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GREEN TENSOR OF MOTION EQUATIONS OF TWO COMPONENTS BIOT'S MEDIUM BY STATIONARY VIBRATIONS

Abstract. Here processes of wave propagation in a two-component Biot's medium are considered which are generated by periodic forces actions. By use Fourier transformation of generalized functions, the Green tensor - a fundamental solutions of oscillation equations of this medium has been constructed. This tensor describes the process of propagation of harmonic waves of a fixed frequency in spaces of dimension N = 1,2,3 under the action of power sources concentrated at the coordinates origin, described by a singular delta -function. Based on it, generalized solutions of these equations are constructed under the action of various sources of periodic perturbations, which are described by both regular and singular generalized functions. For regular acting forces, integral representations of solutions are given that can be used to calculate the stress-strain state of a porous water-saturated medium.

Key words: Biot's medium, solid and liquid components, fundamental solution, generalized direct and inverse Fourier transform, regularization.

Various mathematical models of deformable solids mechanics are used in the study of seismic processes in the earth's crust. The processes of waves propagation are most studied in elastic media. But these models do not take into account many real properties of the ambient array. These are, for example, the presence of groundwater, which complicates the construction and operation of surface and underground structures, affect the magnitude and distribution of stresses. Models, which take into account the water saturation form the earth's crust structures, the presence of gas bubbles, etc., are multi-component medium. A variety of multicomponent media, the complexity of the processes associated with their deformation, lead to a large difference in the methods of analysis and modelling used in the solution of wave problems.

Porous medium saturated with liquid or gas, from the point of view of continuum mechanics, is essentially a two-phase continuous medium, one phase of which is particles of liquid (gas), other solid particles is its elastic skeleton. There are various mathematical models of such media, developed by various authors. The most famous of them are the models of M. Biot, V.N. Nikolaevsky, L.P. Horoshun [1-7]. However, the class of solved tasks to them is very limited and mainly associated with the construction and study of particular solutions of these equations based on the methods of full and partial separation of variables and theory of special functions in the works of Rakhmatullin, H. A., Saatov Ya. U., Filippov I. G., Artykov T. U. [6,7], Erzhanov Zh. S, Ataliev Sh.M., Alexeyeva L.A., Shershnev V.V. [8,9] etc. In this regard, it is important to develop effective methods of solution of boundary value problems for such media with use of modern mathematical methods.

Periodic on time processes are very widespread in practice. By this cause here we consider the process of wave propagation in the Biot's medium, posed by the periodic forces of different types. Based on Fourier transformation of generalized functions we constructed fundamental solutions of oscillation equations of Biot's medium. It is Green's tensor, which describes the process of propagation of harmonic waves at a fixed frequency in the space-time of dimension N=1,2,3, under the action of concentrated at the coordinates origin. By use this tensor we construct generalized solutions of these equations for arbitrary sources of periodic disturbances, which can be described both regular and singular distributions. They can be used to calculate the stress-strain state of a porous water-saturated medium by seismic waves propagation.

1 The parameters and motion equations of a two-components M. Biot medium

The equations of motion of a homogeneous isotropic two-component M. Biot medium are described by the following system of second-order hyperbolic equations [1-3]:

$$\begin{gathered}(\lambda+\mu)\,\mathrm{grad}\,\mathrm{div}\,u_s+\mu\Delta u_s+Q\,\mathrm{grad}\,\mathrm{div}\,u_f+F^s(x,t)=\rho_{11}\ddot{u}_s+\rho_{12}\ddot{u}_f\\ Q\,\mathrm{grad}\,\mathrm{div}\,u_s+R\,\mathrm{grad}\,\mathrm{div}\,u_f+F^f(x,t)=\rho_{12}\ddot{u}_s+\rho_{22}\ddot{u}_f\end{gathered}\tag{1}$$

$(x,t)\in R^N\times[0,\infty)$. Here N is the dimension of the space. At a plane deformation N=2, the total spatial deformation corresponds to N=3, at N=1 the equations describe the dynamics of a porous liquid-saturated rod. We denote $u_s=u_{sj}(x,t)e_j$ a displacements vector of the elastic skeleton, $u_f=u_{fj}(x,t)e_j$ is the displacements vector of a liquid, e_j $(j=1,...,N)$ are the basic orts of the Lagrangian Cartesian coordinate system (everywhere by repeating indices there is summation from 1 to N).

Constants $\rho_{11},\rho_{12},\rho_{22}$ have the dimension of mass density and are associated with the density of the masses of the particles, composing a skeleton ρ_s and a fluid ρ_f , by relationships:

$$\rho_{11}=(1-m)\rho_s-\rho_{12},\quad \rho_{22}=m\rho_f-\rho_{12},$$

where m is a porosity of medium. The constant of the attached density ρ_{12} is related to the dispersion of the deviation of the micro-velocities of the fluid particles in the pores from the average velocity of the fluid flow and depends on the geometry of the pores. Elastic constants λ,μ are the Lame parameters of an isotropic elastic skeleton, and Q, R characterize the interaction of the skeleton with a liquid on the basis of *Biot law for stresses*:

$$\begin{gathered}\sigma_{ij}=(\lambda\partial_k u_k+Q\partial_k U_k)\delta_{ij}+\mu(\partial_i u_i+\partial_j u_i)\\ \sigma=-mp=R\partial_k U_k+Q\partial_k u_k\end{gathered}\tag{2}$$

Here $\sigma_{ij}(x,t)$ are the stress tensor in the skeleton, $p(x,t)$ is a pressure in the fluid. Further we use the notations for partial derivatives: $\partial_k=\dfrac{\partial}{\partial x_k}, u_{j,k}=\partial_k u_j$, $\Delta=\partial_k\partial_k$ is Laplace operator. The external mass forces acting on the skeleton $F^s=F_j^s(x,t)e_j$ and on the liquid component $F^f=F_j^f(x,t)e_j$.

There are three sound speeds in this medium:

$$c_1^2=\frac{\alpha_1+\sqrt{\alpha_1^2-4\alpha_2\alpha_3}}{2\alpha_2},\quad c_2^2=\frac{\alpha_1-\sqrt{\alpha_1^2-4\alpha_2\alpha_3}}{2\alpha_2},\quad c_3=\sqrt{\frac{\rho_{22}\mu}{\alpha_2}}\tag{3}$$

where the next constants were introduced:

$$\alpha_1=(\lambda+2\mu)\rho_{22}+R\rho_{11}-2Q\rho_{12},\quad \alpha_2=\rho_{11}\rho_{22}-(\rho_{12})^2,\quad \alpha_3=(\lambda+2\mu)R-Q^2$$

The first two speeds $c_1, c_2 \left(c_1 > c_2\right)$ describe the velocity of propagation of two types of *dilatational waves*. The second slower dilatation wave is called *the repackaging wave*. A third velocity c_3 corresponds to *shear waves* and at $\rho_{12} = 0$ coincides with velocity of transverse waves propagation in an elastic skeleton ($c_3 < c_1$).

We introduce also two velocities of propagation of dilatational waves in corresponding elastic body and in an ideal compressible fluid:

$$c_s = \sqrt{\frac{\lambda + 2\mu}{\rho_{11}}}, \qquad c_f = \sqrt{\frac{R}{\rho_{22}}}$$

2 Problems of periodic oscillations of the Biot's medium

Construction of motion equation solutions by periodic oscillations is very important for practice since existing power sources of disturbances are often periodic in time and therefore can be decomposed into a finite or infinite Fourier series in the form:

$$F^s\left(x,t\right) = \sum_n F_n^s\left(x\right) e^{-i\omega_n t}, \quad F^f\left(x,t\right) = \sum_n F_n^f\left(x\right) e^{-i\omega_n t} \tag{4}$$

where periods of oscillation of each harmonic $T_n = 2\pi / \omega_n$ are multiple to the general period *T* of oscillation. Therefore, it is enough to consider the case of stationary oscillations, when the acting forces are periodic on time with an oscillation frequency ω:

$$F^s\left(x,t\right) = F^s\left(x\right) e^{-i\omega t}, \quad F^f\left(x,t\right) = F^f\left(x\right) e^{-i\omega t} \tag{5}$$

The solution of the equations (1) can be represented in the similar form:

$$u_s\left(x,t\right) = u_s\left(x\right) e^{-i\omega t}, \quad u_f\left(x\right) = u_f\left(x\right) e^{-i\omega t} \tag{6}$$

where the complex amplitudes of the displacements $u_s\left(x\right)$, $u_f\left(x\right)$ must be determined. If the solution has been known for any frequency щ, then we get similar decomposition for the displacements of the medium:

$$u^s\left(x,t\right) = \sum_n u_n^s\left(x\right) e^{-i\omega_n t}, \quad u^f\left(x,t\right) = \sum_n u_n^f\left(x\right) e^{-i\omega_n t} \tag{7}$$

which give us the solution of problem for forces (4).

We get equations for complex amplitudes by stationary oscillations, substituting (6) into the system (1):

$$\left(\lambda + \mu\right) \operatorname{grad}\operatorname{div} u^s + \mu \Delta u^s + Q \operatorname{grad}\operatorname{div} u^f + F^s\left(x\right) = -\rho_{11}\omega^2 u^s - \rho_{12}\omega^2 u^f$$

$$Q \operatorname{grad}\operatorname{div} u^s + R \operatorname{grad}\operatorname{div} u^f + F^f\left(x\right) = -\rho_{12}\omega^2 u^s - \rho_{22}\omega^2 u^f \tag{8}$$

To construct the solutions of this system we define Green tensor of it.

3 Green tensor of Biot equations by stationary oscillations

Let's construct fundamental solutions of the system (1) in the form:

$$\begin{pmatrix} F^s \\ F^f \end{pmatrix} = \begin{pmatrix} \delta_k^{[j]} e_k \\ \delta_{k+N}^{[j]} e_k \end{pmatrix} \delta\left(x\right) e^{-i\omega t}, \quad k = 1,\ldots,N,\ j = 1,\ldots,2N \tag{9}$$

where δ_k^j is Kronecker symbol. They describe the motion of Biot medium at the action of sources of stationary oscillations, concentrated in the point *x*=0. The upper index of this tensor fixes the current concentrated force and its direction. The lower index corresponds to component of the movement of the skeleton and fluid, respectively $k=1,...,N$ and $k=N+1,...,2N$.

Their complex amplitudes $U_m^j(x,\omega)\ (j,m=1,...,2N)$ satisfy to the next system of equation:

$$(\lambda+\mu)U^k_{j,ji}+\mu U^k_{i,jj}+\omega^2\rho_{11}U^k_i+QU^{k+N}_{j,ji}-\omega^2\rho_{12}U^{k+N}_i+\delta(x)\delta^k_j=0$$
$$QU^k_{j,ji}+\rho_{12}\omega^2U^k_{i},_{tt}+RU^{k+N}_{j,ji}+\rho_{22}\omega^2U^{k+N}_i+\delta(x)\delta^k_{j+N}=0 \qquad (10)$$
$$j=1,...,2N,\quad k=1,...,2N$$

Since fundamental solutions are not unique, we'll construct such, which tend to zero at infinity:

$$U_i^j(x,\omega)\to 0\quad \text{at}\quad \|x\|\to\infty \qquad (11)$$

and satisfy to radiation condition of type of Somerfield radiation conditions [10].
Matrix of such fundamental equations is names *Green tensor* of Eq. (8).

4 Fourie transform of fundamental solutions

To construct $U_m^j(x,\omega)$ we use the Fourier transformation, which for regular functions has the form:

$$F[\varphi(x)]=\bar{\varphi}(\xi)=\int_{R^N}\varphi(x)e^{i(\xi,x)}dx_1...dx_N$$

$$F^{-1}[\bar{\varphi}(\xi)]=\varphi(x)=\frac{1}{(2\pi)^N}\int_{R^N}\bar{\varphi}(\xi)e^{-i(\xi,x)}d\xi_1....d\xi_N$$

where $\xi=(\xi_1,...,\xi_N)$ are Fourier variables. Let's apply Fourier transformation to Eqs (10), and use property of Fourier transform of derivatives [10]:

$$\frac{\partial}{\partial x_j}\quad\leftrightarrow\quad -i\xi_j \qquad (12)$$

Then we get the system of 2N linear algebraic equations for the Fourier components of this tensor:

$$-(\lambda+\mu)\xi_j\xi_j\overline{U}^k_j-\mu\|\xi\|^2\overline{U}^k_j-Q\xi_j\xi_j\overline{U}^k_{j+N}+\rho_{11}\omega^2\overline{U}^k_j+\rho_{12}\omega^2\overline{U}^k_{j+N}+\delta^k_j=0,$$
$$-Q\xi_j\xi_j\overline{U}^k_j-R\xi_j\xi_l\overline{U}^k_{j+N}+\rho_{12}\omega^2\overline{U}^k_j+\rho_{22}\omega^2\overline{U}^k_{j+N}+\delta^k_{j+N}=0,$$
$$j=1,...,N,\quad k=N+1,...,2N \qquad (13)$$

By use gradient-divergence method this system has been solved by us. For this the next basic function were introduced

$$f_{0k}(\xi,\omega)=\frac{1}{c_k{}^2\|\xi\|^2-\omega^2},\quad f_{jk}(\xi,\omega)=\frac{f_{(j-1)k}(\xi,\omega)}{-i\omega},\quad j=1,2; \qquad (14)$$

and the next theorem was proved [11,12].

Theorem 1. *Components of Fourier transform of fundamental solutions have the form*

$$j=\overline{1,N},\quad k=\overline{1,N},$$

$$\overline{U}_j^k=\left(-i\xi_j\right)\left(-i\xi_k\right)\left[\beta_1 f_{21}+\beta_2 f_{22}+\beta_3 f_{23}\right]+\frac{1}{\alpha_2}\left(\rho_{12}\delta_{j+N}^k-\rho_{22}\delta_j^k\right)f_{03}$$

$$\overline{U}_{j+N}^k=\left(-i\xi_j\right)\left(-i\xi_k\right)\left[\gamma_1 f_{21}+\gamma_2 f_{22}+\gamma_3 f_{23}\right]-\frac{\mu}{\alpha_2}\delta_{j+N}^k\left\|\xi\right\|^2 f_{23}-\frac{1}{\alpha_2}\left(\rho_{11}\delta_{j+N}^k+\rho_{12}\delta_j^k\right)f_{03}$$

$$j=1,\dots,N\qquad k=N+1,\dots,2N$$

$$\overline{U}_j^k=\left(-i\xi_j\right)\left(-i\xi_{k-N}\right)\left[\eta_1 f_{21}+\eta_2 f_{22}+\eta_3 f_{23}\right]+\frac{1}{\alpha_2}\left(\rho_{12}\delta_{j+N}^k-\rho_{22}\delta_j^k\right)f_{03}$$

$$\overline{U}_{j+N}^k=\left(-i\xi_j\right)\left(-i\xi_{k-N}\right)\left[\varsigma_1 f_{21}+\varsigma_2 f_{22}+\varsigma_3 f_{23}\right]-\frac{\mu}{\alpha_2}\delta_{j+N}^k\left\|\xi\right\|^2 f_{23}-\frac{1}{\alpha_2}\left(\rho_{11}\delta_{j+N}^k+\rho_{12}\delta_j^k\right)f_{03}$$

where the next constants have been introduced:

$$D_1=\frac{1}{\alpha_2\upsilon_{12}},\quad \upsilon_{lm}=c_l^2-c_m^2,\quad q_1=Q\rho_{12}-(\lambda+\mu)\rho_{12},\quad q_2=\rho_{11}R-Q\rho_{12},$$

$$d_1=\left(\lambda+\mu\right)\rho_{22}-Q\rho_{12},\ d_2=Q\rho_{22}-R\rho_{12},\quad d_{3j}=\rho_{12}c_j^2-Q\ \ (j=1,2)$$

$$\beta_j=(-1)^{(j+1)}\frac{D_1c_j^2}{\alpha_2\upsilon_{3j}}\left(d_1b_{sj}+d_2d_{3j}\right),\quad \beta_3=-\frac{c_3^2}{\alpha_2\upsilon_{31}\upsilon_{32}}\left(d_1b_{s3}+d_2d_{33}\right);$$

$$\gamma_j=(-1)^{j+1}\frac{D_1c_j^2}{\alpha_2\upsilon_{3j}}\left(q_1b_{fj}+q_2d_{3j}\right),\quad \gamma_3=-\frac{D_1c_3^2\upsilon_{12}}{\alpha_2\upsilon_{31}\upsilon_{32}}\left(q_1b_{f3}+q_2d_{33}\right);$$

$$\eta_j=(-1)^{j+1}\frac{D_1c_j^2}{\alpha_2\upsilon_{3j}}\left(d_jd_{3j}+d_2b_{js}\right),\quad \eta_3=-\frac{c_3^2\upsilon_{12}}{\alpha_2\upsilon_{31}\upsilon_{32}}\left(d_1d_{33}+d_2b_{3s}\right);$$

$$\varsigma_j=(-1)^{j+1}\frac{D_1c_j^2}{\alpha_2\upsilon_{3j}}\left(q_1d_{3j}+q_2b_{(4-j)s}\right),\quad \varsigma_3=-\frac{c_3^2\upsilon_{12}}{\alpha_2\upsilon_{31}\upsilon_{32}}\left(q_1d_{33}+q_2b_{3s}\right)\,b_{fj}=\rho_{22}\upsilon_{fj},b_{sj}=\rho_{11}\upsilon_{js}\,.$$

This form is very convenient for constructing originals of Green tensor.

5 Green tensor construction. Radiation conditions

At first let's construct the originals of basic function:

$$\Phi_{0m}(x,\omega)=F^{-1}\left[f_{0m}(\xi,\omega)\right],$$

which, in accordance to its definition (14), satisfy to the equation

$$\left(c_m^2\left\|\xi\right\|^2-\omega^2\right)f_{0m}=1\,. \tag{15}$$

Using the property (12) for derivatives from here we get Helmholtz equation for fundamental solution (accurate within a factor c_m^{-2}) :

$$\left(\Delta + k_m^2\right)\Phi_{0m} + c_m^{-2}\delta\left(x\right) = 0, \quad k_m = \frac{\omega}{c_m} \tag{16}$$

Fundamental solutions of Helmholtz equation which satisfy to Sommerfeld conditions of radiation: at $r \to \infty$

$$\Phi'_{0m}(r) - ik_m\Phi_{0m}(r) = O(r^{-1}), \qquad N = 3,$$
$$\Phi'_{0m}(r) - ik_m\Phi_{0m}(r) = O(r^{-1/2}), \quad N = 2.$$

are well known [10]. They are unique. Using them we obtain:

for $N = 3$ $$\Phi_{0m} = \frac{1}{4\pi rc^2}e^{ik_m r}, \quad k_m = \frac{\omega}{c_m};$$

for $N = 2$ $$\Phi_{0m} = \frac{i}{4c^2}H_0^{(1)}\left(k_m r\right),$$

where $H_j^{(1)}\left(k_m r\right)$ is cylindrical Hankel function of the first kind;

for $N = 1$ $$\Phi_{0m} = \frac{\sin k_m\left|x\right|}{2k_m c_m^2}.$$

These functions (subject to factor $e^{-i\omega t}$)) describe harmonic waves which move from x=**0** to infinity and decay at infinity. Last property is true only for N=2,3. In the case N=1 all fundamental solutions of Eq. (16) :

$$\left(\frac{d^2}{dx^2} + k_m^2\right)\Phi_{0m} + c_m^{-2}\delta\left(x\right) = 0,$$

don’t decay at infinity.

From theorem 1 the next theorem follows.

Theorem 2. The components of Green tensor of Biot’s equations at stationary oscillations with frequency щ, satisfying the conditions of radiation, have the form:

$$for \quad j = \overline{1,N}, \quad k = \overline{1,N},$$
$$U_j^k = -\omega^{-2}\sum_{m=1}^{3}\beta_m\frac{\partial^2\Phi_{0m}}{\partial x_j\partial x_k} + \frac{1}{\alpha_2}\left(\rho_{12}\delta_{j+N}^k - \rho_{22}\delta_j^k\right)\Phi_{03},$$

$$U_{j+N}^k = -\omega^{-2}\sum_{m=1}^{3}\gamma_m\frac{\partial^2\Phi_{0m}}{\partial x_j\partial x_k} + \frac{\mu}{\alpha_2\omega^2}(c_3^{-2}\delta\left(x\right) + k_3^2\Phi_{0m})\delta_{j+N}^k - \frac{1}{\alpha_2}\left(\rho_{11}\delta_{j+N}^k + \rho_{12}\delta_j^k\right)\Phi_{03};$$
$$for \quad j = 1,\dots,N \quad k = N+1,\dots,2N$$

$$U_j^k = -\omega^{-2}\sum_{m=1}^{3}\eta_m\frac{\partial^2\Phi_{0m}}{\partial x_j\partial x_k} + \frac{1}{\alpha_2}\left(\rho_{12}\delta_{j+N}^k - \rho_{22}\delta_j^k\right)\Phi_{03}$$

$$U_{j+N}^k = -\omega^{-2}\sum_{m=1}^{3}\varsigma_m\frac{\partial^2\Phi_{0m}}{\partial x_j\partial x_k} + \frac{\mu}{\alpha_2\omega^2}(c_3^{-2}\delta\left(x\right) + k_3^2\Phi_{0m})\delta_{j+N}^k - \frac{1}{\alpha_2}\left(\rho_{11}\delta_{j+N}^k + \rho_{12}\delta_j^k\right)\Phi_{03}$$

where

$$\frac{d^2\Phi_{0m}}{dx^2} = \frac{1}{2c_m^2 k_m}\left(k_m{}^2\left(\sin k_m|x|\right) - 2k_m\delta(x)\right) \quad for \;\; N=1,$$

$$\frac{\partial^2\Phi_{0m}}{\partial x_j \partial x_k} = -\frac{i}{4c_m^2}\left(0.5k_m^2\left(H_0(k_m r) - H_2(k_m r)\right)r,_j r,_k + kH_1^1(k_m r)r,_{jk}\right) \quad for \;\; N=2,$$

$$\frac{\partial\Phi_{0m}}{\partial x_j \partial x_k} = \frac{1}{4\pi r c_m^2}e^{ikr}\left\{r,_j r,_k\left(\left(ik_m - \frac{1}{r}\right)^2 + \frac{1}{r^2}\right) + r,_{jk}\left(ik_m - \frac{1}{r}\right)\right\} \quad for \;\; N=3;$$

$$k_m = \frac{\omega}{c_m},\; r = \|x\|, \quad r,_j = \frac{x_j}{r}, \quad r,_{ij} = \frac{1}{r}\left(\delta_{ij} - \frac{x_i x_j}{r^2}\right).$$

Proof. By use originals of basic functions , property (12) of derivatives, we can obtain from formulas for $\overline{U}_j^k$ in theorem 1 the originals of all addends, beside that which contain factor $\|\xi\|^2$. But using (16) we have

$$-\Delta\Phi = c_m^{-2}\delta(x) + k_m^2\Phi_{0m} \quad \leftrightarrow \quad \|\xi\|^2 f_{0m} = c_m^{-2} + k_m^2 f_{0m}$$

Then formulas of Theorem 2 follow from formulas of Theorem 1.

Conclusion. Under the action of arbitrary mass forces with frequency ω in the Biot's medium, the solution for complex amplitudes has the form of a tensor-functional convolution:

$$u_j(x,t) = U_j^k(x,\omega) * F_k(x)e^{-i\omega t},\; j,k = \overline{1,2N} \tag{17}$$

Note that mass forces may be different from the space of generalized vector-function, singular and regular. Since Green tensor is singular, contains delta-functions, this convolution are calculated on the rule of convolution in generalized function space. If a support of acting forces are bounded (contained in a ball of finite radius), then all convolutions exist. If supports are not bounded, then the existence condition (17) require some limitations on behavior of forces at infinity which depend on the type of mass forces.

The obtained solutions allow us to study the dynamics of porous water and gas-saturated media at the action of periodic sources of disturbances of a sufficiently arbitrary form. In particular, under the action of certain forces on surfaces, for example cracks, in porous media that can be simulated by simple and double layers on the crack surface.

There is another interesting feature of the Green tensor of the Biot equations, which contains, as one of the terms, the delta function what complicates the application of this tensor for solving boundary value problems based on the boundary element method or boundary untegral equations method [13,14]. Here, when constructing the model, the viscosity of the liquid is not taken into account, which, apparently, leads to the presence of such terms, and requires improvement of this model taking into account the viscosity.

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СТАЦИОНАР ТЕРБЕЛІСТЕР КЕЗІНДЕГІ ЕКІ КОМПОНЕНТТІ М. БИО ОРТАСЫНЫҢ ҚОЗҒАЛЫС ТЕҢДЕУІНІҢ ГРИН ТЕНЗОРЫ

Аннотация. Қатты серпімді және идеалды сұйықтан тұратын екі компонентті Био ортаны қарастырамыз. Мұндай ортаның қозғалысы қатты және сұйық компоненттердің орын ауыстыруы үшін гиперболалық типтегі екінші ретті дифференциалдық теңдеулердің біріккен жүйесімен сипатталады. Әртүрлі типтегі периодты күштер тудыратын био ортадағы толқындардың таралу процестері зерттелген.

Бұл жүйенің стационарлы шешімдері N = 1,2,3 өлшемдер кеңістігіндегі гармоникалық тербелістерді сипаттайды. Бұл жағдайда күрделі тербеліс амплитудасы үшін теңдеулер жүйесі эллиптикалық түрде болады. Жалпыланған функциялардың және оның қасиеттерінің Фурье түрлендіруі негізінде ортаға тербеліс теңдеулерінің іргелі шешімі – сингулярлы дельта функциясымен сипатталған шығу көзіне шоғырланған күш көздерінің әсерінен тұрақты жиіліктегі уақыттық-гармоникалық толқындардың таралуын сипаттайтын Био орта – Грин тензоры тұрғызылды.

Тұрақты және сингулярлы жалпыланған функциялармен сипатталатын периодтық бұзулардың әртүрлі көздерінің әсері кезінде, осы теңдеулердің жалпыланған шешімдері тұрғызылды. Алынған нәтижелерді газ және сұйық қаныққан кеуекті ортадағы толқындық процестерді зерттеу үшін қолдануға болады.

Түйін сөздер: Био ортасы, қатты және сұйық компоненттер, іргелі шешім, жалпыланған тура және кері Фурье түрлендірулері, регуляризация.

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ТЕНЗОР ГРИНА УРАВНЕНИЙ ДВИЖЕНИЯ ДВУХКОМПОНЕНТНОЙ СРЕДЫ М. БИО ПРИ СТАЦИОНАРНЫХ КОЛЕБАНИЯХ

Аннотация. Рассматривается двухкомпонентная среда Био, содержащия твердую упругую компоненту и идеальную жидкую. Движение такой среды описывается связанной системой дифференциальных уравнений в частных производных второго порядка гиперболического типа для перемещений твердой и жидкой компоненты. Исследуются процессы распространения волн в среде Био, порождаемые действующими периодическими силами различного типа.

Строятся стационарные решения этой системы, описывающие гармонические колебания в пространствах размерности N = 1,2,3. В этом случае система уравнений для комплексных амплитуд колебаний является эллиптической. На основе преобразования Фурье обобщенных функций и его свойств построено фундаментальное решение уравнений колебаний среды Био - тензор Грина, который описывает процесс распространения гармонических по времени волн фиксированной частоты при действии сосредоточенных в начале координат силовых источников, описываемых сингулярной дельта-функцией.

На его основе построены обобщенные решения этих уравнений при действии разнообразных источников периодических возмущений, которые описываются как регулярными, так и сингулярными обобщенными функциями. Полученные решения можно использовать для исследования волновых процессов в газо- и жидконасыщенных пористых средах.

Ключевые слова: среда Био, твердая и жидкая компоненты, фундаментальное решение, обобщенное прямое и обратное преобразование Фурье, регуляризация.

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