varepsilon+k_j}\times$$

$$\times\left\{h\sum_{s=1}^{j-1}\left|\eta_{\varepsilon,s}^h\right|h\sum_{m=j+1}^{i}\left|u_{\varepsilon,m}\right|+h\sum_{s=j+1}^{i-1}\left|\eta_{\varepsilon,s}^h\right|h\sum_{m=j+1}^{i}\left|u_{\varepsilon,m}\right|\right\}\le$$

$$\le\frac{C_1T_0Mrd_2h}{d_1\varepsilon}h\sum_{j=1}^{i-1}\left|\eta_{\varepsilon,j}^h\right|\le\frac{C_1T_0Mbrd_2h}{d_1\varepsilon}\left\|\eta_{\varepsilon,i}^h\right\|_{C_h};$$

6) $$\left|\frac{1}{\varepsilon+k_i}exp\left(-h\sum_{s=1}^{i}\frac{G_s}{\varepsilon+k_s}\right)h\sum_{j=1}^{i-1}[M_{i,j}\,(u_{\varepsilon,j})-M_{i,j}(u_j)]\right|\le$$

$$\le\frac{d_2bT_0}{d_1e}(L_2+C_2L_1+L_N)\left\|\eta_{\varepsilon,i}^h\right\|_{C_h}$$

7) $\left|\frac{C_1}{\varepsilon+k_i}exp\left(-h\sum_{s=1}^{i}\frac{G_s}{\varepsilon+k_s}\right)h\sum_{j=1}^{i-1}h\sum_{s=j+1}^{i}N_{s,j}\left(u_j\right)\eta_{\varepsilon,s}^h\right|\le\frac{C_1M_Nb}{ed_1}\left\|\eta_{\varepsilon,i}^h\right\|_{C_h};$

8) $\left|\frac{C_1}{\varepsilon+k_i}exp\left(-h\sum_{s=1}^{i}\frac{G_s}{\varepsilon+k_s}\right)h\sum_{j=1}^{i-1}h\sum_{s=j+1}^{i}\left[N_{s,j}\left(u_{\varepsilon,j}\right)-N_{s,j}\left(u_j\right)\right]u_j\right|\le$
$\le\frac{C_1K_Nbr}{ed_1}\left\|\eta_{\varepsilon,i}^h\right\|_{C_h}.$

On the basis of estimates 1) -8) for the error vector $\eta_{\varepsilon,i}^h$ from (5) we obtain

$$\left|\eta_{\varepsilon,i}^h\right|\le q\left\|\eta_{\varepsilon,i}^h\right\|_{C_h}+\left|\varepsilon H_\varepsilon u_i\right|+\left|R_i\right|, (6)$$

Missing the cumbersome calculations, note that the next estimate for R_i holds true as in [2]:

$$\|R_i\|_{C_h}\le N_2h/\varepsilon+N_3h, 0<N_2,\ N_3=const.$$

Since according to [3]:

$$\left\|\varepsilon H_\varepsilon^h[u_i]\right\|_{C_h}\le N_1\varepsilon, 0<N_1=const,$$

then we get estimate by the grid norm from (6)

$$\left\|\eta_{\varepsilon,i}^h\right\|_{C_h}\le(1-q)^{-1}(N_1\varepsilon+N_2h/\varepsilon+N_3h).$$

Taking into account that $\varepsilon=O(h^\alpha)$, we arrive at the estimate of the theorem, which was to be proved.

Equation (4) is a system of nonlinear, therefore we obtain the following system of equations with respect to $u_{\varepsilon,i}$

$$u_{\varepsilon,i}=\frac{C_1}{\varepsilon+k_i}\left\{h\sum_{j=1}^{i-1}exp\left(-h\sum_{s=j+1}^{i}\frac{G_s}{\varepsilon+k_s}\right)\frac{G_j}{\varepsilon+k_j}+exp\left(-h\sum_{s=1}^{i}\frac{G_s}{\varepsilon+k_s}\right)\right\}\times$$

$$\times h\left\{\left(h\sum_{s=1}^{i-1}K_{i,s}u_{\varepsilon,s}\right)+\left(h\sum_{s=1}^{i-1}N_{i,s}\left(u_{\varepsilon,s}\right)\right)\right\}u_{\varepsilon,i}+$$

$$-\frac{1}{\varepsilon+k_i}h\sum_{j=1}^{i-1}exp\left(-h\sum_{s=j+1}^{i}\frac{G_s}{\varepsilon+k_s}\right)\frac{G_j}{\varepsilon+k_j}\times$$

$$\times\left\{h\sum_{s=1}^{j}\left[M_{j,s}\left(u_{\varepsilon,s}\right)-M_{i,s}\left(u_{\varepsilon,s}\right)\right]-h\sum_{s=j+1}^{i-1}M_{i,s}\left(u_{\varepsilon,s}\right)-C_1h\sum_{s=1}^{j-1}u_{\varepsilon,s}\times\right.$$
$$\times h\sum_{m=j+1}^{i-1}K_{m,s}u_{\varepsilon,m}-C_1h\sum_{s=j+1}^{i-1}u_{\varepsilon,s}\,h\sum_{m=s+1}^{i-1}K_{m,s}u_{\varepsilon,m}-C_1h\sum_{s=1}^{j-1}h\times$$
$$\left.\times\sum_{m=j+1}^{i-1}N_{m,s}\left(u_{\varepsilon,s}\right)u_{\varepsilon,m}-C_1h\sum_{s=j}^{i-1}h\sum_{m=s+1}^{i-1}N_{m,s}\left(u_{\varepsilon,s}\right)u_{\varepsilon,m}+f_j-f_i\right\}+$$

$$+\frac{1}{\varepsilon+k_i}exp\left(-h\sum_{s=1}^{i}\frac{G_s}{\varepsilon+k_s}\right)\left\{h\sum_{j=1}^{i-1}M_{i,j}\left(u_{\varepsilon,j}\right)+C_1h\sum_{j=1}^{i-1}u_{\varepsilon,j}\,h\sum_{s=j+1}^{i-1}K_{s,j}u_{\varepsilon,s}+\right.$$
$$\left.+C_1h\sum_{j=1}^{i-1}h\sum_{s=j+1}^{i-1}N_{s,j}\left(u_{\varepsilon,j}\right)u_{\varepsilon,s}+\varepsilon u_{0,h}+f_i\right\},i=1..n, \qquad (7)$$

Estimate the expression

$$U_{i-1}(u_1,\dots,u_{i-1})=\left\{exp\left(-h\sum_{s=1}^{i}\frac{G_s}{\varepsilon+k_s}\right)+h\sum_{j=1}^{i-1}exp\left(-h\sum_{s=j+1}^{i}\frac{G_s}{\varepsilon+k_s}\right)\times\right.$$
$$\left.\times\frac{G_j}{\varepsilon+k_j}\right\}\frac{C_1h}{\varepsilon+k_i}\left\{\left(h\sum_{s=1}^{i-1}K_{i,s}u_{\varepsilon,s}\right)+\left(h\sum_{s=1}^{i-1}N_{i,s}\left(u_{\varepsilon,s}\right)\right)\right\}$$

putting $C_1=C_0h^2, 0<C_0=const.$

Then

$$|U_{i-1}(u_1,\dots,u_{i-1})|\le\frac{(Mr+M_N)C_1h}{ed_1}+T_0\bar{d}_2b\frac{(Mr+M_N)C_0h}{d_1},$$

$$\bar{d}_2=sup\left|h\sum_{j=1}^{i-1}\left(h\sum_{s=j+1}^{i}\frac{G_s}{\varepsilon+k_s}\right)exp\left(-h\sum_{s=j+1}^{i}\frac{G_s}{\varepsilon+k_s}\right)\right|.$$

If

$$h<\frac{ed_1}{T_0\bar{d}_2bMrC_0}, \quad (8)$$

then $|U_{i-1}(u_1,\dots,u_{i-1})|<1$. If condition (8) is satisfied, the system (7) can be rewritten in the form

$$u_{\varepsilon,i}=\left(1-U_{i-1}(u_1,\dots,u_{i-1})\right)^{-1}\left[-\frac{h}{\varepsilon+k_i}\sum_{j=1}^{i-1}exp\left(-h\sum_{s=j+1}^{i}\frac{G_s}{\varepsilon+k_s}\right)\frac{G_j}{\varepsilon+k_j}\times\right.$$
$$\times\left\{h\sum_{s=1}^{j}\left[M_{j,s}\left(u_{\varepsilon,s}\right)-M_{i,s}\left(u_{\varepsilon,s}\right)\right]-h\sum_{s=j+1}^{i-1}M_{i,s}\left(u_{\varepsilon,s}\right)-C_1h\sum_{s=1}^{j-1}u_{\varepsilon,s}\times\right.$$
$$\times h\sum_{m=j+1}^{i-1}K_{m,s}u_{\varepsilon,m}-C_1h\sum_{s=j+1}^{i-1}u_{\varepsilon,s}\,h\sum_{m=s+1}^{i-1}K_{m,s}u_{\varepsilon,m}-C_1h\sum_{s=1}^{j-1}h\times$$
$$\left.\times\sum_{m=j+1}^{i-1}N_{m,s}\left(u_{\varepsilon,s}\right)u_{\varepsilon,m}-C_1h\sum_{s=j}^{i-1}h\sum_{m=s+1}^{i-1}N_{m,s}\left(u_{\varepsilon,s}\right)u_{\varepsilon,m}+f_j-f_i\right\}+$$
$$+\frac{1}{\varepsilon+k_i}exp\left(-h\sum_{s=1}^{i}\frac{G_s}{\varepsilon+k_s}\right)\left\{h\sum_{j=1}^{i-1}M_{i,j}\left(u_{\varepsilon,j}\right)+C_1h\sum_{j=1}^{i-1}u_{\varepsilon,j}\,h\sum_{s=j+1}^{i-1}K_{s,j}u_{\varepsilon,s}+\right.$$
$$\left.+C_1h\sum_{j=1}^{i-1}h\sum_{s=j+1}^{i-1}N_{s,j}\left(u_{\varepsilon,j}\right)u_{\varepsilon,s}+\varepsilon u_{0,h}+f_i\right\},i=1..n, \qquad (9)$$

It is not difficult to see that (9) is a recursive formula.

Results

This equation is reduced to the Volterra integral equation of the third kind and a numerical method is developed on the basis of that regularized equation. The convergence of the numerical solution to the exact solution of the Volterra integral equation of the first kind is proved, an estimate of the permissible error and a recursive formula of the computational process are obtained.

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МЕТОД ЧИСЛЕННОГО РЕШЕНИЯ НЕЛИНЕЙНЫХ ИНТЕГРАЛЬНЫХ УРАВНЕНИЙ ВОЛЬТЕРРА ПЕРВОГО РОДА

Аннотация. При рассмотрении систем дифференциальных уравнений с весьма общими краевыми условиями, точные методы решения наталкиваются на большие трудности, которые становятся непреодолимыми при рассмотреиии нелинейных задач. В этих случаях приходится обращаться к тем или иным численным методам решения. Важно отметить, что использование численных методов зачастую позволяет отказаться от упрощенной трактовки математической модели процесса. В работе изучаются вопросы численного решения нелинейных интегральных уравнений Вольтерра первого рода с дифференцируемым ядром, которое вырождается в начальной точке диагонали. Рассматриваемое уравнение сводится к интегральному уравнению Вольтерра третьего рода и на основе регуляризованного уравнения разработан численный метод. Доказана сходимость численного решения к точному решению интегрального уравнения Вольтерра первого рода, получены оценка допускаемой погрешности и рекурсивная формула вычислительного процесса.

Ключевые слова: нелинейное интегральное уравнение, систему нелинейных алгебраических уравнений, вектора погрешности, уравнение Вольтерра, малый параметр, численный метод.

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БІРІНШІ ТҮРДЕГІ СЫЗЫҚТЫ ЕМЕС ИНТЕГРАЛДЫ ВОЛЬТЕРРА ТЕҢДЕУЛЕРІН САНДЫҚ ШЕШУ ӘДІСІ

Аннотация. Дифференциалдық теңдеулер жүйесін өте жалпы шекаралық шарттармен қарастырған кезде, сызықты емес проблемаларды қарастыру кезінде шешілмейтін қиындықтарға айналдырудың дәл әдістері. Мұндай жағдайларда белгілі бір сандық әдістерге жүгіну керек. Сандық әдістерді қолдану, процестің математикалық моделін оңайлатылған түсіндіруден бас тартуға мүмкіндік береді. Алғашқы түрдегі сызықты емес Вольтерра интегралдық теңдеулерін диагональды бастапқы нүктесінде нөлге келтіретін дифференциалды ядро сандық шешудің сандық мәселелері қарастырылады. Қарастырылып отырған теңдеу Вольтерра интегралдық теңдеуін үшінші түрге дейін азайтады және реттелген теңдеудің негізінде сандық әдіс әзірленеді. Сандық шешімнің бірінші түрдегі Вольтерра интегралдық теңдеуінің дәл шешіміне дәлелденді, рұқсат етілген қателікті бағалау және есептеу үдерісінің рекурсивті формуласы алынды.

Түйінді сөздер: сызықты емес интегралдық теңдеу, сызықтық алгебралық теңдеулер жүйесі, қателік векторы, Вольтерра теңдеуі, кіші параметр, сандық әдіс.

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