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MULTI-LINE SOLITON SOLUTIONS FOR THE TWO-DIMENSIONAL NONLINEAR HIROTA EQUATION

Abstract. At present, the question of studying multidimensional nonlinear integrable equations in the framework of the theory of solitons is very interesting to foreign and Kazakh scientists. Many physical phenomena that occur in nature can be described by nonlinearly integrated equations. Finding specific solutions to such equations plays an important role in studying the dynamics of phenomena occurring in various scientific and engineering fields, such as solid state physics, fluid mechanics, plasma physics and nonlinear optics. There are several methods for obtaining real and soliton, soliton-like solutions of such equations: the inverse scattering method, the Hirota's bilinear method, Darboux transformation methods, the tanh-coth and the sine-cosine methods. In our work, we studied the two-dimensional Hirota equation, which is a modified nonlinear Schrödinger equation. The nonlinear Hirota equation is one of the integrating equations and the Hirota system is used in the field of study of optical fiber systems, physics, telecommunications and other engineering fields to describe many nonlinear phenomena. To date, the first, second, and n-order Darboux transformations have been developed for the two-dimensional system of Hirota equations, and the soliton, rogue wave solutions have been determined by various methods.

In this article, we consider the two-dimensional nonlinear Hirota equations. Using the Lax pair and Darboux transformation we obtained the first and the second multi-line soliton solutions for this equation and provided graphical representation.

Key words. two-dimensional nonlinear Hirota equations, nonlinear integrable equations, Lax pair, Darboux transformation, multi-line soliton solutions.

Introduction. Solitons are important when considering nonlinear equations. Nonlinearly integrated systems have been hotly debated among scientists. This area has developed extensively, mainly in all areas covered by technology and science. There are several ways to obtain nonlinear soliton solutions of evolutionary equations. These include the nonlinear method of the Hirota equation, the Darboux transformation (DT), the Painleve analysis, and more [1-7].

In this paper, the two-dimensional nonlinear Hirota equation is a modified nonlinear Schrödinger equation considered. The Hirota system is used to describe many nonlinear phenomena. It is widely used in the field of study of optical fiber systems, in the mechanisms of physics, electrical communication and other engineering sciences [8, 9].

Consider the one-component two-dimensional nonlinear Hirota equation as follows [10]:

$$iq_t + \alpha q_{xy} + i\beta q_{xy} - \nu q + i(\omega q)_x = 0, \quad (1)$$

$$\nu_x + 2\alpha\delta(|q|^2)_y - 2i\beta(q_{xy}^* q - q^* q_{xy}) = 0, \quad (2)$$

$$\omega_x - 2\beta\delta(|q|^2)_y = 0, \quad (3)$$

where $q = q(x, y, t)$ is the complex functions and $v = v(x, y, t)$, $w = w(x, y, t)$, $(i = 1, 2)$ are the real functions. It is integrated by the method of the inverse scattering method, thus, for its there is a Lax representation. Its Lax representation is given in the form:

$$\psi_x = A \psi, \tag{4}$$

$$\psi_t = (2\alpha\lambda + 4\beta\lambda^2)\psi_y + B\psi. \tag{5}$$

here λ – spectral parameter, Ψ - independent function, particularly $\Psi(x, y, t; \lambda) = (\Psi_1, \Psi_2, \Psi_3)^T$. A and B matrices

$$\begin{aligned} A &= -i\lambda\sigma_3 + A_0, \\ B &= \lambda B_1 + B_0, \end{aligned} \tag{6}$$

here A_0, B_0 and σ_3 $[2 \times 2]$ dimensional matrices:

$$\begin{aligned} A_0 &= \begin{pmatrix} 0 & q \\ -r & 0 \end{pmatrix}, \\ B_0 &= -\frac{i}{2}v\delta_3 + \begin{pmatrix} 0 & i\alpha q_y - \beta q_{xy} - \omega q \\ i\alpha r_y + \beta r_{xy} + \omega r & 0 \end{pmatrix}, \\ B_1 &= i\omega\delta_3 + 2i\beta\delta_3 A_{0y}, \\ \delta_3 &= \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \end{aligned}$$

Using the Lax pair above, we can construct a Darboux transformation for a two-dimensional nonlinear Hirota equation. Compatibility condition of this system:

$$A_t - B_x + [A, B] - (2\alpha\lambda + 4\beta\lambda^2)A_y = 0. \tag{7}$$

According to the above calculations, we can obtain a two-dimensional nonlinear Hirota system. The spectral parameter λ is obtained as follows:

$$\lambda_t = (2\alpha\lambda + 4\beta\lambda^2)\lambda_y.$$

In this article we will limit ourselves to the situation $\delta = +1$.

In our previous work, we have constructed the first-order Darboux transformation for the two-dimensional nonlinear Hirota equation [1].

$$\begin{aligned} q^{[1]} &= q - 2iS_{12}, \\ v^{[1]} &= v + 4i\alpha s_{11y} + 4\beta(s_{12}q_y^* + s_{12}^*q_y + 2is_{11}s_{11y} - 2is_{12}^*s_{12y}), \\ \omega^{[1]} &= \omega - 4i\beta s_{11y} = \omega + 4i\beta s_{22y}. \end{aligned} \tag{8}$$

here we define the components of the matrix S as follows:

$$\begin{aligned} S &= \frac{1}{\Delta} \begin{pmatrix} \lambda_1|\psi_1|^2 + \lambda_2|\psi_2|^2 & (\lambda_1 - \lambda_2)\psi_1\psi_2^* \\ (\lambda_1 - \lambda_2)\psi_1^*\psi_2 & \lambda_1|\psi_2|^2 + \lambda_2|\psi_1|^2 \end{pmatrix}, \\ \Delta &= |\psi_1|^2 + |\psi_2|^2. \end{aligned}$$

Lets find further multi-line soliton solutions using this Darboux transformation.

Multi-line soliton solutions

Here in order to find one soliton solution we choose seed solution as is the $q = \omega = \nu = 0$. When solving a linear system under zero conditions, we obtain the following basic solution [11]:

$$\psi = \begin{pmatrix} e^{-i\lambda x + f(y,t)} \\ e^{i\lambda x + g(y,t)} \end{pmatrix} \tag{9}$$

here f and g - y and t periodic differential functions.

2.1 The first multi-line soliton solution

we can rewrite the equation (9) to this type:

$$\begin{aligned} f(y,t) &= i\mu y + i(2\alpha\lambda + 4\beta\lambda^2)\mu t + c, \\ g(y,t) &= -i\mu y - i(2\alpha\lambda + 4\beta\lambda^2)\mu t - c. \end{aligned}$$

we use the N-order Darboux transformation to obtain a linear colitis solution:

$$\xi_j = \begin{pmatrix} e^{\theta_j + i\chi_j} \\ e^{-\theta_j + i\chi_j} \end{pmatrix} \tag{10}$$

here

$$\theta_j + i\chi_j = -i\lambda_j x + i\mu_j y + i(2\alpha\lambda_j + 4\beta\lambda_j^2)\mu_j t + c_j,$$

where λ_j, μ_j, c_j are random real constants and $\lambda_j = b_j + id_j, \mu_j = m_j + in_j, c_j = k_j + il_j, (j = 2\tau - 1, \tau = 1, 2, \dots, N)$.

$$\begin{aligned} \theta_j &= d_j x - n_j y - 2(\alpha b_j n_j + \alpha d_j m_j + 4\beta b_j d_j m_j + 2\beta n_j b_j^2 - 2\beta n_j d_j^2)t + k_j \\ \chi_j &= -b_j x + m_j y + 2(\alpha b_j m_j - \alpha d_j n_j - 4\beta b_j d_j n_j + 2\beta m_j b_j^2 - 2\beta m_j d_j^2)t + l_j \end{aligned}$$

$b, d, m, k, l \neq 0, b, d, m, k, l$ - real constants.

Then $q^{[1]}, \nu^{[1]}, \omega^{[1]}$ we obtain a multi-line soliton solution for:

$$\begin{aligned} q^{[1]} &= 2de^{2i\chi_j} \operatorname{sech}(2\theta_1), \\ \nu^{[1]} &= 4\alpha d [\tan 2\theta_1]_y + 8i\beta d^2 ([\tan 2\theta_1]_y - e^{-2i\chi_1} \operatorname{sech} 2\theta_1^* [e^{2i\chi_1} \operatorname{sech} 2\theta_1]_y), \\ \omega^{[1]} &= 4\beta [\tan 2\theta_1]_y. \end{aligned} \tag{11}$$

here

$$(\theta_1 + i\chi_1) + (\theta_1 - i\chi_1) = 2\theta_1 = d_1 x - n_1 y + (4\beta n_1 b_1^2 - 4\beta n_1 d_1^2 - 8\beta m_1 b_1 d_1)t - 2\alpha(n_1 b_1 + m_1 d_1)t + k_1;$$

$$(\theta_1 + i\chi_1) - (\theta_1 - i\chi_1) = 2i\chi_1 = -b_1 x + m_1 y + (4\beta m_1 b_1^2 - 4\beta m_1 d_1^2 - 8\beta n_1 b_1 d_1)t + 2\alpha(m_1 b_1 - n_1 d_1)t + l_1.$$

Let's take $q^{[1]}$ as an example to analyze the distribution features of soliton, some parameters of which are affected. Using the calculated expressions, we can deduce from the equation the following physical properties of the solution (11 a): the amplitude $[2d_1]$, the wave vector $K = (K_x, K_y) = (2d_1, -2n_1)$, the frequency $\nu = [-4(d_1 - b_1) \cdot 2\alpha(n_1 b_1 + m_1 d_1)]$, the soliton direction $g = \frac{d_1}{n_1}$. From the figure it can be seen

that the distribution of a linear soliton d_j depends on the change in magnitude.

To visualize the results of the first multi-line soliton solution, we construct graphs of the expressions $q^{[1]}, \nu^{[1]}$ and $\omega^{[1]}$ obtained with the help of Maple 2018:

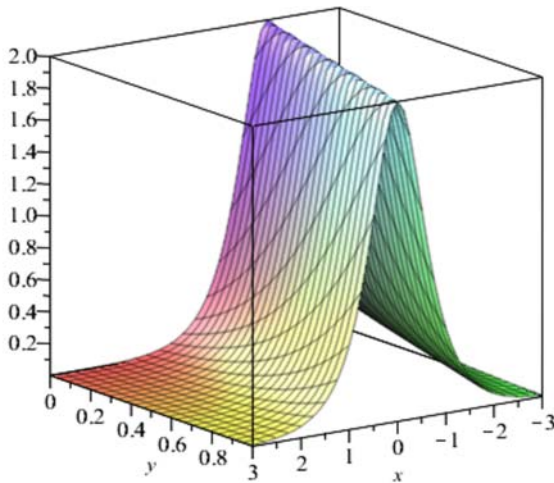


Figure 1 - $\alpha, d_1, l_1 = 1, m_1 = 2, \beta = 2, b_1 = 1.3$ and $t=0$ soliton solution for $q^{[1]}$

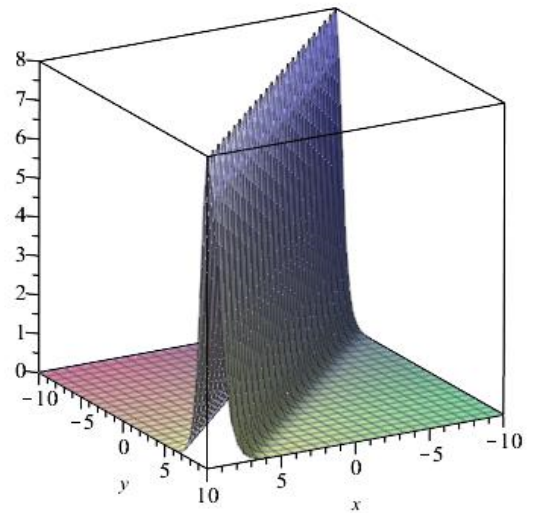


Figure 2 - $\alpha, d_1, l_1 = 1, \beta = 2, b_1 = 1.3, m_1 = 2, k_1 = 1, 5$ and $t=0$ soliton solution for $v^{[1]}$

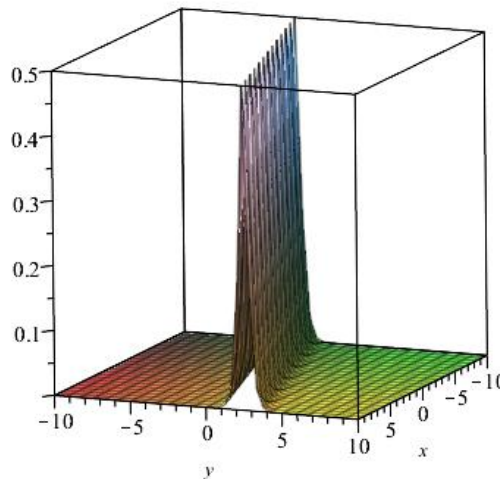


Figure 3 - $\alpha, d_1, l_1 = 1, \beta = -2, b_1 = 1.3, m_1 = 2, n_1, k_1 = 1, 5$ and $t=0$ soliton solution for $\omega^{[1]}$

We obtained first-order multi-linear soliton solutions using the Darboux transformation for the two-dimensional nonlinear Hirota equation with given seed solution conditions.

2.2 The second multi-line soliton solution

Now we obtain three two-line soliton solutions for $N = 2$.

$$q^{[2]} = \frac{2i \left(e^{\theta_1 - \theta_1^* + \theta_3 + \theta_3^*} a_1 + e^{-*(\theta_1 + \theta_1^*) - \theta_3 - \theta_3^*} a_2 + e^{-(\theta_1 + \theta_1^*) + \theta_3 - \theta_3^*} a_2 - e^{-(\theta_1 + \theta_1^*) + \theta_3 - \theta_3^*} a_2^* \right)}{\left(e^{2(\theta_1 + \theta_3)} + e^{-2(\theta_1 + \theta_3)} \right) \beta_1 + \left(e^{2(\theta_1 - \theta_3)} + e^{-2(\theta_1 - \theta_3)} \right) \beta_2 + \left(e^{-2i(\chi_1 + \chi_3)} \right) \beta_3}, \tag{12}$$

$$\begin{aligned} a_1 &= 2id_1(b_1 - id_1 - b_3 - id_3)(b_1 + id_1 - b_3 - id_3), & \beta_1 &= (b_3 - id_3 - b_1 - id_1)(b_1 + id_1 - b_3 + id_3), \\ a_2 &= 2id_3(b_3 - id_3 - b_1 - id_1)(b_3 + id_3 - b_1 - id_1), & \beta_2 &= (b_3 - id_3 - b_1 - id_1)(b_1 - id_1 - b_3 - id_3), \\ & & \beta_3 &= -2i(d_3 - d_1). \end{aligned}$$

$v^{[2]}, \omega^{[2]}$ values are considered in the same way.

$$\begin{aligned}
 v^{[2]} &= -4iP + 8i\beta(R - Q) \\
 \omega^{[2]} &= -4i\beta \frac{\gamma_1 \left(e^{2(\theta_1 + \theta_1 y + \theta_3 + \theta_{3y})} \right) + \gamma_2 \left(e^{2(\theta_1 + \theta_1 y - \theta_3 - i\chi_{3y})} \right) + \gamma_1^2 \left(e^{-2(\theta_1 + \theta_1 y + \theta_3 + i\chi_{3y})} \right) + \gamma_2^2 \left(e^{-2(\theta_1 + \theta_1 y - \theta_3 - \theta_{3y})} \right)}{\beta_1 \left(e^{2(\theta_3 + \theta_{3y} - \theta_1 y - i\chi_{3y})} \right) + \beta_2 \left(e^{2(\theta_{3y} - i\chi_{3y})} \right)} + \\
 &+ \frac{\gamma_3 \left(e^{2i(\chi_1 - \chi_3)} \right) + \gamma_3^2 \left(e^{-2i(\chi_1 - \chi_3)} \right)}{\beta_3 \left(e^{-4i(\chi_1 - \chi_3)} \right)}
 \end{aligned}$$

To visualize the results obtained during the calculations on the second-order multi-linear soliton, we construct graphs of the formulas $q^{[2]}$, $v^{[2]}$ and $\omega^{[2]}$ obtained with the help of Maple 2018:

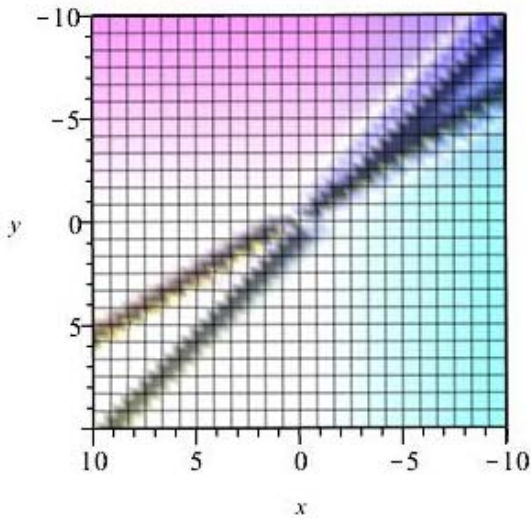


Figure 4 - $d_1, n_3, l_1, m_3, k_3 = 1, k_1 = -1$,
 $d_3 = 1.2, b_3 = 3, \alpha, \beta, l_3 = 2, b_1, n_1 = 1, 5$,
 and $t=0$ soliton solution for $q^{[2]}$

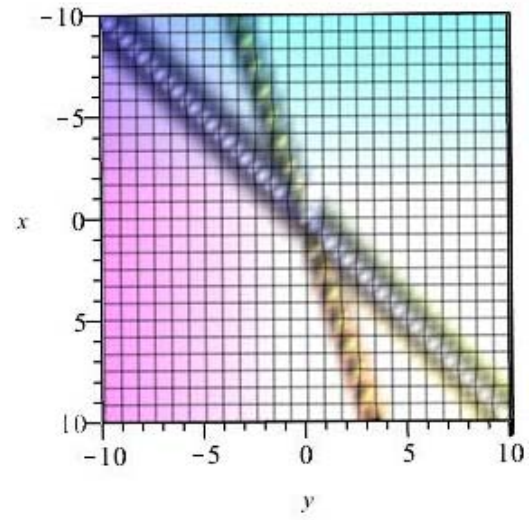


Figure 5 - $n_1, b_1, d_1, l_1, d_3, n_3, m_3, k_3, \gamma_2 = 1$,
 $\delta_3, \beta_1 = 1, \beta, \alpha, m_1, l_3, \delta_1, \gamma_1, \beta_2 = 2, b_3, \delta_2, \gamma_3$,
 $\beta_3 = 3, k_1 = -1$ and $t=0$ soliton solution for $v^{[2]}$

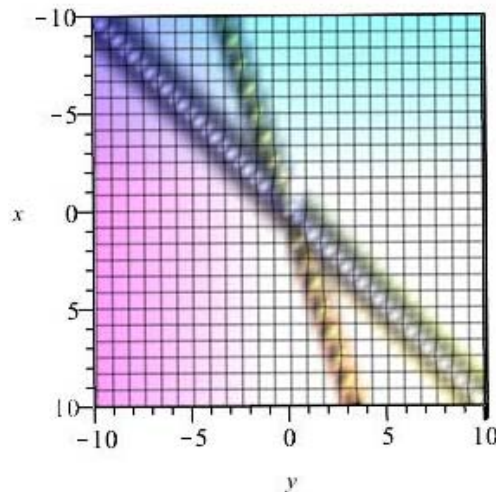


Figure 6 - $n_1, n_3, k_3, \gamma_2 = 1, k_1 = -1, \beta, \alpha, m_1, l_3, \delta_1, \gamma_1, \beta_2 = 2, b_1 = 4, b_3, \delta_2, \gamma_3, \beta_3, \beta_1 = 3$,
 $m_3 = 2.5, d_1, \delta_3, l_1 = 1, 5, d_3 = 1.6$ and $t=0$ soliton solution for $\omega^{[2]}$

In this section, we obtained second-order multi-linear soliton solutions for the two-dimensional nonlinear Hirota equation with initial conditions. 3 decisions were obtained by introducing certain

designations. As a result, we constructed graphs of the exact expressions to visualize the results of the second multi-line soliton solution.

Conclusions. In this article, we considered the first and second multi-line soliton solutions for two-dimensional nonlinear Hirota equations using the Lax pair and Darboux transformation. We also provided graphical representation. The obtained result is used for further research of nonlinear integrable equations when finding different soliton solutions.

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ЕКІ ӨЛШЕМДІ СЫЗЫҚТЫ ЕМЕС ХИРОТА ТЕНДЕУІНЕ АРНАЛҒАН КӨП СЫЗЫҚТЫ ШЕШІМДЕР

Аннотация. Қазіргі уақытта солитондар теориясы шеңберінде көп өлшемді сызықты емес интегралданатын тендеулерді зерттеу мәселесі шетелдік және Қазақстандық ғалымдар үшін үлкен қызығушылық тудырады. Табиғатта кездесетін көптеген физикалық құбылыстарды сызықты емес интегралданатын тендеулер арқылы сипаттауға болады. Мұндай тендеулердің нақты шешімдерін табу қатты дене физикасы, сұйықтар механикасы, плазма физикасы және сызықты емес оптика секілді түрлі ғылыми және инженерлік салаларда кездесетін құбылыстардың динамикасын зерттеуде маңызды рөл атқарады. Осындай тендеулердің нақты және солитондық, солитон тәрізді шешімдерін алудың біршама әдісі бар: кері шашырау есебі әдісі, бисызықты Хирота әдісі, Дарбу түрлендіру әдісі, кері шашырау есебі әдісі тангенс және синус-косинус әдістері сияқты көптеген әдістер ұсынылған.

Бұл жұмыста Шредингердің модификацияланған сызықты емес тендеуі болып табылатын екі өлшемді Хирота тендеуі зерттелінді. Сызықты емес Хирота тендеуі интегралданатын тендеудің бірі болып табылады және Хирота жүйесі көптеген сызықты емес құбылыстарды сипаттау үшін оптикалық талшықтар жүйесінің зерттеу аймағында, физика ғылымында, электрлік байланыс және басқа да инженерлік салаларда қолданылады. Осы уақытта дейін екі өлшемді Хирота тендеулер жүйесі үшін бірінші, екінші, n -ретті Дарбу түрлендіруі құрылған және солитондық, киратушы толқынды шешімдері әр түрлі әдістермен анықталған.

Бұл мақалада екі өлшемді сызықты емес Хирота тендеулерін қарастырамыз. Лакс жұбын және Дарбу түрлендіруін қолдана отырып, біз осы тендеуге арналған бірінші және екінші көп сызықты солитондық шешімдер алынды және осы шешімді графикалық бейнелеу мақсатында Maple 2018 бағдарламасы көмегімен әртүрлі уақытта көп сызықты солитондардың графиктері құрылды.

Осы мақалада қолданылған әдісті басқа да сызықты емес комплексті тендеулерді зерттеу барысында, олардың солитондық шешімдерді табу үшін пайдалануға болады. Сонымен қатар, алынған нәтижені Хирота тендеуін одан әрі зерттеу үшін қолдануға болады.

Түйін сөздер. Екі өлшемді сызықты емес Хирота тендеулері, сызықты емес интегралданатын тендеулер, Лакс жұбы, Дарбу түрлендіруі, мульти-сызықты солитон шешімдері.

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МУЛЬТИЛИНЕЙНЫЕ РЕШЕНИЯ ДЛЯ ДВУМЕРНОГО НЕЛИНЕЙНОГО УРАВНЕНИЯ ХИРОТЫ

Аннотация. В настоящее время вопрос изучения многомерных нелинейных интегрируемых уравнений в рамках теории солитонов имеет большой интерес для зарубежных и казахстанских ученых. Многие физические явления, происходящие в природе, можно описать нелинейно-интегрированными уравнениями. Нахождение конкретных решений таких уравнений играет важную роль в изучении динамики явлений, происходящих в различных областях науки и техники, таких как физика твердого тела, механика жидкости, физика плазмы и нелинейная оптика. Существует несколько методов получения реальных и солитонных, солитоноподобных решений таких уравнений: метод обратной задачи рассеяния, билинейный метод Хироты, метод преобразования Дарбу, методы тангенса и синуса-косинуса.

В данной работе изучено двумерное уравнение Хироты, которое является модифицированным нелинейным уравнением Шредингера. Нелинейное уравнение Хироты является одним из интегрируемых уравнений, а система Хироты используется в области исследования волоконно-оптических систем, физики, телекоммуникаций и других инженерных областях для описания многих нелинейных явлений. К настоящему времени разработаны преобразования Дарбу первого, второго и n -го порядка для двумерной системы уравнений Хироты, а солитонные решения в виде разрушительных волн определены различными методами.

В статье рассмотрены двумерные нелинейные уравнения Хироты. С использованием пары Лакса и преобразования Дарбу получены первое и второе мульти-линейные солитонные решения этого уравнения и для графического представления этого решения с помощью программы Maple 2018 были построены графики многолинейных солитонов в разное время.

Метод, использованный в данной статье, может быть использован для изучения других нелинейных комплексных уравнений с целью поиска их солитонных решений. Кроме того, полученный результат может быть использован для дальнейшего изучения уравнения Хироты.

Ключевые слова: двумерные нелинейные уравнения Хироты, нелинейные интегрируемые уравнения, пара Лакса, преобразование Дарбу, мультилинейные солитонные решения.

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REFERENCES

- [1] Yesmakhanova K., Shaikova G., Bekova G., Soliton solutions of the Hirota's system // AIP Conference Proceedings. Vol.1759. 2016. P. 020147. <https://doi.org/10.1063/1.4959761>
- [2] Myrzakulov R., Nugmanova G.N.(2013) Integriruemye obobshcheniya uravneniya Landau-Lifshica // Master PO, Kazakhstan. ISBN 978-601-301-063-2 (In Russ.).
- [3] Zaharov V.E., Tahtadzhyan L.A. Ekvivalentnost' nelinejnogo uravneniya SHredingera i uravneniya ferromagnetika Gejzenberga // Teor.Mat.Fiz. Vol. 38, 1979. P. 26-34. (In Russ.).
- [4] Zhadyranova A.A. Hierarchy of WDVV associativity equations for $n=3$ and $N=2$ case when $V_0=0$ with new system a_i, b_i, c_i // News of the National Academy of Sciences of the Republic of Kazakhstan. Physical-mathematical series. <https://doi.org/10.32014/2019.2518-1726.39> 4(326). 2019. P. 14–21.
- [5] Zhadyranova A.A., Myrzakul Zh.R. Soliton surface associated with the WDVV equation for $n = 3$ case // 2019 J. Phys.: Conf. Ser. 1391(1) 012105.
- [6] Zaharov V.E., Manakov S.V. Mnogomernye nelinejnye integriruemye sistemy i metody ih reshenij // Zap. nauch. sem. LOMI. Vol.133. 1984. P. 77-91. (In Russ.).
- [7] Myrzakul A., Myrzakulov R. Integrable motion of two interacting curves, spin systems and the Manakov system // International Journal of Geometric Methods in Modern Physics C. Vol.14, 2017. P. 1750115. <https://doi.org/10.1142/S0219887817501158>
- [8] Nugmanova G., Myrzakul A. (2019) Integrability of the Two-layer Spin System // Geom., Integr. and Quant. Proc. Of XXth Int. Conf., Sofia. P.208-214.
- [9] Myrzakulov R., Nugmanova G., Syzdykova R. Gauge equivalence Between (2+1)-Dimensional Continuous Heisenberg Ferromagnetic Models and Nonlinear Schrodinger-Type Equations // J. Phys.A: Math and Gen. Vol.31, 1998. P. 9535-9545 <https://iopscience.iop.org/article/10.1088/0305-4470/31/47/013>
- [10] Shaikova G., Yesmakhanova K., Bekova G. Conservation laws of the Hirota-Maxwell-Bloch system and its reductions // Journal of Physics Conference Series. Vol.936. 2017. 012098. <https://doi.org/10.1088/1742-6596/936/1/012098>
- [11] Run Zhou, Hui-Qin Hao, Rong-Rong Jia. New soliton solutions for the (2+1)-dimensional Schrödinger-Maxwell-Bloch equation // Superlattices and Microstructures. V.113. 2018. P. 409-418 <https://doi.org/10.1016/j.spmi.2017.11.019>