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M-FUNCTION NUMBERS: CYCLES AND OTHER EXPLORATIONS. PART 2

Abstract. This paper establishes the cyclic properties of the M-Function, which we define as a function, [M(n)], that takes a positive integer, adds to it the sum of its digits and the number produced by reversing its digits, and then divides the entire sum by three. Our definition of the M-Function is influenced by D. R. Kaprekar's work on a remarkable class of positive integers, called self- numbers, and his procedure, [K(n)], of adding to any positive integer the sum of its digits [1]. We analyze the distribution of numbers that make the defined M-Function behave like a cyclic function, and observe that many such "cycles" form arithmetic sequences. We examine the distribution of numbers that produce integer ratios between the outputs of Kaprekar's and the M-Function functions, [K(n)/M(n)]. We also prove that the set of numbers with equal outputs to both Kaprekar's and M-Function functions, [K(n)=M(n)], is infinite.

Key words: M-Function, D.R.Kaprekar, self-numbers.

5. Integer ratios between two numbers from n, $K(n)$, $M(n)$. Kaprekar defined a *digitaddition* as $K(n) = n + S(n) > n$. Hence, the difference in numbers $K(n)$ and n is equal to $K(n) - n = S(n)$. $S(n)$ is a lot less than n when n is large, hence the equation $t * n = K(n)$ when $t \geq 2, t \in N$, does not have any solutions.

Because $K(n) > n$, the investigation of the equation $t * K(n) = n$, when $t \geq 2$ will not produce any solutions: $t * K(n) > K(n) > n$.

Therefore, we will study integer ratios between numbers n and $M(n)$ in paragraph 5.1, and between $K(n)$ and $M(n)$ in paragraph 5.2.

5.1. Integer ratios between numbers n and $M(n)$. As defined earlier, if $n = M(n)$, the number n is stationary. So, we need to consider two cases:

$t * n = M(n)$, when $t \geq 2$, $t \in N$, and

$$n = t * M(n), \quad \text{when } t \geq 2, \ t \in N.$$

A) Consider $t * n = M(n)$, when $t \geq 2, t \in N$.

Proposition 5. If $t \geq 4$, the equation $t * n = M(n)$ will not have solutions.

Proof. Let $t \geq 4$, and $t * n = M(n)$.

Then:

$$t * n = \frac{1}{3}(n + S(n) + \bar{n}).$$

Multiply by 3, and we get $3t * n = n + S(n) + \bar{n}$.

Re-arranging, we get $(3t - 1) * n = S(n) + \bar{n}$. (Equation 5)

Note that $3t - 1 \geq 11$ when $t \geq 4$.

Multiplying both sides by n, we get: $(3t-1)n \geq 11n > S(n) + \bar{n}$.

(Note, $10n > \bar{n}$ and $n \geq S(n)$, thus $11n > S(n) + \bar{n}$.)

Hence, when $t \geq 4$, Equation 5 doesn't have any solutions, and proposition 5 has been proven.

Let's consider the two remaining situations.

i) $t = 2 \rightarrow \quad 2 * n = M(n)$. Substituting in the definition of M(n) and re-arranging the equation, we can simplify the equation in the following steps:

$$2 * n = \frac{1}{3}(n + S(n) + \bar{n}).$$

$$5 * n = S(n) + \qquad \text{(Equation 6)}$$

Using a C++ program, we solve Equation 6 and get the following results for numbers to 10^{10}:

18	1206	120006	12000006	1200000006
126	12006	1200006	120000006	

Proposition 6. When $d(n) > 10$, all numbers with the type $a_d = 12\underbrace{0 \ldots 0}_{d-3}6$ are solutions of Equation 6. In other words, $2 * a_d = M(a_d)$ for all $d(n) > 10$.

The proof is similar to the proof of Proposition 1.

ii) $t = 3 \rightarrow \quad 3 * n = M(n)$. Substituting in the definition of M(n) and re-arranging the equation, we can simplify the equation in the following steps:

$$3n = \frac{1}{3}(n + S(n) + \bar{n}).$$

$$8n = S(n) + n \qquad \text{(Equation 7)}$$

Using a C++ program, no solutions were found for Equation 7 for numbers up to 10^{10}.

B) Consider $n = t * M(n)$, when $t \geq 2$, $t \epsilon N$.

Proposition 7. If $t \geq 3$ such that $t \in N$, the equation $n = t * M(n)$ will not have solutions.

Proof. Since $t \geq 3$, we know the following:

$$t * M(n) = t * \tfrac{1}{3}(n + S(n) + \bar{n}) \geq n + S(n) + \bar{n}.$$

(Note, $n + S(n) + \bar{n} > n$.)

Hence, when $t \geq 3$, the equation $n = t * M(n)$, doesn't have any solutions.

It remains to consider only 1 case:

$t = 2$, which means we consider the equation

$$n = 2 * M(n).$$

By definition of M(n), it is equivalent to: $n = \frac{2}{3}(n + S(n) + \bar{n})$.

$$n = 2 * S(n) + 2 * \bar{n} \qquad \text{(Equation 8)}$$

Using a C++ program, we found numbers that satisfy Equation 8 for numbers up to 10^{10}:

72	9054	120060	4920642	90000054	1200000060
180	12060	492642	9000054	120000060	4920000642
954	49842	900054	12000060	492000642	9000000054
1260	90054	1200060	49200642	900000054	

Proposition 8. When $d(n) > 10$, the following numbers with types $b_d = 12\underbrace{0 \ldots 0}_{d-4}60$, $c_d = 492\underbrace{0 \ldots 0}_{d-6}642$, $e_d = 9\underbrace{0 \ldots 0}_{d-3}54$ are solutions of Equation 8.

In other words, $b_d = 2 * M(b_d)$, $c_d = 2 * M(c_d)$, $e_d = 2 * M(e_d)$.

The proof is similar to the proof of Proposition 1.

Conjecture 4. When $d(n) > 10$, Equations 6 and 8 don't have any other solutions, except the solutions specified at propositions 6 and 8, respectively. Equation 7 doesn't have solutions in the set N.

5.2. Integer ratios between numbers $\boldsymbol{K(n)}$ and $\boldsymbol{M(n)}$. Earlier, we investigated the equation $K(n) = M(n)$. Hence, in this section we investigate 2 specific cases:

$$t * K(n) = M(n), \qquad \text{when } t \geq 2, \ \ t \in N,$$
$$K(n) = t * M(n), \qquad \text{when } t \geq 2, \ \ t \in N.$$

A) Let's consider $t * K(n) = M(n),$ when $t \geq 2$ and $t \in N$.

Proposition 9. Equation $t * K(n) = M(n)$ doesn't have any solutions when $t \geq 4, t \in N$.

Proof. Let's simplify this equation:

$$t * K(n) = M(n)$$
$$t * \big(n + S(n)\big) = \frac{1}{3}(n + S(n) + \bar{n})$$

Multiply by 3:

$$3t * n + 3t * S(n) = n + S(n) + \bar{n}$$
$$(3t - 1) * n + (3t - 1) * S(n) = \bar{n}$$

Because we are considering $t \geq 4$, $3t - 1 \geq 11$.

Hence $$(3t - 1) * n + (3t - 1) * S(n) \geq 11 * (n + S(n)) > \bar{n}$$

If number n has d digits, number $11 * \big(n + S(n)\big)$ will have at least d+1 digits.

Thus, when $t \geq 4$, the equation $t * K(n) = M(n)$ doesn't have any solutions.

It remains to consider two other subcases: when $t = 2$ and $t = 3$.

i) $t = 2 \rightarrow 2 * K(n) = M(n)$. We can simplify the equation further:

$$2\big(n + S(n)\big) = \frac{1}{3}(n + S(n) + \bar{n})$$
$$5n = -5 * S(n) + \bar{n} \qquad \text{(Equation 9)}$$

ii) $t = 3 \rightarrow 3 * K(n) = M(n)$. We can also simplify this equation:

$$3\big(n + S(n)\big) = \tfrac{1}{3}(n + S(n) + \bar{n}).$$
$$8n = -8 * S(n) + \bar{n} \qquad \text{(Equation 10)}$$

Using a C++ program, no solutions were found for Equations 9 and 10 for numbers up to 10^{10}.

Conjecture 5. Equations 9 and 10 don't have any solutions in N, which means that n ∈ N doesn't exist for the following 2 equations:

$$2 * K(n) = M(n), \qquad 3 * K(n) = M(n).$$

B) Let's consider $K(n) = t * M(n)$, when $t \geq 2$, and $t \epsilon N$.

Proposition 10. When $t \geq 3$, the equation $K(n) = t * M(n)$ doesn't have solutions in the set *N*.

Proof. We can simplify the following equation by substituting in the definitions of K(n) and M(n), and performing algebraic manipulation:

$$K(n) = t * M(n)$$
$$n + S(n) = \tfrac{t}{3}(n + S(n) + \bar{n}).$$

Since we are considering the case

$t \geq 3$, we can set up the following inequalities:

$$\frac{t}{3}(n + S(n) + \bar{n}) \geq n + S(n) + \bar{n} > n + S(n).$$

Hence, when $t \geq 3$, $t * M(n) > K(n)$ and the equation $K(n) = t * M(n)$ doesn't have solutions ($t \in N$).

It remains to consider just 1 case.

i) $t = 2 \rightarrow K(n) = 2 * M(n)$. We can simplify the equation:

$$n + S(n) = \frac{2}{3}(n + S(n) + \bar{n}).$$

$$n = -S(n) + 2 * \bar{n} \qquad \text{(Equation 11)}$$

Using a C++ program, we get the following solutions to Equation 11 for numbers up to 10^{10}:

1	201	8004	200001	4999952	60000003	1079999350
2	402	8734	340071	5400072	74000073	1400000070
3	603	9474	400002	6000003	80000004	2000000001
4	804	14070	540072	6999943	94000074	3079999351
5	1470	20001	600003	7400073	140000070	3400000071
6	2001	34071	740073	8000004	200000001	4000000002
7	2731	40002	800004	8999944	340000071	5079999352
8	3471	54072	940074	9400074	400000002	5400000072
9	4002	60003	1400070	14000070	540000072	6000000003
21	4732	74073	2000001	20000001	600000003	7079999353
42	5472	80004	2999941	34000071	740000073	7400000073
63	6003	94074	3400071	40000002	800000004	8000000004
84	6733	140070	4000002	54000072	940000074	9079999354
	7473					9400000074

Proposition 11. When $d(n) > 10$, the following types of numbers are solutions to Equation 11:

$a_d = 14\underbrace{0\ldots0}_{d-4}70$, $b_d = 2\underbrace{0\ldots0}_{d-2}1$, $c_d = 34\underbrace{0\ldots0}_{d-4}71$, $e_d = 4\underbrace{0\ldots0}_{d-2}2$, $f_d = 54\underbrace{0\ldots0}_{d-4}72$, $g_d = 6\underbrace{0\ldots0}_{d-2}3$, $h_d = 74\underbrace{0\ldots0}_{d-4}73$, $l_d = 8\underbrace{0\ldots0}_{d-2}4$, $q_d = 94\underbrace{0\ldots0}_{d-4}74$

Which, equivalently, means that all of these numbers satisfy the equation

$$K(n) = 2 * M(n).$$

The proof is similar to the proof of Proposition 1.

Note. The equation $K(n) = 2 * M(n)$ satisfies all 9 types of numbers described in proposition 11, starting with $d(n) \geq 4$, (i.e. starting with four-digit numbers). However, there are also other four-digit, seven-digit and ten-digit solutions to Equation 11:

2731	6733	2999941	6999943	1079999350	5079999352
4732	8734	4999942	8999944	3079999351	7079999353

We can't observe a clear relationship between these 3 groups of numbers.

Recall the following fact: cycles with length 10 can only contain four-digit, seven-digit or ten-digit numbers. According to all of our discovered facts, our investigation of function $M(n)$ for four-digit, seven-digit and ten-digit numbers take special place.

In the case of Equation 11, we can't observe a general pattern of solutions for numbers up to 10^{10}. Hence, we cannot formulate a conjecture, but we can instead formulate a problem.

Problem 1. It is possible to find all solutions to Equation 11 for all $n \in N$, when $d(n) > 10$.

6. The distribution and properties of sets of m-generated numbers. Let N be the set of positive integer numbers. Let us choose any $n \epsilon N$ and denote the sum of its digits by $S(n)$. The number $M(n) = \frac{1}{3}(n + S(n) + \bar{n})$ is called a *m-generated number* and the inputted number *n* is its *generator*. Positive integer that has no generator is called a *m-self number*. Let's denote E as the set of all m-self numbers, and G as the set of all m −generated numbers. Clearly,

$$N = G \cup E$$

In contrast to Kaprekar function's set of generated numbers, m −generated numbers are much rare than m −self numbers. The distribution of the set of m −generated numbers in the set of natural numbers N brings strong interest to us.

Using a C++ program, we can find $m-$ generated numbers for numbers up to 10^8, and compile the following table.

Table 2 - The distribution of set of $m-$ generated numbers for numbers to 10^8

$d(n)$	Interval	Quantity of generated numbers	% percentage out of all numbers
1	[1, 9]	9	100%
2	[10, 99]	29	32.22%
3	[100, 999]	188	20.89%
4	[1000, 9999]	594	6.60%
5	[10000, 99999]	3668	4.076%
6	[100000, 999999]	11352	1.261%
7	[1000000, 9999999]	69819	0.776%
8	[10 000 000, 99 999 9999]	215985	0.240%

Let's calculate $\sqrt[7]{215985/9} \approx 4.224$. This means that, if we increase the order to 1, the quantity of m −generated numbers in the next order $d(n)$ will be increased, on average, by a factor of 4.224. However, the percentage of m −generated numbers from all natural numbers decreases from 100% among one-digit numbers to 0,24% among eight-digit numbers. The percentage of m −generated numbers, on average, decreases by a factor of 2,37 when the order $d(n)$ decreased by one, because $\sqrt[7]{100/0{,}24} \approx 2{,}37$.

The decreasing in the proportion of m −generated numbers out of all natural numbers while the order of numbers is increasing suggests that the average number of m −generators for one m −generated number is increasing. $m-$ generated numbers with more $m-$ generators appear when the order of numbers is increasing.

Conjecture 6. For any $k \epsilon N$, there exist $m-$ generated numbers, and the quantity of $m-$ generators greater than or equal to k.

Let's consider the distribution of m-generated numbers by intervals of hundreds, thousands, ten-thousands, hundred-thousands, millions, and ten-millions.

Table 3

Intervals	Quantity
[100, 199]	30
[200, 299]	28
[300, 399]	31
[400, 499]	32
[500, 599]	31
[600, 699]	20
[700, 799]	6
[800, 899]	6
[900, 999]	4

Table 4

Intervals	Quantity
[1000, 1999]	57
[2000, 2999]	57
[3000, 3999]	72
[4000, 4999]	84
[5000, 5999]	85
[6000, 6999]	65
[7000, 7999]	60
[8000, 8999]	60
[9000, 9999]	54

Table 5

Intervals	Quantity
[10000, 19999]	570
[20000, 29999]	570
[30000, 39999]	598
[40000, 49999]	625
[50000, 59999]	624
[60000, 69999]	350
[70000, 79999]	114
[80000, 89999]	114
[90000, 99999]	103

Table 3

Intervals	Quan-tity
[100000, 199999]	1083
[200000, 299999]	1083
[300000, 399999]	1364
[400000, 499999]	1646
[500000, 599999]	1646
[600000, 699999]	1206
[700000, 799999]	1140
[800000, 899999]	1140
[900000, 999999]	1044

Table 4

Intervals	Quan-tity
[1000000, 1999999]	10830
[2000000, 2999999]	10830
[3000000, 3999999]	11366
[4000000, 4999999]	11906
[5000000, 5999999]	11900
[6000000, 6999999]	6671
[7000000, 7999999]	2166
[8000000, 8999999]	2165
[0000000, 9999999]	1985

Table 5

Intervals	Quan-tity
[10000000, 19999999]	20577
[20000000, 29999999]	20577
[30000000, 39999999]	25980
[40000000, 49999999]	31383
[50000000, 59999999]	31379
[60000000, 69999999]	22915
[70000000, 79999999]	21660
[80000000, 89999999]	21659
[90000000, 99999999]	19855

From the investigation of the distribution, we can observe that by the order $d(n)$ where $1 \leq d(n) \leq 8$, the quantity of $m-$ generated numbers increases.

7. "Neighboring" numbers. If we consider a variety of $m-$generated numbers, we can observe 2 or more consecutive numbers belonging to the defined set G. So, if numbers $a_1 + i \in G$, such that $i = 0,1,\dots,k-1$, we will call them ($\{a_1, a_1+1, \dots, a_1+k-1\}$) "neighboring" with length k, where $k \geq 2$.

Using a C++ program, we find "neighboring" numbers up to 10^9 and study them. We completed a table showing the distribution of "neighboring" numbers up to 10^9:

Table 9 - The table of "neighboring" numbers' distribution up to 10^9

$d(n)$	Intervals	The length of "neighboring" numbers							
		2	3	4	5	6	7	8	9
1	[1,9]	0	0	0	0	0	0	0	1
2	[10,99]	7	0	0	0	0	0	0	0
3	[100,999]	87	2	0	0	0	0	0	0
4	[1000,9999]	127	17	2	0	0	0	0	0
5	[10000,99999]	26	5	0	0	0	0	0	0
6	[100000,999999]	28	5	0	0	0	0	0	0
7	[1000000,9999999]	77	0	0	0	0	0	0	0
8	[10000000,99999999]	21918	27	0	0	0	0	0	0
9	[100000000,999999999]	138510	0	0	0	0	0	0	0

On this list (considering lengths 2-9), there exist "neighboring" numbers only with length $k = \{2,3,4,9\}$. Among the "neighboring" numbers with length 9, there is just one: $\{1,2,3,4,5,6,7,8,9\}$. Among "neighboring" numbers with length 4, there are a total of 2: $\{3671,3672,3673,3674\}$ and $\{4340,4341,4342,4343\}$. According to the table, "neighboring" numbers of length 2 are more frequent than such numbers of length 3.

Let's consider the following table of the greatest and the smallest "neighboring" numbers in terms of each order $d(n)$ of m –generated numbers.

Table 10 - The table of smallest and largest "neighboring" numbers up to 10^9

$d(n)$	The smallest "neighboring" numbers	The greatest "neighboring" numbers
2	{40,41}	{96,97}
3	{102,103}	{963,964}
4	{1000,1001,1002}	{6566,6567,6568}
5	{10002,10003}	{65535,65536}
6	{358903, 358904}	{655204,655205}
7	{35852237, 3585238}	{6551873,6551874}
8	{35521871,35521872}	{99966311, 99966312}
9	{100033705, 100033706}	{429966350, 429966351}

According to all the gathered facts about “neighboring” numbers, we can propose the following conjectures.

Conjecture 7. The “neighboring” numbers with length $k \geq 4$ don’t exist among the set of $m-$generated numbers with order $d(n) > 9$.

Conjecture 8. There is an infinite amount of “neighboring” numbers with length $k = 2$ among the set of $m-$generated numbers.

Conjecture 9. There is an infinite amount of “neighboring” numbers with length $k = 3$ among the set of $m-$generated numbers.

Conclusion. Following Kaprekar, three years ago we came up with a simple procedure of generating positive integers

$$n \to M(n) = \frac{1}{3}(n + S(n) + \bar{n}).$$

Although the case $n < K(n)$ occurs for all positive integers n, in the M-Function, it is possible to have 3 different cases: $n < M(n)$, $n = M(n)$ and $n > M(n)$. Through investigation, new facts and notions were discovered. Based on the results of investigation, 9 conjectures and 1 problem were formulated.

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М-ФУНКЦИЯ САНДАРЫ: ЦИКЛДЕР ЖӘНЕ БАСҚА ЗЕРТТЕУЛЕР

Аннотация. Индиялық математик Д.Р. Капрекар өзі ашқан“ Капрекар Константасы” - 6174 санымен аса танымал.

Капрекардың тағы бір ашқан жаңалығы – өзіндік туындаған сандар класы белгілі американдық ғылым насихаттаушысы Мартин Гарднердің «Уақыт бойынша саяхат» [1] атты кітабында баяндалған. Кез-келген натурал n санын аламыз және ол санға цифрларының қосындысы $S(n)-$ді қосамыз. Шыққан сан $K(n) = n + S(n)$ туындаған сан, ал алғашқы сап n – оның генераторы деп аталады. Мысалы, егер 53 санын алсақ, онда туындаған сан $53 + 3 + 5 = 61$ саны болады.

Туындаған санның генераторларының саны бірден артық болуы мүмкін. Екі генераторы бар ең кіші сан 101, ал оның генераторлары 91 и 100 сандары. Өзіндік туындаған сандар - генераторлары жоқ сандар. «The American Mathematical Monthly» [2] журналында жарияланған мақалада өзіндік туындаған сандардың шексіз көп екендігі және өзіндік туындаған сандар туындаған сандарға қарағанда өте сирек кездесетіндігі дәлелденген.

Капрекар ашқан осы жаңалықтар көптеген математиктерді қызықтырды. Әртүрлі елдерде «Капрекар Константасының» , өзіндік туындаған және туындаған сандары жиындарының жаңа қасиеттерін жан-жақты зерттеген көптеген мақалалар, математикалық ғылыми жобалар және программалық өнімдер жарық көрді.

Мен Капрекарға сүйене отырып натурал сандарды алудың жаңа әдісін таптым: $n \to M(n) = \frac{1}{3}(n + S(n) + \bar{n})$, мұнда $\bar{n}$ - сол цифрлармен, бірақ кері бағытта жазылған сан. $M(n)$ саны әрқашан бүтін сан болады, себебі $n, S(n), \bar{n}$ сандарының 3- ке бөлгендегі қалдықтары әрқашан тең болады. Егер Капрекар жағдайында $n < K(n)$ теңсіздігі кез –келген натурал n сандарында орындалса, менің құрған функциямда әртүрлі қатынастар болады, яғни барлық 3 жағдай да орын алады : $n < M(n), n = M(n)$ и $n > M(n)$.

Менің тапқан жаңа натурал сан алу функциясы $n \to M(n)$ әрі қарапайым, табиғи және ол Капрекар $n \to K(n)$ функциясының аналогы болып табылады.

Мақаланың 1-ші бөлімінде m-циклдарының таралуы және олардың қасиеттері зерттеледі. (Егер $M^l(n) = n$ теңдігі орындалатындай ең кіші натурал сан l болса, онда $n \to M(n) \to M^2(n) \to \ldots \to M^{l-1}(n) \to M^l(n) \to n$ сандары m-циклді құрайды. Ал $K(n)$ жағдайында циклдар туындамайды, себебі. $n < K(n) < K^2(n)$ ....). Сонымен қатар, $K(n) = M(n)$ функцияларының теңдігі сұрағы және $n, K(n)$ и $M(n)$ сандарының қандай да бір ретпен арифметикалық прогрессия құрайтын сұрақтары зерттелген.

Мақаланың 2-ші бөлімінде $n, K(n)$ и $M(n)$ сандарының арасындағы еселік қатынастар қарастырылған. Яғни, қандай натурал t сандарында, $t \geq 2$, $K(n) = tM(n), tK(n) = M(n), n = t\,M(n), n * t = M(n)$ теңдіктері орындалатындығы зерттелген. (Айта кетейік, n және $K(n)$ сандарының арасында еселік қатынастар болуы мүмкін емес). Сонымен қатар $m-$туындаған сандар жиының таралуы және қасиеттері зерттелген.

(m – туындаған сандар $m-$өзіндік туындаған сандарға қарағанда өте сирек кездеседі. Сондықтан m – туындаған сандар жиынын зерттеу m –өзіндік туындаған сандар жиынына қарағанда маңыздырақ). Осы бөлімде “көрші”, яғни қатарлас тұрған m –туындаған сандар жиыны зерттелген.

Зерттеу барысында 1 мәселе және 9 гипотезалар тұжырымдалған.

Түйін сөздер: М-функция, Д.Р.Капрекар, өзіндік туындаған сандар.

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ЧИСЛА М-ФУНКЦИИ: ЦИКЛЫ И ДРУГИЕ ИССЛЕДОВАНИЯ

Аннотация. Индийский математик Д.Р. Капрекар особенно известен своим открытием “Константу Капрекара”- числом 6174.

Другое выдающиеся открытие Капрекара, описанная известным американским популяризатором науки Мартином Гарднером в своей книге “Путешествие во времени”[1], это класс самопорожденных чисел. Выберем любое натуральное число n и прибавим к нему сумму его цифр $S(n)$. Полученное число $K(n) = n + S(n)$ называется *порожденным*, а исходное число n – его *генератором*. Например, если выберем число 53, порожденное им число равно $53 + 3 + 5 = 61$. Порожденное число может иметь более одного генератора. Наименьшее число с двумя генераторами равно 101, и его генераторами являются числа 91 и 100. Самопорожденное число – это число, у которого нет генератора. В статье журнала «The American Mathematical Monthly»[2] доказывалось, что существует бесконечно много самопорожденных чисел, но встречаются они гораздо реже, чем порожденные числа. Эти открытия Капрекара заинтересовали многих математиков и в разных странах мира появились много научных статьей, научных проектов по математике, программных продуктов, в которых исследовались различные новые свойства “Константы Капрекара” и множеств самопорожденных чисел и порожденных чисел.

Следуя Капрекару, я нашел новый способ получения натуральных чисел: $n \to M(n) = \frac{1}{3}(n + S(n) + \bar{n})$, где $\bar{n}$ - число, записанное теми же цифрами, но в обратном порядке. Число $M(n)$ будет всегда целым, так как числа $n, S(n), \bar{n}$ дают одинаковые остатки при делении на 3. Если в случае Капрекара неравенство $n < K(n)$ выполняется при всех натурального n, то в моем случае положение разнообразнее, т.е. возможны все 3 случая: $n < M(n)$, $n = M(n)$ и $n > M(n)$.

Моя функция получения новых натуральных чисел $n \to M(n)$ простая, естественная и она является аналогом функции Капрекара $K(n)$. В 1-й части данной статьи исследованы распределение m-циклов и их свойства. (Если l – наименьшее натуральное такое, что $M^l(n) = n$, то числа $n \to M(n) \to M^2(n) \to \ldots \to M^{l-1}(n) \to M^l(n) \to n$ образуют m-цикл. В случае функции $K(n)$ циклы невозможны, т.к. $n < K(n) < K^2(n)$ ....). Также изучены вопрос равенства чисел $K(n) = M(n)$ и вопрос образования арифметической прогрессии в некотором порядке числами $n, K(n)$ и $M(n)$.

Во 2-й части статьи изучены кратные отношения между числами $n, K(n)$ и $M(n)$. Т.е. исследованы вопросы: при каких t натуральном, $t \geq 2$, возможны равенства $K(n) = tM(n), tK(n) = M(n), n = t\,M(n), n * t = M(n)$. (Отметим, что кратные отношения между числами n и $K(n)$ невозможны). Также исследованы распределение и свойства множества m – порожденных чисел (m –порожденныех чисел встречаются гораздо реже, чем $m-$самопорожденные, поэтому изучение множества m – порожденных намного важнее, чем изучение класса m – самопорожденных чисел). В этой части исследовано множество “соседних”, т.е. последовательных m – порожденных чисел.

В процессе исследования сформулированы 1 проблема и 9 гипотез.

Ключевые слова: М-функция, Д.Р.Капрекар, самопорожденные числа.

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