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M-FUNCTION NUMBERS: CYCLES AND OTHER EXPLORATIONS. PART 1

Abstract. This paper establishes the cyclic properties of the M-Function, which we define as a function, [M(n)], that takes a positive integer, adds to it the sum of its digits and the number produced by reversing its digits, and then divides the entire sum by three. Our definition of the M-Function is influenced by D. R. Kaprekar's work on a remarkable class of positive integers, called self- numbers, and his procedure, [K(n)], of adding to any positive integer the sum of its digits [1]. We analyze the distribution of numbers that make the defined M-Function behave like a cyclic function, and observe that many such "cycles" form arithmetic sequences. We examine the distribution of numbers that produce integer ratios between the outputs of Kaprekar's and the M-Function functions, [K(n)/M(n)]. We also prove that the set of numbers with equal outputs to both Kaprekar's and M-Function functions, [K(n)=M(n)], is infinite.

Key words: M-Function, D.R.Kaprekar, self-numbers.

1. Introduction. Indian mathematician D.R. Kaprekar is especially known for the discovery of the "Kaprekar Constant." His another prominent work, described by the famous American science writer Martin Gardner in his book "Time travel and other mathematical bewilderments," [1] is called self-numbers, discovered by Kaprekar in 1949. Choose any integer n and add to it the sum of its digits S (n). The resulting number K (n) = n + S (n) is called a *digitaddition*, and the initial number n is called its *generator*. A digitaddition can have several generators. A self-number is a positive integer that does not have a generator. In the "Columbian Numbers" article published by mathematicians Recaman and Bange in "The American Mathematical Monthly" [2] magazine in 1974, it was proven that there are infinitely many self-numbers.

The discoveries that D. R. Kaprekar made engaged not only serious mathematics scholars and researchers, but also astonished many high school students - Kaprekar's core discoveries do not require knowledge of concepts outside of a normal high-school curriculum to understand. Followed by Kaprekar's discoveries, many scientific articles, scientific projects in mathematics, and software products globally examined various new properties of the "Kaprekar Constant" and the sets of self-numbers and digitadditions. There were many math Olympiad problems based on Kaprekar's remarkable class of positive integers and their properties. Thus, Kaprekar's discoveries inspired and drew the attention of mathematicians of many levels.

This paper outlines the investigation of a function similar to Kaprekar's function, a function defined as the M-Function. M-Function takes a positive integer, adds to it the sum of its digits and the number produced by reversing its digits, and then divides the entire sum by three:

For positive integer n → M (n) = (n + S (n) + r (n)) / 3, where r (n) is a number with the digits of n in reverse order. For example, if n = 358, M (358) = (358 + (3+5+8) + 853) / 3 = 409.

In the case of Kaprekar's digitaddition procedure, the inequality n < K (n) holds true for all positive integers n. However, in the case of the M-Function, all three inequality cases are possible: n < M (n), n = M (n), n > M (n). As it turns out, there could be many interesting properties and consistencies that fascinatingly flow out of the defined M-Function. The procedure for obtaining new positive integers n →

M (n) is a simple mathematical operation, yet it can produce fascinating properties. The research also involved a study of relationships between Kaprekar's function and the M-Function, and raises general questions concerning a particular number set (distribution problems) or the mutual relationship of several number sets (such as the coincidence of elements of two sets, multiple ratios of elements of integer sets).

The research outlined in this paper focuses on discovering fundamental mathematical dependencies, properties and theories within Number Theory field. Hence, the openings of the research are valuable to the ever-growing field of Number Theory. Perhaps, its results would have no immediate practical applications, however, as the scientific development progresses forward, there might be a number of applications of M-Functions beyond the fundamental Number Theory in fields like computer science and computational biology.

2. Distribution of *m*-cycles and their properties. Let's give some definitions. Let N be the set of positive integers. For all $n \in N$, let $S(n)$ be the sum of digits of n, $\bar{n}$ be the number produced by reversing n's digits.

Let the number $d(n)$ be the "order" of the number, the quantity of digits of n. Then, for any n such that $n \epsilon N$, the condition $10^{d(n)-1} \leq n < 10^{d(n)}$ is true.

The definition of the M-Function is $M(n) = \frac{1}{3}(n + S(n) + \bar{n})$ and $K(n) = n + s(n)$.

If $n = M(n)$ then n is called a stationary number.

Let l be the least positive integer such that $M^l(n) = n$ for some $n \epsilon N$. Then, the number l is the length of the cycle:

$$n \to M(n) \to M^2(n) \to \ldots \to M^{l-1}(n) \to M^l(n) = n.$$

Table 1 Table of the distribution of *m* – cycles for numbers 1 to 10^{10}

	The length of the cycle (l)																		
$d(n)$	1	2	3	4	5	6	9	10	12	13	15	16	18	19	21	23	24	∑ (total)	%
1	9																	9	100
2	4		1															5	5.556
3	4		3		1													8	$8.89 * 10^{-1}$
4	12	3	2	2				1										20	$2.22 * 10^{-1}$
5	8	1	12			4	2											27	$3 * 10^{-2}$
6	8		9			6							1					24	$2.67 * 10^{-3}$
7	12	10	16			3		17	2			1			1	1	1	64	$7.11 * 10^{-4}$
8	8		12			2	1		1				1		2			27	$3.0 * 10^{-5}$
9	8	7	21			7	4		1		2		3		1		1	55	$6.11 * 10^{-6}$
10	12	2	17	2	1	3	5	3	3	1	2		3	2	1		1	58	$6.44 * 10^{-7}$
∑	85	23	93	4	2	25	12	21	7	1	4	1	8	2	5	1	3	297	

Using a C++ program, we can compute all $m-$ cycles for numbers from 1 to 10^{10}, and make a list of them. Based on this data, we can compose a table of distribution of m-cycles in the set N (Table 1). Their total quantity is 297. Also, we can observe from the table that the length l of m-cycles can be of any value from 1 to 24, except the numbers 7, 8, 11, 14, 17, 20 and 22. m-cycles with length $l = 10$ occur just among the numbers when $d(n) = \{4, 7, 10\}$. As seen from the table, the proportion of m-cycles out of all positive integers in the given order decreases rapidly as d(n) increases. In the table below, the percentage (%) represents the proportion of m-cycles out of all positive integers in the given order $d(n)$. So, among ten-digit numbers (we are considering $9*10^9$ integers), there are 58 m-cycles, which means that the percentage of m-cycles out of all 10-digit positive integers is $6.44*10^{-7}$%.

Conjecture 1. m-cycles with length greater than $l = 24$ don't exist. m-cycles with length $l = 10$ occur only in the numbers that have order $d(n) = 3k + 1$, where $k \epsilon N$.

Considering the table, we might be curious in looking at m – cycles with length $l = 10$: they might possess interesting properties.

Among the four-digit numbers, there is only one m –cycle with length $l = 10$:

$$1297 \rightarrow 3079 \rightarrow 4267 \rightarrow 3970 \rightarrow 1594 \rightarrow 2188 \rightarrow 3673 \rightarrow 2485 \rightarrow 2782 \rightarrow 1891 \rightarrow 1297$$

These numbers form an arithmetic progression, where $a_1 = 1297$ - is the initial term, $d = 297$- is the common difference.

Hence:

$1297 = a_1$, $3079 = a_1 + 6d = a_7$, $4267 = a_1 + 10d = a_{11}$,
$3970 = a_1 + 9d = a_{10}$, $1594 = a_1 + d = a_2$, $2188 = a_1 + 3d = a_4$,
$3673 = a_1 + 8d = a_9$, $2485 = a_1 + 4d = a_5$, $2782 = a_1 + 5d = a_6$,
$1891 = a_1 + 2d = a_3$.

If we write in terms of a(n) where a1 = 1297 and the nth term belongs to the arithmetic sequence, then we can write the sequence of the cycle as follows:

$$a_1 \rightarrow a_7 \rightarrow a_{11} \rightarrow a_{10} \rightarrow a_2 \rightarrow a_4 \rightarrow a_9 \rightarrow a_5 \rightarrow a_6 \rightarrow a_3 \rightarrow a_1$$

Note that the term a_8= 3376 is not in the sequence, which we can define as a stationary number ($a_8 = M(a_8)$).

We can draw a circle and label points 1-11 on the circle (in order), each equally spaced. We can construct a hendecagon, an eleven-sided polygon, by connecting points in the order of the cycle (that is, connect a1 to a7, then a7 to a11, etc.), excluding a8. We can draw a "mirror" AB with a length of the circle's diameter. We can observe an interesting picture: the mirror is symmetric with respect to the diameter AB, where the endpoint A is 8! (Figure 1).

We can draw another circle, but this time with the order of the cycle in terms of an arithmetic progression (points 1, 7, 11, … , 6, 3), also equally spaced. We can construct two pentagons, connecting every other point for one pentagon, and connecting the remaining points on the other. (Figure 2).

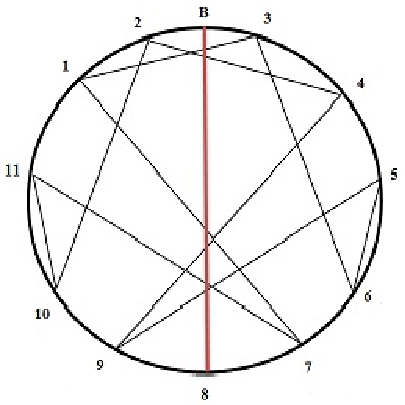

Figure 1

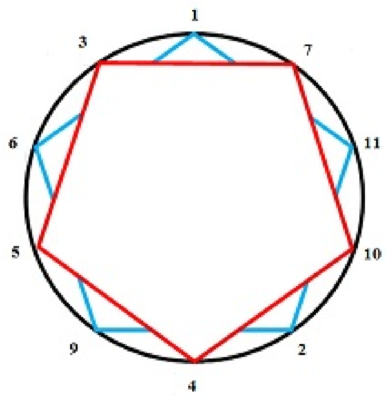

Figure 2

We can sum the vertices of each pentagon, and notice that the sums are equal:

$$1 + 11 + 2 + 9 + 6 = 7 + 10 + 4 + 5 + 3 = 29$$

In terms of the arithmetic sequence, this found property means the following:

$$a_1 + a_{11} + a_2 + a_9 + a_6 = a_7 + a_{10} + a_4 + \ a_5 + a_3.$$

If we designate numbers by the sequence of the cycle, it will have the following order:

$$u_1 \to u_2 \to u_3 \to u_4 \to u_5 \to u_6 \to u_7 \to u_8 \to u_9 \to u_{10} \to u_1$$

and the given property will be written as:

$$u_1 + u_3 + u_5 + u_7 + u_9 = u_2 + u_4 + u_6 + u_8 + u_{10}.$$

Now, let's look at m −cycles with $l = 10$ for seven-digit numbers: there are 17 of them. There are only 8 $m -$ cycles where the numbers form an arithmetic progression in some sequence.

1) There are 4 m −cycles with a common difference of $d = 32670$ that are related to each other with a difference of 1000002

$a_1 = 1102982$, a_8 , a_9, a_6, a_4, a_{10}, a_3, a_2, a_5 , a_7.

+1000002

$b_1 = 2102984$, b_8, b_9, b_6, b_4, b_{10}, b_3, b_2 , b_5 , b_7.

+1000002

$c_1 = 3102986$, c_8, c_9, c_6, c_4, c_{10}, c_3, c_2, c_5, c_7.

+1000002

$e_1 = 4102988$, e_8, e_9, e_6, e_4, e_{10}, e_3, e_2, e_5, e_7.

Where $b_i = a_i + 1000002$, $c_i = a_i + 2000004$, $e_i = a_i + 3000006$, $i = \overline{1,10}$. Note that 1000002, 2000004, 3000006 are stationary numbers!

2) There are also 4 m −cycles with a common difference of $d = 27270$:

$f_1 = 1127282$, f_8, f_9, f_6, f_4, f_{10}, f_3, f_2, f_5, f_7.

+1000002

$g_1 = 2127284$, g_8, g_9, g_6, g_4, g_{10}, g_3, g_2, g_5, g_7.

+1000002

$h_1 = 3127286$, h_8, h_9, h_6, h_4, h_{10}, h_3, h_2, h_5, h_7.

+1000002

$j_1 = 4127288$, j_8, j_9, j_6, j_4, j_{10}, h_3 , j_2, j_5 , j_7.

Where $g_i = f_i + 1000002$, $h_i = f_i + 2000004$, $j_i = f_i + 3000006$, $i = \overline{1,10}$.

All 8 m-cycles of length 10 form a similar arithmetic sequence:

$$a_1 \to a_8 \to a_9 \to a_6 \to a_4 \to a_{10} \to a_3 \to a_2 \to a_5 \to a_7 \to a_1 \ .$$

Placing points 1-10 around the circle in order and connecting the points in the order of the arithmetic sequence, we get another symmetry with respect to diameter AB (Figure 3). The endpoints of the diameter, B and A, are centered in the midpoint of an arc between 5 and 6, and between 10 and 1, respectively. We can draw another circle, but this time putting the points in the order of the arithmetic sequence terms. We get the following circle with 10 "slices" (Figure 4).

We see that the sum of diametrically opposite 2 numbers is always equal to

$$1 + 10 = 8 + 3 = 9 + 2 = 6 + 5 = 4 + 7 = 11.$$

If we consider the sequence in terms of a cycle rather than in terms of an arithmetic progression, then we can describe it as follows:

$$u_1 \to u_2 \to u_3 \to u_4 \to u_5 \to u_6 \to u_7 \to u_8 \to u_9 \to u_{10} \ .$$

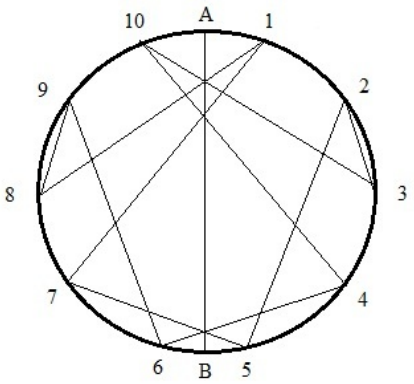

Figure 3

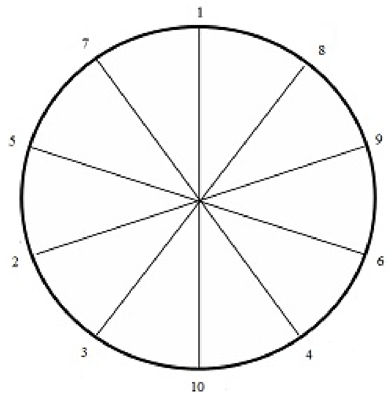

Figure 4

The observed pattern means that for all 8 m-cycles, the following equation holds true:

$$u_1 + u_6 = u_2 + u_7 = u_3 + u_8 = u_4 + u_9 = u_5 + u_{10}.$$

We verified the aforementioned statement algebraically.

3. Solutions to the equation K(n) = M(n). Let's investigate the following question: for what values of n, where $n \in N$, will the output of Kaprekar's function $K(n)$ be equal to the output of the M-Function $M(n)$? Let $K(n) = M(n)$ for some $n \in N$. Then, we can simplify the equation:

$$n + S(n) = \frac{1}{3}(n + S(n) + \bar{n}),$$

$$3n + 3S(n) = n + S(n) + \bar{n}\,.$$

Here we get the equation

$$2n = -2 * S(n) + \bar{n}\,. \qquad \text{(Equation 1)}$$

By solving Equation 1 using a C++ program, we obtain the following results for numbers up to 10^{10}:

27	24894	450009	24600294	450000009
459	45009	2460294	45000009	2460000294
4509	246294	4500009	246000294	4500000009

As evident in the list of solutions, starting with 6-digit numbers up until 10 digits, there are only 2 solutions for each order $d(n)$, having the following types:

$$\alpha_d = 246\underbrace{0\ldots0}_{d-6}294, \qquad \beta_d = 45\underbrace{0\ldots0}_{d-3}9\,.$$

Proposition 1. For all integer values $n \in N$ of $d(n)$ where $d(n) > 10$, numbers α_d and β_d satisfy Equation 1. Hence, there is an infinite number of solutions to the equation K (n) = M (n).

Proof. 1). We can express a solution from the type α_d in general form as follows:

$$\alpha_d = 246\underbrace{0\ldots0}_{d-6}294.$$

We can multiply it by 2, and get the following:

$$2\alpha_d = 492\underbrace{0\ldots0}_{d-6}588.$$

Then, we know that $2 * S(\alpha_d) = 2(2 + 4 + 6 + 2 + 9 + 4) = 54,$

and $\overline{\alpha_d} = 492\underbrace{0\ldots0}_{d-6}642.$

Recall the simplified equation above, and substitute n with α_d :

$$2\alpha_d = -2 * S(\alpha_d) + \overline{\alpha_d}.$$

Algebraically, we show that the equality holds true and hence, we prove Proposition 1.

Consequently, $K(\alpha_d) = M(\alpha_d)$ when $d(n) > 10$, which means we are convinced that there are infinitely many solutions to the equation K(n) = M(n).

2) We can express a solution from the type β_d in general form as follows:

$$\beta_d = 45\underbrace{0\ldots0}_{d-3}9.$$

We can multiply it by 2, and get the following:

$$2\beta_d = 9\underbrace{0\ldots0}_{d-3}18.$$

Then, we know that

$$2 * S(\beta_d) = 2(4+5+9) = 36,$$

and

$$\overline{\beta_d} = 9\underbrace{0\ldots0}_{d-3}54.$$

Again, recall the simplified equation, and substitute n with β_d :

$$2\,\beta_d = -2 * S(\beta_d) + \overline{\beta_d}.$$

Algebraically, we show that the equality holds true and hence, we prove Proposition 1.

Consequently, $K(\beta_d) = M(\beta_d)$ for $d(n) > 10$, which is another evidence that there are infinitely many solutions to the equation K(n) = M(n).

Conjecture 2. Equation 1 does not have any other solutions except α_d and β_d when $d(n) > 10$.

4. Arithmetic progression with $\boldsymbol{n}$, $\boldsymbol{K(n)}$, $\boldsymbol{M(n)}$ terms. We are intrigued to know for what $n \in N$ do numbers $n,\ K(n),\ M(n)$ form an arithmetic progression in a certain order. Since $n < K(n)$ for all $n \in N$, there are only 3 distinct orders possible to form an increasing arithmetic progression.

4.1. Let $n,\ K(n),\ M(n)$ be the order of an arithmetic progression. Then, by definition

$$n + M(n) = 2K(n),$$

which we can simplify to

$$n + \frac{1}{3}(n + S(n) + \bar{n}) = 2\big(n + S(n)\big).$$

Algebraically rearranging the equation, we can express it as follows:

$$2n = -5 * S(n) + \bar{n}\ . \qquad \text{(Equation 2)}$$

Using a C++ program, we found the following solutions to Equation 2 for numbers up to 10^{10}:

18	15003	186273	15000003	186000273
153	18873	1500003	18600273	1500000003
1503	150003	1860273	150000003	1860000273

Proposition 2. All numbers in the form $a_d = 15\underbrace{0\ldots0}_{d-3}3$ and $b_d = 186\underbrace{0\ldots0}_{d-6}273$, where $d(n) > 10$, satisfy Equation 2.

Hence, the numbers $a_d, K(a_d),\ M(a_d)$ and $b_d, K(b_d),\ M(b_d)$ form arithmetic progressions.

The proof is similar to the proof of Proposition 1: we recall the simplest form of the equation, plug in the values for a_d and b_d, and show the proof algebraically.

4.2. Let $n,\ M(n),\ K(n)$ be the order of an arithmetic progression. Then, by definition

$$n + K(n) = 2 * M(n),$$

which we can simplify to

$$n + \left(n + S(n)\right) = \frac{2}{3}(n + S(n) + \bar{n}).$$

Algebraically rearranging the equation, we can express it as follows:

$$4n = -S(n) + 2\bar{n}\,. \qquad \text{(Equation 3)}$$

Using a C++ program, we found numbers that satisfy Equation 3 for numbers up to 10^{10}:

387	36027	3600027	360000027
3627	360027	36000027	3600000027

Proposition 3. All numbers in the form $c_d = 36\underbrace{0\ldots0}_{d-4}27$, where $d(n) > 10$, satisfy Equation 3. Hence, the numbers $c_d, M(c_d),\ K\,(c_d)$ form an arithmetic progression.

Again**, the proof** is similar to the proof of Proposition 1.

4.3. Let *M(n), n, K(n)* be the order of an arithmetic progression. Then, by definition

$$M(n) + K(n) = 2n.$$

which we can simplify to

$$\frac{1}{3}(n + S(n) + \bar{n}) + \left(n + S(n)\right) = 2n.$$

Algebraically rearranging the equation, we can express it as follows:

$$2n = 4S(n) + \bar{n}\,. \qquad \text{(Equation 4)}$$

Using a C++ program, we found solutions for Equation 4 for numbers up to 10^{10}:

45	2124	42048	420048	2670435	21000024	47700459	420000048	2670000435
234	4248	210024	477459	4200048	26700435	210000024	477000459	4200000048
468	21024	267435	2100024	4770459	42000048	267000435	2100000024	4770000459

Proposition 4. When $d(n) > 10$, all numbers of the following types

$$\mathrm{e}_d = 21\underbrace{0\ldots0}_{d-4}24\ , \qquad \mathrm{f}_d = 267\underbrace{0\ldots0}_{d-6}435\ ,$$

$$\mathrm{g}_d = 42\underbrace{0\ldots0}_{d-4}48\ , \qquad \mathrm{h}_d = 477\underbrace{0\ldots0}_{d-6}459$$

are solutions to Equation 4.

Consequently, the following numbers:

$\mathrm{M}(\mathrm{e}_d),\ e_d,\ \mathrm{K}\,(\mathrm{e}_d)$; $\mathrm{M}(\mathrm{f}_d),\ f_d,\ \mathrm{K}\,(\mathrm{f}_d)$; $\mathrm{M}(\mathrm{g}_d),\ g_d,\ \mathrm{K}\,(\mathrm{g}_d)$;

$\mathrm{M}(\mathrm{h}_d),\ h_d,\ \mathrm{K}\,(\mathrm{h}_d)$ - all form an arithmetic progression when $d(n) > 10$.

The proof is similar to the proof of Proposition 1.

According to our results for equations 2, 3, and 4, we can devise the following conjecture:

Conjecture 3. When $d(n) > 10$, equations 2, 3, and 4 don't have any other solutions except solutions specified in propositions 2, 3, 4.

С. Мақышов

М-ФУНКЦИЯ САНДАРЫ: ЦИКЛДАР ЖӘНЕ БАСҚА ЗЕРТТЕУЛЕР

Аннотация. Индиялық математик Д.Р. Капрекар ашқан "Капрекар Константасы" - 6174 санымен аса танымал.

Капрекардың тағы бір ашқан жаңалығы өзіндік туындаған сандар класы белгілі америкалық ғылым насихаттаушысы Мартин Гарднердің «Уақыт бойынша саяхат» [1] атты кітабында баяндалған. Кез келген натурал n санын аламыз және ол санға цифрларының қосындысы $S(n)$−ді қосамыз. Шыққан сан $K(n)=n+S(n)$ туындаған сан, ал алғашқы сап n – оның генераторы деп аталады. Мысалы, егер 53 санын алсақ, онда туындаған сан 53+3+5=61 саны болады.

Туындаған санның генераторларының саны бірден артық болуы мүмкін. Екі генераторы бар ең кіші сан 101, ал оның генераторлары 91 және 100 сандары. Өзіндік туындаған сандар – генераторлары жоқ сандар. «The American Mathematical Monthly» [2] журналында жарияланған мақалада өзіндік туындаған сандардың шексіз көп екендігі және өзіндік туындаған сандар туындаған сандарға қарағанда өте сирек кездесетіндігі дәлелденген.

Капрекар ашқан осы жаңалықтар көптеген математиктерді қызықтырды. Әртүрлі елдерде «Капрекар Константасының», өзіндік туындаған және туындаған сандары жиындарының жаңа қасиеттерін жан-жақты зерттеген көптеген мақалалар, математикалық ғылыми жобалар мен программалық өнімдер жарық көрді.

Мен Капрекарға сүйене отырып, натурал сандарды алудың жаңа әдісін таптым: $n \to M(n)=13(n+S(n)+\overline{n})$, мұндағы $\overline{n}$ – сол цифрлармен, бірақ кері бағытта жазылған сан. $M(n)$ саны әрқашан бүтін сан болады, себебі $n, S(n), \overline{n}$ сандарының 3-ке бөлгендегі қалдықтары әрқашан тең болады. Егер Капрекар жағдайында $n<K(n)$ теңсіздігі кез келген натурал n сандарында орындалса, менің құрған функциямда әртүрлі қатынастар болады, яғни барлық 3 жағдай да орын алады : $n<M(n)$, $n=M(n)$ и $n>M(n)$.

Мен тапқан жаңа натурал сан алу функциясы $n \to M(n)$ әрі қарапайым, табиғи және Капрекар $n \to K(n)$ функциясының аналогы.

Мақаланың 1-бөлімінде m-циклдарының таралуы және олардың қасиеттері зерттеледі (егер $Ml(n)=n$ теңдігі орындалатындай ең кіші натурал сан l болса, онда $n \to M(n) \to M2(n) \to \ldots \to Ml-1(n) \to Ml(n) \to n$ сандары m-циклді құрайды. Ал $K(n)$ жағдайында циклдар туындамайды, себебі . $n<K(n)<K2(n)\ldots.$). Сонымен қатар $K(n)=M(n)$ функцияларының теңдігі сұрағы және n, $K(n)$ және $M(n)$ сандарының кандай да бір ретпен арифметикалық прогрессия құрайтын сұрақтары зерттелген.

Мақаланың 2-бөлімінде $n, K(n)$ және $M(n)$ сандарының арасындағы еселік қатынастар қарастырылған. Яғни, қандай натурал t сандарында, $t \geq 2$, $K(n)=tM(n)$, $tK(n)=M(n)$, $n=t\ M(n)$, $n \cdot t=M(n)$ теңдіктері орындалатындығы зерттелген (айта кетейік, n және $K(n)$ сандары арасында еселік қатынастар болуы мүмкін емес). Сонымен қатар m−туындаған сандар жиынының таралуы және қасиеттері зерттелген (m−туындаған сандар m−өзіндік туындаған сандарға қарағанда өте сирек кездеседі. Сондықтан m−туындаған сандар жиынын зерттеу m−өзіндік туындаған сандар жиынына қарағанда маңыздырақ). Осы бөлімде "көрші", яғни қатарлас тұрған m−туындаған сандар жиыны зерттелген.

Зерттеу барысында 1 мәселе және 9 гипотеза тұжырымдалған.

Түйін сөздер: М-функция, Д.Р. Капрекар, өзіндік туындаған сандар.

С. Макышов

ЧИСЛА М-ФУНКЦИИ: ЦИКЛЫ И ДРУГИЕ ИССЛЕДОВАНИЯ

Аннотация. Индийский математик Д.Р. Капрекар особенно известен своим открытием "Константу Капрекара"- числом 6174. Другое выдающееся открытие Капрекара, описанное известным американским популяризатором науки Мартином Гарднером в своей книге "Путешествие во времени"[1] – это класс самопорожденных чисел. Выберем любое натуральное число n и прибавим к нему сумму его цифр $S(n)$. Полученное число $K(n) = n + S(n)$ называется *порожденным*, а исходное число n – его *генератором*. Например, если выберем число 53, порожденное им число равно $53 + 3 + 5 = 61$.

Порожденное число может иметь более одного генератора. Наименьшее число с двумя генераторами равно 101, и его генераторами являются числа 91 и 100. Самопорожденное число – это число, у которого нет генератора. В статье журнала «The American Mathematical Monthly»[2] доказывалось, что существует бесконечно много самопорожденных чисел, но встречаются они гораздо реже, чем порожденные числа.

Эти открытия Капрекара заинтересовали многих математиков, и в разных странах мира появились много научных статьей, научных проектов по математике, программных продуктов, в которых исследовались различные новые свойства “Константы Капрекара” и множеств самопорожденных чисел и порожденных чисел.

Следуя Капрекару, я нашел новый способ получения натуральных чисел: $n \to M(n) = \frac{1}{3}(n + S(n) + \bar{n})$, где $\bar{n}$ – число, записанное теми же цифрами, но в обратном порядке. Число $M(n)$ будет всегда целым, так как числа $n, S(n), \bar{n}$ дают одинаковые остатки при делении на 3. Если в случае Капрекара неравенство $n < K(n)$ выполняется при всех натуральных n, то в моем случае положение разнообразнее, т.е. возможны все 3 случая: $n < M(n)$, $n = M(n)$ и $n > M(n)$.

Моя функция получения новых натуральных чисел $n \to M(n)$ простая, естественная, и она является аналогом функции Капрекара $K(n)$. В 1-й части данной статьи исследованы распределение m-циклов и их свойства. (Если l – наименьшее натуральное такое, что $M^l(n) = n$, то числа $n \to M(n) \to M^2(n) \to \ldots \to M^{l-1}(n) \to M^l(n) \to n$ образуют m-цикл. В случае функции $K(n)$ циклы невозможны, т.к. $n < K(n) < K^2(n)$ ....). Также изучен вопрос равенства чисел $K(n) = M(n)$ и вопрос образования арифметической прогрессии в некотором порядке числами n, $K(n)$ и $M(n)$.

Во 2-й части статьи изучены кратные отношения между числами n, $K(n)$ и $M(n)$. Т.е. исследованы вопросы: при каких t натуральном, $t \geq 2$, возможны равенства $K(n) = tM(n)$, $tK(n) = M(n)$, $n = t\,M(n)$, $n * t = M(n)$. (Отметим, что кратные отношения между числами n и $K(n)$ невозможны). Также исследованы распределение и свойства множества $m-$порожденных чисел ($m-$порожденных чисел встречаются гораздо реже, чем $m-$ самопорожденные, поэтому изучение множества $m-$порожденных намного важнее, чем изучение класса $m-$ самопорожденных чисел). В этой части исследовано множество “соседних”, т.е. последовательных $m-$порожденных чисел.

В процессе исследования сформулированы 1 проблема и 9 гипотез.

Ключевые слова: М-функция, Д.Р.Капрекар, самопорожденные числа.

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REFERENCES

[1] Gardner, M., “Time travel and other mathematical bewilderments,” New York, 1988.

[2] Recaman, B., and Bange, D. W., Columbian numbers, American Mathematical Monthly 81 (1974), 407-409.

[3] Weisstein, E. W., *Self Numbers*, in “MathWorld. A Wolfram Web Resource,” http://mathworld.wolfram.com/SelfNumber.html

[4] Su, Francis E., et al., "Kaprekar's Constant." Math Fun Facts. http://www.math.hmc.edu/funfacts

[5] Grozdev, S., Nenkov, V., “Kaprekar’s Constant,” Mathematics Plus, 22 (85), number 1, 2014, 65-68.

[6] Baizhanov B.S., Baizhanov S.S. Some questions on external definability. News of the Nationl Academy of sciences of the Republic of Kazakhstan, №6,2019, 146-150. https://doi.org/10.32014/2019.2518-1726.81