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NUMERICAL SOLUTION OF TWO-DIMENSIONAL PROBLEMS OF THERMOVISCOELASTICITY

Abstract. This paper considers the numerical implementation of two-dimensional thermoviscoelastic waves. The elastic collision of an aluminum cylinder with a two-layer plate of aluminum and iron is considered. In work [1] the difference schemes and algorithm of their realization are given. The most complete reviews of the main methods of calculation of transients in deformable solids can be found in [2, 3, 4], which also indicates the need and importance of generalized studies on the comparative evaluation of different methods and identification of the areas of their most rational application. In the analysis and physical interpretation of numerical results in this work it is also useful to use a priori information about the qualitative behavior of the solution and all kinds of information about the physics of the phenomena under study. Here is the stage of evolution of contact resistance of collision – plate, stress profile.

Keywords: two-dimensional thermoviscoelastic waves, stability of the difference scheme, convergence of the solution of the difference problem, indenter, deformation, tensor, stresses.

Introduction. To test the numerical method proposed in [1], we consider the elastic collision of an aluminum cylinder of unit radius with a two-layer plate made of aluminum ($h_1 = 0{,}75$) and iron ($h_2 = 0{,}25$).

Figure 1 shows the normal voltage distribution σ_z (figure 1, a) and vertical speed υ (figure 1, b) on the axis of symmetry, at the points where the wave motion up to the moment of time $t = 1{,}0$ is one-dimensional (self-similar). The information is presented in dimensionless form, where the linear dimensions are related to the thickness of the plate h, time – to $\frac{h}{a}$ (a– velocity of propagation of longitudinal waves in aluminum), stresses are divided by $\rho \cdot a^2$, and velocities – by a. The solid curve corresponds to the results of calculations with a step $\Delta h = 0{,}02$, solid with a point – $\Delta h = 0{,}01$; the dashed line refers to the Lax-Wendroff Scheme at $\Delta h = 0{,}02$; the dashed line is the exact solution.

It can be seen from the graphs that with a sufficiently large number of integration nodes, the first-order accuracy scheme for a strong gap gives a fairly clear and steep stress and velocity profile. The convergence of difference solutions is monotonic, which allows their refinement by means of Richardson extrapolation [5]. The "smearing" zone of the jump decreases proportionally to the decrease Δh, and at $\Delta h = 0{,}02$ it practically disappears, the numerical solution coincides with the exact one. The results presented here are consistent with the data of [6].

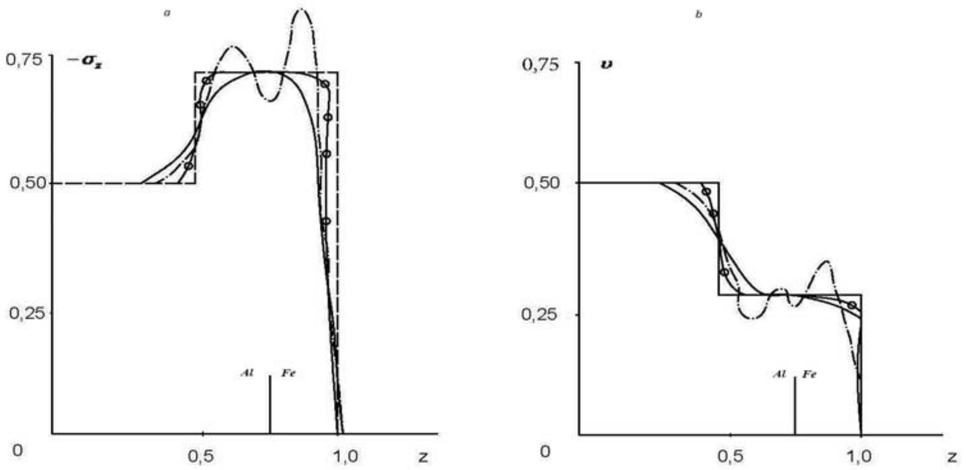

Figure 1(a, b) - distribution of normal stress $-\sigma_z$ and vertical velocity υ

The non-monotonicity of the second-order accuracy scheme generates very significant oscillations in the vicinity of the front, which are increasingly amplified as the waves reflect from the outer boundaries of the plate. With decreasing grid spacing, the amplitude of oscillations decreases, while the width of the zone of "spreading" at the same time narrows. Note that non-monotonicity scheme occurs only before front. Oscillations are absent after the front. The amplitude of the oscillations in the vicinity of the jump can be minimized by introducing a specially selected viscosity, however, as shown by Lax and Wendroff, it is impossible to get rid of these oscillations at all. In addition, the introduction of artificial viscosity reduces the accuracy of the difference scheme in the areas of smoothness of the solution. In some cases, the attribution of different viscosities when they disappear can lead to different limit solutions [7]. The advantages of high accuracy schemes, which include the Lax - Wendroff Scheme, are manifested in the calculation of smooth solutions.

The results presented here clearly illustrate the advantages of a monotone (non-oscillating) scheme for the calculation of jump motion. Countable phenomena, manifested in the through account of discontinuous (generalized) solutions by schemes of increased order of accuracy without special allocation of features, significantly complicate the analysis of the wave pattern, especially in cases where the exact location of the discontinuities and their configuration are unknown. In addition, it is hardly advisable to use a high approximation order to compute a solution that does not have the corresponding smoothness. It should also be borne in mind that for any difference scheme, the order of approximation, and hence the convergence of the solution in the areas of the influence of smearing discontinuities in the general case is close to the first [8].

In the contact problem under consideration, the computational domain has a rather complex geometry, so the study of the a posteriori accuracy of the desired functions, especially near the angular points where the stresses may have features, required a special series of test computational experiments to obtain convergent solutions on a sequence of grinding grids. Control of the calculation results was also carried out by calculating the total thermomechanical energy, which should remain unchanged in time and be equal to the kinetic energy of the translational and rotational movements originally stored by the stamp. Note that when constructing the difference grid, the partition was chosen so that its nodes do not fall on the contact boundaries (hence, in the angular points). The reduction of the integration step over space makes it possible to determine the numerical solution outside an arbitrarily small neighborhood of singular points (this approach eliminates the need for the averaging procedure of the solution at these points, which is inevitable with other methods).

Figure *2, a* shows the distribution of the contact voltage σ_z that occurs at the time $t = 0{,}25$ of collision of an iron cylinder with a radius of 0.5 on a two-layer plate $0{,}75\,Al$ $-0.25\,Fe$. Curves are obtained at $\Delta h = 0{,}02$ (1); $0{,}07$ (2); 0,005 (3).

It can be seen from the graphs that when the grid thickens, the discrete functions differ slightly and these differences are noticeable only in the diffraction region, where the unsteady field is essentially two-dimensional. In the zone of uniaxial stress state the calculated value $\sigma_z = -0{,}72$ coincides with the exact solution [9], which is effectively calculated only in the region of one-dimensional wave motion:

$$\sigma_z = -V_0 \frac{\rho_0 a_0 \rho_1 a_1}{\rho_0 a_0 + \rho_1 a_1}.$$

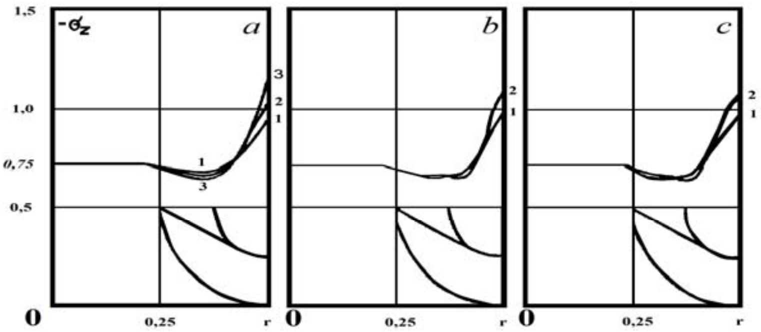

Figure 2(a, b, c) - distribution of the contact stresses $-\sigma_z$ resulting from point-in-time $t = 0{,}25$

Comparative analysis of the compared numerical results indicates their reliability and proximity to the exact solution. On the basis of a posteriori estimates, it can be argued that the individual components of the solution vector are with a relative error of 7 - 10% on the grid with a step $\Delta h = 0{,}02$ in the norm L_2, and the accuracy of determining the mean at the site of contact stresses $\sigma_x = 2\pi \int_0^R \sigma_z(r, o, t) r dr$ is slightly higher: 1 - (-0,588); 2 - (-0,591); 3 - (-0,592).

The imbalance of total energy is only a fraction of a percent on grid 3.

The results of calculations of contact voltage $-\sigma_z$ according to the Lax-Wendroff ($A = 0$) scheme and the scheme with minimal dispersion ($A = 1{,}3$) on grids 1 and 2 reflect figure 2,b,c. Comparison of the curves shows that both schemes give similar grids, in good agreement with the numerical data of the scheme of first order of accuracy ($q^* = A = 0$). The schemes of the increased order of accuracy allow revealing small features of a wave field when distribution of stresses is rather smooth. However, in the area of pronounced discontinuity, the description of wave motion is preferable using a monotone scheme, as evidenced by the results obtained $r = 0$ in (figure 3). The amplitude of false oscillations in the scheme of minimal dispersion is minimized by a special selection of the coefficient on the model problem of collision of semi-infinite plates, for which the exact analytical solution in the form of series of trigonometric functions is known [10, 11].

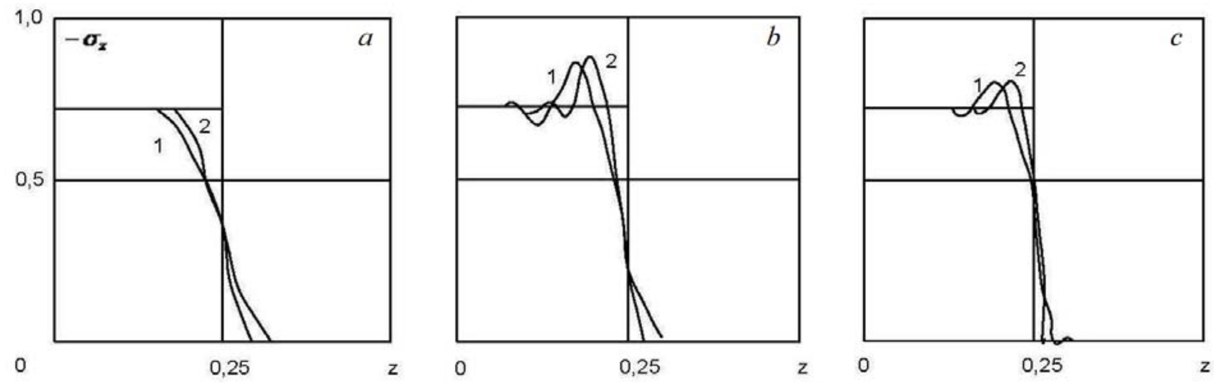

Figure 3 - Description of the wave motion is preferable using a monotone scheme, at $r = 0$

Stress profiles σ_y for dimensionless time moment $t = 0{,}5$ in sections $x:0(a)$; $0{,}2\ (б)$; $0{,}4\ (в)$ are proved in figure 4. The exact distribution is plotted by a dashed line, the solid curve refers to the scheme proposed in [1], and the dashed line refers to the scheme of the minimum dispersion. The diagrams show that the numerical solutions obtained are close enough to the exact solution, tabulation of which is also associated with some errors, which are inevitable, apparently, due to machine rounding.

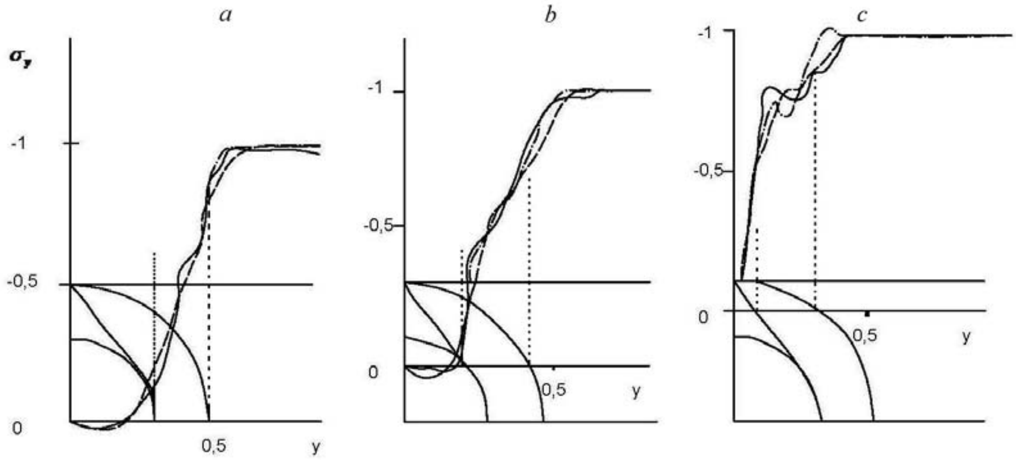

Figure 4 - Stress profiles σ_y for dimensionless instant $t = 0{,}5$ in cross sections by $x:0(a); 0{,}2(b); 0{,}4(c)$

The parametric scheme, which depends on q^* and A and includes schemes of the first and second order of accuracy, allows to calculate the desired solution depending on its local differential properties. In the region of sufficient smoothness of the solution, it is advisable to use a scheme with minimal dispersion, and in the vicinity of jumps or large gradients of the wave field - a monotone scheme. The disadvantage of using the scheme of the second order of accuracy is due to the need for special allocation of contact boundaries, near which interpolation formulas with unilateral derivatives are used. Thus, even within the framework of the approach used, when auxiliary quantities are used to construct a numerical algorithm, it is not possible to construct a homogeneous difference scheme of an increased order of accuracy. Numerous calculations carried out by the parametric scheme have shown that its stability is always provided under the following conditions

$$\tau\left(\frac{1}{\tau_r}+\frac{1}{\tau_z}+\frac{\eta_m}{2}\right)\le 1 .$$

Test calculations on grinding grids allowed us to estimate the actual rate of convergence of difference solutions and determine the optimal size of the space-time grid, which achieved acceptable accuracy with minimal machine time. The noted features of discrete solutions are characteristic and take place in the study of other dynamic contact problems with discontinuous boundary conditions.

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ЕКІӨЛШЕМДІ ТЕРМОТҰТҚЫРЛЫ-СЕРПІМДІЛІК ЕСЕБІНІҢ САНДЫҚ ШЕШІМІНІҢ ЖҮЗЕГЕ АСЫРЫЛУЫ

Аннотация. Жұмыста екіөлшемді термотұтқырлы-серпімділік толқыны сандық шешімінің жүзеге асырылуы қарастырылды. Алюминий мен темірден жасалған екіқабатты плитасы бар алюминий цилиндрінің серпінді соққысы қарастырылған.

[1] Жұмыста айырымдық схемасы мен оларды жүзеге асыру алгоритмі келтірілген. Деформацияланатын қатты денелерде өтпелі үдерістерді есептеудің негізгі әдістерінің толық шолу жағдайы [2, 3, 4] көрсетілген, онда түрлі әдістерді салыстырмалы бағалау мәселелеріне және неғұрлым ұтымды қолдану салаларын анықтауға арналған жалпылама зерттеулер қажеттілігі мен маңыздылығы айқындалған.

Жұмыста сандық нәтижелерді талдау және физикалық интерпретациялау кезінде шешімнің сапалық тәртібі туралы априорлы ақпаратты және зерттелетін құбылыстар физикасы туралы барлық ықтимал мәліметтер пайдаланылды. Есептік және эксперименталды деректерді сәйкестендіру шешім дәлдігінің неғұрлым объективті өлшемі болып саналады, оған математикалық модельдің толық адекваттық тексеру жағдайы кіреді.

Түйін сөздер: екіөлшемді термотұтқырлы-серпімді толқындар, айырымдық тізбегінің тұрақтылығы, айырымдық есеп шешімінің жинақтылығы, индентор, деформация, тензор, кернеу.

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ЧИСЛЕННАЯ РЕАЛИЗАЦИЯ РЕШЕНИЯ ДВУМЕРНЫХ ЗАДАЧ ТЕРМОВЯЗКОУПРУГОСТИ

Аннотация. В данной работе рассмотрена численная реализация двумерных термовязкоупругих волн. Рассмотрено упругое соударение алюминиевого цилиндра с двухслойной плитой из алюминия и железа.

В работе [1] приведены разностные схемы и алгоритм их реализации. Наиболее полные обзоры основных методов расчета переходных процессов в деформируемых твердых телах можно найти в [2, 3, 4], где указывается также необходимость и важность обобщающих исследований, посвященных вопросам сравнительной оценки различных методов и выявлению областей их наиболее рационального применения.

При анализе и физической интерпретации численных результатов в данной работе полезно также использовать априорную информацию о качественном поведении решения и всевозможные сведения о физике исследуемых явлений. Сопоставление расчетных и экспериментальных данных служит наиболее объективным критерием точности решения, включающем в себя и проверку адекватности математической модели явлению в целом.

Ключевые слова: двумерные термовязкоупругие волны, устойчивость разностной схемы, сходимость решения разностной задачи, индентор, деформация, тензор, напряжения.

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