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TRANSLATIONAL-ROTATIONAL MOTION OF A NONSTATIONARY AXISYMMETRIC BODY

Abstract. A nonstationary two-body problem is considered such that one of the bodies has a spherically symmetric density distribution and is central, while the other one is a satellite with axisymmetric dynamical structure, shape, and variable oblateness. Newton's interaction force is characterized by an approximate expression of the force function up to the second harmonic. The masses of the central body and the satellite vary isotropically at different rates and do not occur reactive forces and additional rotational moments. The nonstationary axisymmetric body have an equatorial plane of symmetry. Thus, it has three mutually perpendicular planes of symmetry. The axes of its intrinsic coordinate system coincide with the principal axes of inertia and they are directed along the intersection lines of these three mutually perpendicular planes. This position remains unchangeable during the evolution. Equations of motion of the satellite in a relative system of coordinates are considered. The translational-rotational motion of the nonstationary axisymmetric body in the gravitational field of the nonstationary ball is studied by perturbation theory methods. The equations of secular perturbations reduces to the fourth order system with one first integral. This first integral is considered and three-dimensional graphs of this first integral are plotted using the Wolfram Mathematica system.

Key words: variable mass, translational-rotational motion, axisymmetric body, secular perturbations.

1. Introduction.

At present the observational data of astronomy show to the nonstationarity of real cosmic systems associated with the effects of varying the masses of gravitating bodies with time, with variations in sizes and shapes of bodies themselves and some other physical characteristics in the process of evolution [1]-[3]. In this regard, creating mathematical models of motion of nonstationary celestial bodies becomes relevant.

The purpose of this paper is to consider the first integral and to plot three-dimensional graphs of this integral. Solving this problem involves rather cumbersome symbolic computations, which are best to perform using computer algebra systems. In this work, all necessary symbolic computations are carried out using the Wolfram Mathematica system [4].

2. The physical statement of the problem.

Let us consider the following conditions:

1. The first body is «central», it is a ball with variable mass $m_1 = m_1(t)$, with variable density, spherical distribution and variable radius $l_1 = l_1(t)$;

2. The second body is a «satellite» with a mass $m_2 = m_2(t)$ has an axisymmetric dynamical structure and a shape with a characteristic linear size $l_2 = l_2(t)$ and its second-order moments of inertia are variable and given functions of time. Such a satellite is characterized by variable oblateness, and its principal central moments of inertia A, B, C satisfy the relations

$$A(t) = B(t) \neq C(t)\,,\ \frac{C(t) - A(t)}{C(t)} \neq const\,; \tag{1}$$

3. The nonstationary axisymmetric body have an equatorial plane of symmetry. Thus, it has three mutually perpendicular planes of symmetry [1-4];

4. The masses of the central body and the satellite vary isotropically at different rates $\dot{m}_1/m_1 \neq \dot{m}_2/m_2$ and do not occur reactive forces and additional rotational moments;

5. The axes of its intrinsic coordinate system coincide with the principal axes of inertia and they are directed along the intersection lines of these three mutually perpendicular planes. This position remains unchangeable during the evolution;

6. We restrict ourselves to an approximation of the force function up to the second harmonic inclusive.

3. Equations of motion in the relative coordinate system.

The relative translational motion of the center of mass of the satellite around the central body is described by the equations

$$\tilde{m}\ddot{x} = \frac{\partial U}{\partial x}\,,\ \tilde{m}\ddot{y} = \frac{\partial U}{\partial y}\,,\ \tilde{m}\ddot{z} = \frac{\partial U}{\partial z}\,, \tag{2}$$

where x, y, z are the coordinates of the center of mass of T_2 in the relative coordinate system O_1xyz with the origin at the center of T_1, $\tilde{m} = m_1 m_2 / (m_1 + m_2)$ is the reduced mass, the force function of Newton's interaction of the two bodies has the form

$$U = f\frac{m_1 m_2}{R} + \tilde{U}\,,\ R = x^2 + y^2 + z^2\,, \tag{3}$$

$$\tilde{U} = f m_1 \frac{2A(t) + C(t) - 3J}{2R^3}\,, \tag{4}$$

and the moment of inertia J of the axisymmetric body with respect to the vector $\overrightarrow{O_1O_2}$ connecting the centers of mass of the two bodies is determined by the expression

$$J = A\left(\alpha^2 + \beta^2\right) + C\gamma^2 \tag{5}$$

where α, β, γ are the direction cosines of $\overrightarrow{O_1O_2}$ with the axes of the intrinsic coordinate system of the satellite coinciding with its principal central axes of inertia.

The equation of perturbed translational motion has the form [3]

$$\ddot{\vec{R}} + f\frac{m_1 + m_2}{R^3}\vec{R} - b\vec{R} = grad_{\vec{R}} W\,, \tag{6}$$

where

$$W = -\frac{1}{2}bR^2 + \frac{m_1 + m_2}{m_1 m_2}\tilde{U}\,,\ b = b(t_0) = \frac{\ddot{\sigma}}{\sigma}\,,\ \sigma = \sigma(t) = \frac{m_1(t_0) + m_2(t_0)}{m_1(t) + m_2(t)}\,, \tag{7}$$

The rotational motion of the satellite around its own center of mass in the Euler variables is governed by the equations [3,5]

$$\begin{aligned}
&\frac{d}{dt}(Ap) - (A - C)qr = \frac{\sin\varphi}{\sin\theta}\left[\frac{\partial U}{\partial\psi} - \cos\theta\frac{\partial U}{\partial\varphi}\right] + \cos\varphi\frac{\partial U}{\partial\theta}\,,\\
&\frac{d}{dt}(Aq) - (C - A)rp = \frac{\cos\varphi}{\sin\theta}\left[\frac{\partial U}{\partial\psi} - \cos\theta\frac{\partial U}{\partial\varphi}\right] - \sin\varphi\frac{\partial U}{\partial\theta}\,,\\
&\frac{d}{dt}(Cr) = 0,
\end{aligned} \tag{8}$$

$$p = \dot{\psi}\sin\varphi\sin\theta + \dot{\theta}\cos\varphi\,,\;\; q = \dot{\psi}\cos\varphi\sin\theta - \dot{\theta}\sin\varphi\,,\;\; r = \dot{\psi}\cos\theta + \dot{\varphi}\,. \tag{9}$$

where p, q, r are the projections of the angular velocity of the satellite onto the axes of its intrinsic coordinate system and φ, ψ, θ are the Euler angles [6–8]. The problem in this statement is very complicated; therefore, we will use the methods of perturbation theory to study it [3].

4. Differential equations of secular perturbations.

The equations of translational-rotational motion are described in the analogue of the Delaunay-Andoyer variables. In this case, the unperturbed motion is analogous of Euler-Poinsot motion – the rotational motion of the free non-stationary axisymmetric body around its own center of inertia [1-3].

Calculation of secular perturbations reduces to the fourth-order system with one first integral [9]

$$\dot{H} = \left(\frac{E(t)}{\tilde{m}(t)}\right)\left[\frac{\partial I}{\partial h}\right],\quad \dot{h} = -\left(\frac{E(t)}{\tilde{m}(t)}\right)\left[\frac{\partial I}{\partial H}\right],\quad \dot{H}' = E(t)\left[\frac{\partial I}{\partial h'}\right],\quad \dot{h}' = -E(t)\left[\frac{\partial I}{\partial H'}\right], \tag{10}$$

$$E(t) = -\frac{3fm_1(C-A)}{8\sigma^3 a^3(1-e)^{3/2}},\;\; \tilde{m}(t) = \frac{m_1 m_2}{m_1 + m_2}\,. \tag{11}$$

$$I = I(H, h, H', h') = I_0 = const\,. \tag{12}$$

$$\begin{aligned} I = \frac{1}{2GG'^3}\Big(& GG'^3\left(3+\cos\left(2\left(h'-\Omega\right)\right)+\cos^2 I'\cos^2\left(h'+\Omega\right)+3\cos^2 I'\cos\left(2J'\right)\cos^2\left(h'+\Omega\right)-\right.\\ & -\cos^2 J'\left(1+\cos\left(2\left(h'-\Omega\right)\right)+2\cos\left(2\left(h'+\Omega\right)\right)\right)\Big)-\frac{1}{2}GG'^3\cos^2 i\,\Big(4\cos^2\left(h'-\Omega\right)+\\ & +\cos^2 I'\left(1+3\cos 2J'\right)\left(3+\cos\left(2\left(h'+\Omega\right)\right)\right)-2\cos^2 J'\left(3+\cos\left(2\left(h'-\Omega\right)\right)+2\cos\left(2\left(h'+\Omega\right)\right)\right)-\\ & -4\cos i\cos I'\cos\left(h'+\Omega\right)\sqrt{G^2\sin^2 i}\left(-2G'^2\cos^2 J'\sqrt{G'^2\sin^2 I'}+\sqrt{G'^2\sin^2 J'}\sqrt{G'^4\sin^2 I'\sin^2 J'}\right)\Big)\Big) \end{aligned} \tag{13}$$

Upon solving system (10), we integrate the remaining equations [9]

$$\dot{L}_{sec} = 0,\qquad \dot{G}_{sec} = 0,\qquad \dot{L}'_{sec} = 0,\qquad \dot{G}'_{sec} = 0, \tag{14}$$

$$\dot{l}_{sec} = -\frac{\partial W_{sec}}{\partial L'_{sec}},\qquad \dot{g}_{sec} = -\frac{\partial W_{sec}}{\partial G_{sec}},\qquad \dot{l}'_{sec} = \frac{\partial F_{sec}}{\partial L'_{sec}},\qquad \dot{g}'_{sec} = \frac{\partial F_{sec}}{\partial G'_{sec}}. \tag{15}$$

$$\begin{aligned} W_{sec} = & \frac{\mu_0^2}{2\sigma^2(t)}\left(\frac{1}{L^2}\right)+\frac{m_1+m_2}{m_1 m_2}\left(\frac{fm_1(C-A)}{2\sigma^3}\left[\frac{1}{a^3\left(1-e^2\right)}\right]\right)-\\ & -\frac{m_1+m_2}{m_1 m_2}\left(\frac{3fm_1(C-A)}{2\sigma^3}\left[\frac{I}{4a^3\left(1-e^2\right)^{3/2}}\right]\right)-\frac{1}{2}b\sigma^2\left[a^2\left(1+\frac{3}{2}e^2\right)\right] \end{aligned} \tag{16}$$

$$\begin{aligned} F_{sec} = & \frac{1}{2A}\left(G'^2\right)+\frac{1}{2}\left(\frac{1}{C}-\frac{1}{A}\right)\left(L'^2\right)+\frac{fm_1(C-A)}{2\sigma^3}\left[\frac{1}{a^3\left(1-e^2\right)}\right]-\\ & -\frac{3fm_1(C-A)}{2\sigma^3}\left[\frac{I}{4a^3\left(1-e^2\right)^{3/2}}\right]-\frac{1}{2}b\sigma^2\left[a^2\left(1+\frac{3}{2}e^2\right)\right] \end{aligned} \tag{17}$$

5. Graphs of the first integral

The resulting first integral (13) depends on the variables Ω, i, I', h'. We set a constant value for one variable and use the other three variables to plot the three-dimensional graph. To simplify the calculations we use the dimensionless variables in the integral (13).

Introduce the following dimensionless variables [10]. Dimensionless time t is defined by $\tilde{t} = \omega_0 t$, where $\omega_0 = \sqrt{\mu_0} / a^{3/2}$, $\mu_0 = f(m_{10} + m_{20})$.

Dimensionless masses and moments of inertia are given by $\tilde{m}_1(\tilde{t}) = m_1(t)/m_{10}$, $\tilde{m}_2(\tilde{t}) = m_2(t)/m_{10}$, $\tilde{A}(\tilde{t}) = A(t)/A(t_0)$, $\tilde{C}(\tilde{t}) = C(t)/C(t_0)$.

$m_{10} = m_1(t_0)$, $m_{20} = m_2(t_0)$, $t_0 -$ an initial time . In equations (14), the Delaunay-Andoyer elements are replaced by the corresponding dimensionless values $\tilde{G} = G/\sqrt{\mu_0 a}$, $\tilde{G}' = G'/m_{10} a^2 \omega_0$.

These data are sufficient to consider and obtain the graphs. The integral (13) in dimensionless values has the form

$$
\begin{aligned}
I = \frac{1}{2\tilde{G}\tilde{G}'^3}\Big(&\tilde{G}\tilde{G}'^3\big(3+\cos\big(2(h'-\Omega)\big)+\cos^2 I'\cos^2(h'+\Omega)+3\cos^2 I'\cos(2J')\cos^2(h'+\Omega)-\\
&-\cos^2 J'\big(1+\cos\big(2(h'-\Omega)\big)+2\cos\big(2(h'+\Omega)\big)\big)\big)-\frac{1}{2}\tilde{G}\tilde{G}'^3\cos^2 i\,\big(4\cos^2(h'-\Omega)+\\
&+\cos^2 I'(1+3\cos 2J')\big(3+\cos\big(2(h'+\Omega)\big)\big)-2\cos^2 J'\big(3+\cos\big(2(h'-\Omega)\big)+2\cos\big(2(h'+\Omega)\big)\big)-\\
&-4\cos i\cos I'\cos(h'+\Omega)\sqrt{\tilde{G}^2\sin^2 i}\Big(-2\tilde{G}'^2\cos^2 J'\sqrt{\tilde{G}'^2\sin^2 I'}+\sqrt{\tilde{G}'^2\sin^2 J'}\sqrt{\tilde{G}'^4\sin^2 I'\sin^2 J'}\Big)\Big)
\end{aligned}
\qquad (18)
$$

We use here the following values of the physical parameters of Sun (central body) and Earth (satellite). $M_\odot$ is the mass of the Sun, $M_\oplus$ is the mass of the Earth, A_0, C_0 is Earth's principal moments of inertia for initial conditions [11]

$$
\begin{gathered}
m_{10} = m_1(t_0) = 1M_\odot, \qquad m_{20} = m_2(t_0) = 3\cdot 10^{-6} M_\odot, \qquad e = 0.017, \\
A_0 = A(t_0) = 0.3295 M_\oplus a^2, \qquad C_0 = C(t_0) = 0.3306 M_\oplus a^2, \\
\tilde{G} = 0.999, \quad \tilde{G}' = 4.372, \quad \tilde{h}' = \pi/18, \quad \Omega = \pi/9, \quad i = \pi/3, \quad I' = \pi/9, \quad J = \pi/6.
\end{gathered}
\qquad (19)
$$

Using the Mathematica system, three-dimensional graphs of the integral (18) are obtained, which are shown in figures 1-4.

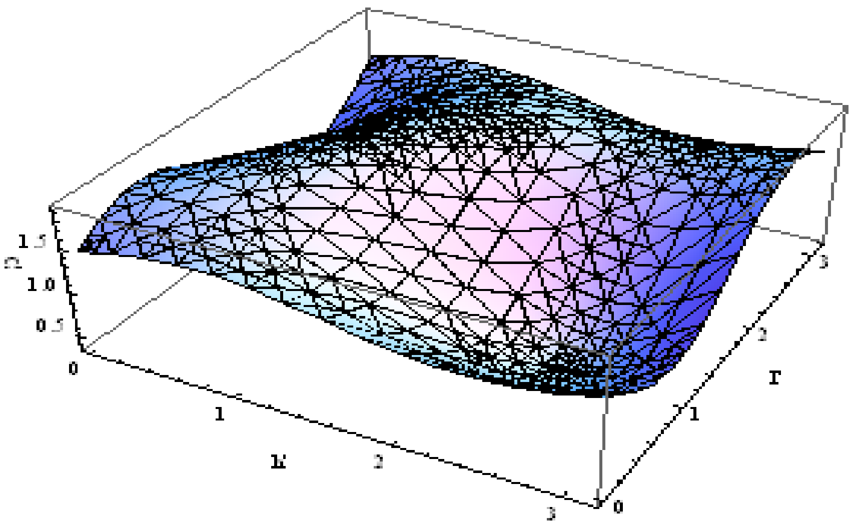

Figure 1 - Graph of the integral, when $i = \pi/3 = const$, the others according to (19)

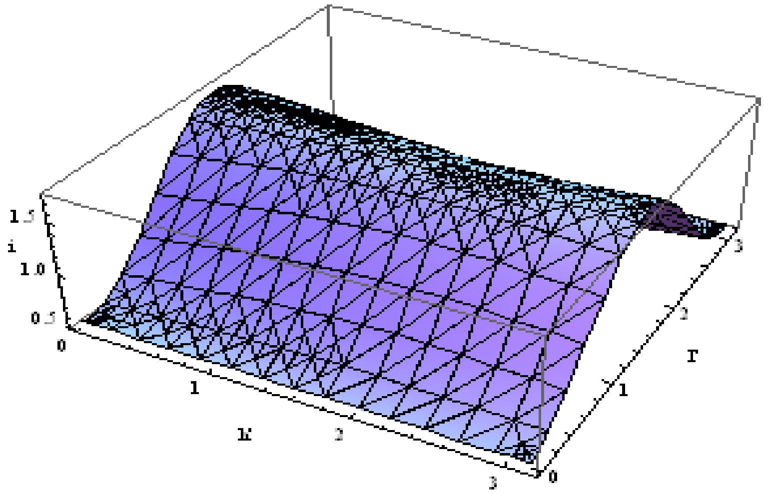

Figure 2 - Graph of the integral, when $\Omega = \pi/9 = const,$ the others according to (19)

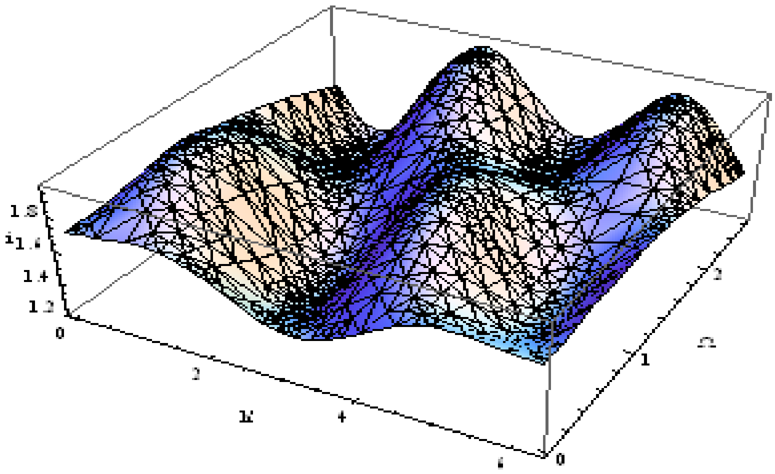

Figure 3 - Graph of the integral, when $I' = \pi/9 = const,$ the others according to (19)

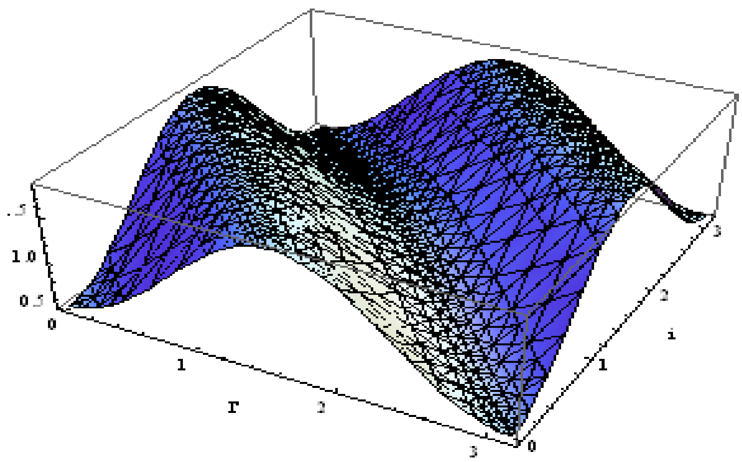

Figure 4 - Graph of the integral, when $h' = \pi/18 = const,$ the others according to (19)

These figures are more informative and illustrative, a detailed analysis of which will be dealt with in other papers.

6. Conclusion.

The translational-rotational motion of the nonstationary axisymmetric body in the gravitational field of a nonstationary ball is studied by perturbation theory methods. The equations of secular perturbations reduce to the fourth order system with one first integral. The first integral is considered in dimensionless variables and three-dimensional graphs of this integral plotted using Wolfram Mathematica.

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БЕЙСТАЦИОНАР ӨСТІК СИММЕТРИЯЛЫ ДЕНЕНІҢ ІЛГЕРІЛЕМЕЛІ-АЙНАЛМАЛЫ ҚОЗҒАЛЫСЫ

Аннотация. Мақалада өзара гравитацияланушы бейстационар екі дене қарастырылады: бірінші дене – «центрлік», яғни тығыздығы сфера бойынша үлестірілген шар, ал екінші дене – «серік», яғни динамикалық құрылымы және пішіні өстік симметриялы. Ньютонның өзара әсерлесу күші екінші гармониканы ескергендегі күштік функцияның жуық өрнегімен сипатталған. Денелердің массасы мен өлшемі әртүрлі қарқында изотропты түрде өзгереді, сондықтанда қосымша реактивті күш және қосымша айналдырушы моменттер туындамайды. Бейстационар өстік симметриялы дене экваторлық симметрия жазықтығына ие. Сондықтан, оның үш өзара перпендикуляр симметрия жазықтығы бар. Дененің бас инерция өстерімен сәйкес келетін өзіндік координаталар жүйесінің өстерін осы үш өзара перпендикуляр жазықтықтардың қиылысу сызығы бойымен бағыттаймыз. Денеге қатысты осы өстердің бағыты эволюция барысында өзгеріссіз қалады. Серіктің қозғалыс теңдеуі салыстырмалы координаталар жүйесінде алынған. Бейстационар шардың гравитациялық өрісіндегі өстік симметриялы бейстационар дененің ілгерілемелі-айналмалы қозғалысы ұйытқу теориясының тәсілдерімен алынған. Ғасырлық ұйытқулардың теңдеулері бір интегралды төртінші ретті жүйеге келтірілген. Осы бірінші интеграл қарастырылған және осы бірінші интегралдың үш өлшемді графиктері Wolfram Mathematica пакетінің көмегімен тұрғызылған.

Түйін сөздер: айнымалы масса, ілгерілемелі-айналмалы қозғалыс, өстік симметриялы дене, ғасырлық ұйытқу.

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ПОСТУПАТЕЛЬНО-ВРАЩАТЕЛЬНОЕ ДВИЖЕНИЕ НЕСТАЦИОНАРНОГО ОСЕСИММЕТРИЧНОГО ТЕЛА

Аннотация. Рассматривается нестационарная задача двух тел, одно из которых имеет сферически симметричное распределение плотности и является “центральным”, а второе – “спутник”, обладающий осесимметричным динамическим строением, формой и переменным сжатием. Ньютоновская сила взаимодействия характеризуется приближенным выражением силовой функции с точностью до второй гармоники. Массы центрального тела и спутника изменяются изотропно в различных темпах, при этом не появляются реактивные силы и дополнительные вращательные моменты. Нестационарное осесимметричное тело обладает экваториальной плоскостью симметрии. Следовательно, оно обладает тремя взаимно перпендикулярными плоскостями симметрии. Оси собственной системы координат совпадают главными осями инерции и их направим вдоль линий пересечения трех взаимно перпендикулярных плоскостей. Это положение в ходе эволюции остается неизменными. Получены уравнения движения спутника в относительной системе координат. Поступательно-вращательное движение нестационарного осесимметричного тела в гравитационным поле нестационарного шара изучено методами теории возмущений. Уравнения вековых возмущений сводится к системе четвертого порядка с одним первым интегралом. Рассмотрены этот первый интеграл и построены трехмерные графики этого первого интеграла с помощью пакета Wolfram Mathematica.

Ключевые слова: переменная масса, поступательно-вращательное движение, осесимметричное тело, вековые возмущения.

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