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ИЗВЕСТИЯ

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EVOLUTION EQUATIONS OF THE RESTRICTED THREE-BODY PROBLEM WITH VARIABLE MASSES

Abstract. In classical celestial mechanics, the theory of perturbation based on Keplerian motion is well developed. Various kinds of perturbed motion equations are described by canonical equations, Lagrange equations and Newton's equations. The masses of bodies are assumed constant and the bodies are point or spherical. Classical perturbation theory is a basic tool in the study of many astronomical problems. Observational astronomy shows that the masses of real cosmic bodies change over time. Real celestial bodies are neither spherical nor solid. Celestial bodies are non-stationary and their masses, sizes, shapes and structures change over time as they evolve. There are now a number of studies with a community bibliography of papers on the celestial mechanics of bodies of variable mass. However, the perturbation theory for unsteady space systems is not sufficiently developed. Therefore, we considered a gravitational system consisting of three spherical celestial bodies, with spherical density distributions, with variable masses in the restricted formulation. We have considered the general case where the masses of the bodies change non-isotropically at different rates, in the presence of reactive forces. We will assume that a body with infinitesimal mass has no effect on the motions of the two main bodies. The problem was investigated by methods of perturbation theory based on aperiodic motion along the quasi-conic section developed for nonstationary gravitating systems. Evolution motion equations of a less massive body and a small body in oscillating elements, in the relative coordinate system, have been obtained.

Key words: restricted three-body problem, non-isotropic mass change, reactive forces, aperiodic motion along a quasi-conic section, perturbed motion equations in the form of Newton's equation.

1. Introduction. Modern celestial mechanics mainly applies the perturbation theory based on Keplerian motion. Classical perturbation theory is the basic tool in the study of many astronomical problems. Various kinds of perturbed motion equations are described by the canonical equations, the Lagrange and Newton equations. In this case, the masses of bodies are assumed constant, the bodies are point or spherical. However, observational astronomy shows that real celestial bodies are neither spherical nor solid. Their masses, sizes, shapes and structures change over time.

The novelty of our work is that we considered a model problem proposed as an initial approximation for the problems of celestial mechanics of bodies of variable mass in the form of Newton's equations. On the basis of this model problem, methods of perturbation theory are developed and new forms of perturbed motion equations of the restricted three-body problem with variable masses in the presence of reactive forces are obtained.

2. Problem statement and motion equations in the relative coordinate system. Consider a gravitational system consisting of three spherical celestial bodies, with spherical (or point) density distributions, with variable masses. Let, T_0 be the more massive body, T_1 the less massive body, then the main bodies. Let us denote by T_2 the body with a very small mass, then the body with a small mass. Accordingly, we denote the masses, which are functions of time and are considered known

$$m_0 = m_0(t), \qquad m_1 = m_1(t), \qquad m_2 = m_2(t). \tag{2.1}$$

Let's assume that the masses of bodies decrease due to separating particles and increase due to joining particles. In this case, in general, the relative velocity of the separating particles from the body differs from the relative velocity of the joining particles to the body. Let us consider the general case when the masses of bodies change non-isotropically at different rates, in the presence of reactive forces [1]

$$\frac{\dot m_0}{m_0} \neq \frac{\dot m_1}{m_1}, \qquad \frac{\dot m_0}{m_0} \neq \frac{\dot m_2}{m_2}, \qquad \frac{\dot m_1}{m_1} \neq \frac{\dot m_2}{m_2}. \tag{2.2}$$

Let's assume that the body with a small mass m_2, does not affect to the motion of the two main bodies with masses m_0 and m_1, and this is the statement restricted of the considered problem of three bodies with variable masses

$$m_2 << m_0, \qquad m_2 << m_1. \tag{2.3}$$

The motion equations of the two main bodies in the relative coordinate system, with the origin in the center of the body T_0 (the central body) will be written in the form

$$\ddot{\vec r}_1 + f\left(m_0 + m_1\right)\frac{\vec r_1}{r_1^3} = \vec F_1, \tag{2.4}$$

$$\vec F_1 = \vec F_1(t), \qquad r_1 = \sqrt{x_1^{\,2} + y_1^{\,2} + z_1^{\,2}}. \tag{2.5}$$

Accordingly, the motion equations of a infinitesimal mass body in the Newtonian field of attraction of two main bodies in the relative system of coordinates looks like

$$\ddot{\vec r}_2 + fm_0\frac{\vec r_2}{r_2^{\,3}} = \vec\Pi_2 + \vec F_2, \tag{2.6}$$

$$\vec\Pi_2 = grad_{\mathbf{r}_2} U, \quad U = fm_1\left(\frac{1}{r_{12}} - \frac{x_1x_2 + y_1y_2 + z_1z_2}{r_1^3}\right), \qquad r_2 = \sqrt{x_2^{\,2} + y_2^{\,2} + z_2^{\,2}}, \tag{2.7}$$

$$\vec F_2 = \vec F_2(t), \qquad r_{12} = \sqrt{\left(x_1 - x_2\right)^2 + \left(y_1 - y_2\right)^2 + \left(z_1 - z_2\right)^2} = r_{21}. \tag{2.8}$$

In equations (2.4) and (2.6) f is the gravitational constant, $\vec F_1$, $\vec F_2$ is the net reactive force.

In the orbital coordinate system the perturbing forces (2.5), (2.7), (2.8) can be written in the form

$$\vec F_1 = \vec F_1(t) = F_{1r}(t)\cdot\vec e_r + F_{1\tau}(t)\cdot\vec e_\tau + F_{1n}(t)\cdot\vec e_n, \tag{2.9}$$

$$\vec F_2 = \vec F_2(t) = F_{2r}(t)\cdot\vec e_r + F_{2\tau}(t)\cdot\vec e_\tau + F_{2n}(t)\cdot\vec e_n, \tag{2.10}$$

$$\vec\Pi_2 = \vec\Pi_2\left(t, x_1, y_1, z_1, x_2, y_2, z_2\right) = \Pi_{2r}\cdot\vec e_r + \Pi_{2\tau}\cdot\vec e_\tau + \Pi_{2n}\cdot\vec e_n, \tag{2.11}$$

where $\vec F_{ir}$ the radial (directed along the radius vector), $\vec F_{i\tau}$ transversal (perpendicular to the radius vector, lying on the plane of the instantaneous orbit and directed toward the motion side) and $\vec F_{in}$ normal (perpendicular to the plane of the instantaneous orbit) components of the reactive forces, similarly for $\Pi_{2r}, \Pi_{2\tau}, \Pi_{2n}$. The moving orthogonal trihedron of unit vectors $\vec e_r$, $\vec e_\tau$, $\vec e_n = \vec e_r \times \vec e_\tau$ form the right triplet [2]-[6].

The obtained motion equations of the restricted three-body problem with variable masses in the presence of reactive forces (2.4), (2.6) are in general very difficult. Therefore we investigate the problem by the methods of the perturbation theory based on the aperiodic motion along the quasi-conic section developed for such nonstationary gravitating systems [7].

3. Perturbation theory based on aperiodic motion along a quasi-conic section. A number of problems in celestial mechanics of bodies with variable masses can be described by equations of the form as the two-body problem [8] with variable masses in the presence of perturbing forces

$$\ddot{\vec{r}} + f\nu\frac{\vec{r}}{r^3} - \tilde{a}\dot{\vec{r}} - \tilde{b}\vec{r} = \vec{W}\,, \tag{3.1}$$

where $\vec{r} = \vec{r}(x, y, z)$ - radius vector, $\dot{\vec{r}} = \vec{\mathrm{v}}$ - velocity, $\nu = \tilde{\nu}_1(t) + \tilde{\nu}_2(t) = \nu(t)$, $\tilde{a} = \tilde{a}(t)$, $\tilde{b} = \tilde{b}(t)$ given functions of their arguments. $\vec{W} = \vec{W}(t,\ \vec{r},\ \dot{\vec{r}}) = \vec{W}(W_r, W_\tau, W_n)$ - perturbing function.

When

$$\tilde{a} = \frac{1}{2}\left(\frac{\dot{\nu}}{\nu} + \frac{\dot{\gamma}}{\gamma}\right), \qquad \tilde{b} = \frac{\ddot{\gamma}}{\gamma} - \frac{1}{2}\left(\frac{\dot{\nu}}{\nu} + \frac{\dot{\gamma}}{\gamma}\right)\frac{\dot{\gamma}}{\gamma}, \qquad \vec{W} = 0\,, \tag{3.2}$$

where $\gamma = \gamma(t)$ is an arbitrary dimensionless time function, the concrete form of it we determine depending on the concrete form of the equation (3.1) of the considered problem, we have a perturbed motion.

For our goals, the following system of oscillating elements is preferable [9]

$$a\,,\ e\,,\ i\,,\ \pi\,,\ \Omega\,,\ \lambda\,, \tag{3.3}$$

which are analogues of the semi-major axis, eccentricity, inclination, longitude of the pericenter, longitude of the ascending node, and mean longitude in the orbit of the corresponding Keplerian elements, in the case of quasi-elliptical motion *(e(t) < 1)*.

To investigate our restricted three-body problem with variable masses varying non-isotropically at different rates, in the presence of reactive forces in the relative coordinate system (2.6), we further use perturbed motion equations in Newton form

$$\dot{a} = \frac{2a^2 e\sin\theta}{p}\cdot\tilde{F}_r + \frac{2a^2}{r}\cdot\tilde{F}_\tau\,, \qquad p = a\left(1 - e^2\right) \tag{3.4}$$

$$\dot{e} = \sin\theta\cdot\tilde{F}_r + \left(\cos\theta + \frac{e + \cos\theta}{1 + e\cdot\cos\theta}\right)\cdot\tilde{F}_\tau\,, \tag{3.5}$$

$$\frac{di}{dt} = \frac{\cos u}{1 + e\cdot\cos\theta}\cdot\tilde{F}_n\,, \tag{3.6}$$

$$\dot{\pi} = -\frac{\cos\theta}{e}\cdot\tilde{F}_r + \frac{\sin\theta}{e}\left(1 + \frac{r}{\gamma p}\right)\cdot\tilde{F}_\tau + \frac{r}{\gamma p}\sin u\cdot\mathrm{tg}\frac{i}{2}\cdot\tilde{F}_n\,, \tag{3.7}$$

$$\dot{\Omega} = \frac{\sin u}{(1 + e\cos\theta)\sin i}\cdot\tilde{F}_n\,, \tag{3.8}$$

$$\dot{\lambda} = n\left(\frac{m_0}{m_{00}}\right)^2 - 2\frac{r}{\gamma p}\sqrt{1 - e^2}\cdot\tilde{F}_r + \frac{r}{\gamma p}\sin u\cdot tg\frac{i}{2}\cdot\tilde{F}_n + \frac{1}{1 + \sqrt{1 - e^2}}\left[-\cos\theta\cdot\tilde{F}_r + \left(1 + \frac{r}{\gamma p}\right)\sin\theta\cdot\tilde{F}_\tau\right], \tag{3.9}$$

where

$$\tilde{F}_r = W_r\cdot\frac{\sqrt{p}}{\sqrt{\mu_0}}\left(\frac{\gamma m_0}{m}\right)^{1/2}, \quad \tilde{F}_\tau = W_\tau\cdot\frac{\sqrt{p}}{\sqrt{\mu_0}}\left(\frac{\gamma m_0}{m}\right)^{1/2}, \quad \tilde{F}_n = W_n\cdot\frac{\sqrt{p}}{\sqrt{\mu_0}}\left(\frac{\gamma m_0}{m}\right)^{1/2}, \tag{3.10}$$

which were obtained earlier in the work [10].

4. The restricted three-body problem with variable masses changing at different rates in the presence of reactive forces in the osculating elements. Proceeding from motion equations of the problem under consideration in the relative coordinate system (2.4)-(2.6), let us rewrite them as perturbed motion equations on the basis of aperiodic motion along quasi-conic section. To simplify the perturbed motion equations, let us choose an arbitrary function $\gamma = \gamma(t)$ in formulas (3.1)-(3.10) in the following concrete form

$$\left(\frac{\dot{\nu}}{\gamma}+\frac{\dot{\gamma}}{\gamma}\right)=0, \quad \gamma=\frac{\nu(t_0)}{\nu(t)}=\gamma(t). \tag{4.1}$$

Introducing the notation $m_0 + m_1 = \nu_1$, we rewrite equations (2.4) in the form

$$\ddot{\vec{r}}_1 + f\nu_1\frac{\vec{r}_1}{r_1^3}-\frac{\ddot{\gamma}_1}{\gamma_1}\vec{r}_1=\vec{W}_1, \qquad \vec{W}_1=-\frac{\ddot{\gamma}_1}{\gamma_1}\vec{r}_1+\vec{F}_1=\vec{W}_1\left(W_{1r},W_{1\tau},W_{1n}\right), \tag{4.2}$$

$$W_{1r}=-\frac{\ddot{\gamma}_1}{\gamma_1}r_1+F_{1r}(t), \qquad W_{1\tau}=F_{1\tau}(t), \qquad W_{1n}=F_{1n}(t), \tag{4.3}$$

$$\gamma_1=\frac{\nu_1(t_0)}{\nu_1(t)}=\gamma_1(t), \tag{4.4}$$

where $\vec{W}_1\left(W_{1r},W_{1\tau},W_{1n}\right)$ - perturbing force.

Then, the perturbed motion equations of the less massive main body, as a general two-body problem with variable masses in the presence of reactive forces, on the basis of aperiodic motion along the quasi-conic section (4.2) in the osculating elements (3.3), have the form

$$\dot{a}_1=\frac{2a_1^{\,2}e_1\sin\theta_1}{p_1}\cdot W_{1r}^*+\frac{2a_1^2}{r_1}\cdot W_{1\tau}^*, \tag{4.5}$$

$$\dot{e}_1=\sin\theta_1\cdot W_{1r}^*+\left(\cos\theta_1+\frac{e_1+\cos\theta_1}{1+e_1\cos\theta_1}\right)\cdot W_{1\tau}^*, \tag{4.6}$$

$$\frac{di_1}{dt}=\frac{\cos u_1}{1+e_1\cos\theta_1}\cdot W_{1n}^*, \tag{4.7}$$

$$\dot{\pi}_1=-\frac{\cos\theta_1}{e_1}\cdot W_{1r}^*+\frac{\sin\theta_1}{e_1}\left(1+\frac{r_1}{\gamma_1 p_1}\right)\cdot W_{1\tau}^*+\frac{r_1}{\gamma_1 p_1}\sin u_1\cdot \mathrm{tg}\frac{i_1}{2}\cdot W_{1n}^*, \tag{4.8}$$

$$\dot{\Omega}_1=\frac{\sin u_1}{\left(1+e_1\cos\theta_1\right)\sin i_1}\cdot W_{1n}^*, \tag{4.9}$$

$$\begin{aligned}\dot{\lambda}_1=n_1\left(\frac{m_0+m_1}{m_{00}+m_{01}}\right)^2-2\frac{r_1}{\gamma_1 p_1}\sqrt{1-e_1^2}\cdot W_{1r}^*+\frac{r_1}{\gamma_1 p_1}\mathrm{sin}u_1\cdot \mathrm{tg}\frac{i_1}{2}\cdot W_{1n}^*+\\ +\frac{1}{1+\sqrt{1-e_1^2}}\left[-\cos\theta_1\cdot W_{1r}^*+\left(1+\frac{r_1}{\gamma_1 p_1}\right)\sin\theta_1\cdot W_{1\tau}^*\right],\end{aligned} \tag{4.10}$$

$$W_{1r}^*=W_{1r}\left(\frac{\sqrt{p_1}}{\sqrt{f\nu_{10}}}\left(\frac{\gamma_1\nu_{10}}{\nu_1}\right)^{1/2}\right), \quad W_{1\tau}^*=W_{1\tau}\left(\frac{\sqrt{p_1}}{\sqrt{f\nu_{10}}}\left(\frac{\gamma_1\nu_{10}}{\nu_1}\right)^{1/2}\right), \quad W_{1n}^*=W_{1n}\left(\frac{\sqrt{p_1}}{\sqrt{f\nu_{10}}}\left(\frac{\gamma_1\nu_{10}}{\nu_1}\right)^{1/2}\right). \tag{4.11}$$

Now consider the perturbed motion equations of a massless body in the attraction field of two main bodies

$$\ddot{\vec{r}}_2 + fm_0\frac{\vec{r}_2}{r_2^3}-\frac{\ddot{\gamma}_2}{\gamma_2}\vec{r}_2=\vec{W}_2, \qquad \vec{W}_2=-\frac{\ddot{\gamma}_2}{\gamma_2}\vec{r}_2+\vec{\Pi}_2+\vec{F}_2=\vec{W}_2\left(W_{2r},W_{2\tau},W_{2n}\right), \tag{4.12}$$

$$W_{2r}=-\frac{\ddot{\gamma}_2}{\gamma_2}r_2+\Pi_{2r}+F_{2r}(t), \qquad W_{2\tau}=\Pi_{2\tau}+F_{2\tau}(t), \qquad W_{2n}=\Pi_{2n}+F_{2n}(t), \tag{4.13}$$

$$\gamma_2=\frac{m_0(t_0)}{m_0(t)}=\gamma_2(t), \tag{4.14}$$

where $\vec{W}_2\left(W_{2r},W_{2\tau},W_{2n}\right)$ - perturbing force.

Accordingly, the motion equations of a infinitesimal mass body (4.12) in the osculating elements (3.3) have the form

$$\dot{a}_2 = \frac{2a_2^2 e_2 \sin\theta_2}{p_2} \cdot W_{2r}^* + \frac{2a_2^2}{r_2} \cdot W_{2\tau}^*, \tag{4.15}$$

$$\dot{e}_2 = \sin\theta_2 \cdot W_{2r}^* + \left(\cos\theta_2 + \frac{e_2 + \cos\theta_2}{1 + e_2 \cos\theta_2} \right) \cdot W_{2\tau}^*, \tag{4.16}$$

$$\frac{di_2}{dt} = \frac{\cos u_2}{1 + e_2 \cos\theta_2} \cdot W_{2n}^*, \tag{4.17}$$

$$\dot{\pi}_2 = -\frac{\cos\theta_2}{e_2} \cdot W_{2r}^* + \frac{\sin\theta_2}{e_2}\left(1 + \frac{r_2}{\gamma_2 p_2}\right) \cdot W_{2\tau}^* + \frac{r_2}{\gamma_2 p_2} \sin u_2 \cdot \mathrm{tg}\frac{i_2}{2} \cdot W_{2n}^*, \tag{4.18}$$

$$\dot{\Omega}_2 = \frac{\sin u_2}{\left(1 + e_2 \cos\theta_2\right) \sin i_2} \cdot W_{2n}^*, \tag{4.19}$$

$$\begin{aligned} \dot{\lambda}_2 = n_2 \left(\frac{m_0}{m_{00}} \right)^2 - 2\frac{r_2}{\gamma_2 p_2}\sqrt{1 - e_2^2} \cdot W_{2r}^* + \frac{r_2}{\gamma_2 p_2} \sin u_2 \cdot tg\frac{i_2}{2} \cdot W_{2n}^* + \\ + \frac{1}{1 + \sqrt{1 - e_2^2}} \left[-\cos\theta_2 \cdot W_{2r}^* + \left(1 + \frac{r_2}{\gamma_2 p_2}\right) \sin\theta_2 \cdot W_{2\tau}^* \right], \end{aligned} \tag{4.20}$$

$$\begin{aligned} W_{2r}^* = W_{2r} \left(\frac{\sqrt{p_2}}{\sqrt{fm_{00}}} \left(\frac{\gamma_2 m_{00}}{m_0} \right)^{1/2} \right), \qquad W_{2\tau}^* = W_{2\tau} \left(\frac{\sqrt{p_2}}{\sqrt{fm_{00}}} \left(\frac{\gamma_2 m_{00}}{m_0} \right)^{1/2} \right), \\ W_{2n}^* = W_{2n} \left(\frac{\sqrt{p_2}}{\sqrt{fm_{00}}} \left(\frac{\gamma_2 m_{00}}{m_0} \right)^{1/2} \right). \end{aligned} \tag{4.21}$$

5. Averaging and obtaining evolution equations. The evolution equations of a less massive body has the form

$$\dot{a}_1^{\sec} = 2a_1 \cdot F_{1\tau}(t) \left(\frac{\sqrt{p_1}}{\sqrt{f\nu_{10}}} \left(\frac{\gamma_1 \nu_{10}}{\nu_1} \right)^{1/2} \right), \tag{5.1}$$

$$\dot{e}_1^{\sec} = e_1 \frac{a_1}{p_1} \left(\frac{5 + e_1^2}{2} \right) \left(\frac{\sqrt{p_1}}{\sqrt{f\nu_{10}}} \left(\frac{\gamma_1 \nu_{10}}{\nu_1} \right)^{1/2} \right) F_{1\tau}(t), \tag{5.2}$$

$$\frac{di_1^{\sec}}{dt} = -e_1 \cos\omega_1 \frac{3a_1}{2} \left(\frac{\sqrt{p_1}}{\sqrt{f\nu_{10}}} \left(\frac{\gamma_1 \nu_{10}}{\nu_1} \right)^{1/2} \right) F_{1n}(t), \tag{5.3}$$

$$\dot{\Omega}_1^{\sec} = -e_1 \frac{3a_1}{2} \frac{\sin\omega_1}{\sin i_1} F_{1n}(t) \left(\frac{\sqrt{p_1}}{\sqrt{f\nu_{10}}} \left(\frac{\gamma_1 \nu_{10}}{\nu_1} \right)^{1/2} \right), \tag{5.4}$$

$$\dot{\pi}_1^{\sec} = \left(-\frac{3a_1}{2\gamma_1} \left(\ddot{\gamma}_1 + e_1 \frac{\sin\omega_1}{p_1} \cdot \mathrm{tg}\frac{i_1}{2} F_{1n}(t) \right) - \left(1 + \frac{e_1^2}{2} \right) \frac{a_1}{\gamma_1 p_1} F_{1\tau}(t) + F_{1r}(t) \right) \left(\frac{\sqrt{p_1}}{\sqrt{f\nu_{10}}} \left(\frac{\gamma_1 \nu_{10}}{\nu_1} \right)^{1/2} \right), \tag{5.5}$$

$$\dot{\lambda}_1^{\sec} = n_1\left(\frac{m_0+m_1}{m_{00}+m_{01}}\right)^2 - \left[\left(2+3e_1^2\right)\sqrt{1-e_1^2}\,\frac{\ddot{\gamma}_1}{\gamma_1^2}\frac{a_1^2}{p_1} - \right.$$

$$-\left(\left(2+e_1^2\right)\frac{a_1}{\gamma_1 p_1}\sqrt{1-e_1^2} - \frac{e_1}{1+\sqrt{1-e_1^2}}\right)F_{1r}(t) - \tag{5.6}$$

$$\left.-\frac{3}{2}\frac{a_1e_1}{\gamma_1}\left(\frac{\ddot{\gamma}_1}{1+\sqrt{1-e_1^2}} + \frac{\sin\omega_1}{p_1}\cdot\mathrm{tg}\frac{i_1}{2}F_{1n}(t)\right)\right]\left(\frac{\sqrt{p_1}}{\sqrt{f\nu_{10}}}\left(\frac{\gamma_1\nu_{10}}{\nu_1}\right)^{1/2}\right).$$

Now let us consider the evolution equations of a infinitesimal mass body. It is known that to calculate the secular perturbations of the first order, the perturbations generated by each force can be calculated separately, then they should be summed up.

Therefore, the secular perturbations generated by the Newtonian gravitational force

$$\mathbf{\Pi}_2 = grad_{\mathbf{r}_2}U, \qquad U = fm_1\left(\frac{1}{r_{12}} - \frac{x_1x_2+y_1y_2+z_1z_2}{r_1^3}\right), \qquad r_2 = \sqrt{x_2^{\,2}+y_2^{\,2}+z_2^{\,2}}, \tag{5.7}$$

can be calculated separately. In our problem, as in the classical restricted three-body problem, the secular perturbations are generated only by the main part of the perturbative function of Newtonian gravity (5.7)

$$U = fm_1\left(\frac{1}{r_{12}}\right) = U_{prim.} = fm_1\left(\frac{1}{\Delta_{12}}\right), \qquad \Delta_{12} = r_{12}. \tag{5.8}$$

Averaging the expressions (5.8) by λ_1 we get

$$U_{\lambda_1}^{\sec} = \tilde{U} = \frac{fm_1}{2\pi}\int_0^{2\pi}\frac{d\lambda_1}{\Delta_{12}}. \tag{5.9}$$

Then we can write

$$\tilde{\Pi}_{2r} = \frac{\partial\tilde{U}}{\partial\mathbf{e}_r} \qquad \tilde{\Pi}_{2\tau} = \frac{\partial\tilde{U}}{\partial\mathbf{e}_\tau} \qquad \tilde{\Pi}_{2n} = \frac{\partial\tilde{U}}{\partial\mathbf{e}_n}, \tag{5.10}$$

in the right side of it is necessary to calculate the derivatives along the directions of $\mathbf{e}_r$, $\mathbf{e}_\tau$, $\mathbf{e}_n$ from the scalar quantity (5.9).

In the adopted notations (5.9), (5.10), the equations of evolution equations for the analog of the semi-major axis looks like

$$\dot{a}_2^{\sec} = \frac{1}{2\pi}\int_0^{2\pi}\left\{\frac{2a_2^2e_2}{p_2}\sin\theta_2\left[-\frac{\ddot{\gamma}_2}{\gamma_2}r_2 + F_{2r}(t)\right]\left(\frac{\sqrt{p_2}}{\sqrt{fm_{00}}}\left(\frac{\gamma_2m_{00}}{m_0}\right)^{1/2}\right)+\right.$$

$$\left.+\frac{2a_2^2}{r_2}F_{2\tau}(t)\left(\frac{\sqrt{p_2}}{\sqrt{fm_{00}}}\left(\frac{\gamma_2m_{00}}{m_0}\right)^{1/2}\right)\right\}d\lambda_2 + \tag{5.11}$$

$$+\left(\frac{\sqrt{p_2}}{\sqrt{fm_{00}}}\left(\frac{\gamma_2m_{00}}{m_0}\right)^{1/2}\right)\frac{1}{2\pi}\int_0^{2\pi}\left\{\frac{2}{n_2a_2}\left(\frac{n_2a_2^2e_2}{\left(1-e_2^2\right)}\sin\theta_2\left[\tilde{\Pi}_{2r}\right] + \frac{n_2a_2^3}{r_2}\left[\tilde{\Pi}_{2\tau}\right]\right)\right\}d\lambda_2.$$

The expression (5.9) can be interpreted, according to the Gaussian interpretation, as the potential of a quasi-elliptic spiral with parameters $(a_1,\ e_1,\ i_1,\ \Omega_1,\ \pi_1)$ and with mass m_1 per outer point T_2.

Let us show that in equation (5.11) the expression in the curly bracket in the second summand is zero.

It is true that in deriving the Lagrange perturbed motion equations from Newton's equation, for secular perturbations generated by the Newtonian gravitational force, the well-known relation takes place

$$\frac{\partial U}{\partial \lambda_2} = +\frac{n_2 a_2^2 e_2 \sin\theta_2}{1-e^2}\cdot \varPi_{2r} + \frac{n_2 a_2^3}{r_2}\cdot \varPi_{2\tau}. \tag{5.12}$$

In the left part of equation (5.12), after averaging over λ_2, it is clear that

$$\frac{\partial \tilde{U}}{\partial \lambda_2} = 0. \tag{5.13}$$

Therefore, at calculating the first-order secular perturbations, in the perturbed motion equations in the form of Newton's equations (5.11), it takes place

$$\frac{1}{2\pi}\int_0^{2\pi}\left\{\frac{n_2 a_2^2 e_2 \sin\theta_2}{1-e^2}\cdot \tilde{\varPi}_{2r} + \frac{n_2 a_2^3}{r_2}\cdot \tilde{\varPi}_{2\tau}\right\} d\lambda_2 = 0. \tag{5.14}$$

Therefore, the second component in equation (5.11) is zero

$$\left(\frac{\sqrt{p_2}}{\sqrt{fm_{00}}}\left(\frac{\gamma_2 m_{00}}{m_0}\right)^{1/2}\right)\frac{2}{n_2 a_2}\left\{\frac{1}{2\pi}\int_0^{2\pi}\left[\frac{n_2 a_2^2 e_2}{\left(1-e_2^2\right)}\sin\theta_2\left[\varPi_{2r}\right] + \frac{n_2 a_2^3}{r_2}\left[\varPi_{2\tau}\right]\right] d\lambda_2\right\} = 0.$$

Thus, equation (5.11) has the form

$$\begin{aligned}\dot{a}_2^{\sec} = \frac{1}{2\pi}\int_0^{2\pi}\Bigg\{&\frac{2a_2^2 e_2}{p_2}\sin\theta_2\left(-\frac{\ddot{\gamma}_2}{\gamma_2}r_2 + F_{2r}(t)\right)\left(\frac{\sqrt{p_2}}{\sqrt{fm_{00}}}\left(\frac{\gamma_2 m_{00}}{m_0}\right)^{1/2}\right) + \\ &+\frac{2a_2^2}{r_2}F_{2\tau}(t)\left(\frac{\sqrt{p_2}}{\sqrt{fm_{00}}}\left(\frac{\gamma_2 m_{00}}{m_0}\right)^{1/2}\right)\Bigg\} d\lambda_2.\end{aligned} \tag{5.15}$$

Expanding the integrals into series according to the formulas of unperturbed motion up to the second degree of eccentricity, and averaging by λ_2 from equation (5.15) we get

$$\dot{a}_2^{\sec} = 2F_{2\tau}(t)\left(1+\frac{e_2^2}{2}\right)\left(\frac{\sqrt{p_2}}{\sqrt{fm_{00}}}\left(\frac{\gamma_2 m_{00}}{m_0}\right)^{1/2}\right). \tag{5.16}$$

By averaging, we obtain evolutionary equations for the other osculating elements

$$\begin{aligned}\dot{e}_2^{\sec} = \Bigg[&-a_2\frac{\ddot{\gamma}_2}{\gamma_2} - e_2\left(\tilde{\varPi}_{2\tau}^{\sec} + F_{2\tau}(t)\right) + e_2\frac{a_2}{p_2}\left(1+\frac{e_2^2}{2}\right)\left(\tilde{\varPi}_{2\tau}^{\sec} + F_{2\tau}(t)\right) - \\ &-e_2\frac{3a_2}{2p_2}\left(\tilde{\varPi}_{2\tau}^{\sec} - F_{2\tau}(t)\right)\Bigg]\left(\frac{\sqrt{p_2}}{\sqrt{fm_{00}}}\left(\frac{\gamma_2 m_{00}}{m_0}\right)^{1/2}\right),\end{aligned} \tag{5.17}$$

$$\frac{di_2^{\sec}}{dt} = -\frac{3}{2}\frac{a_2}{p_2}e_2\cos\omega_2\left(\tilde{\varPi}_{2n}^{\sec} + F_{2n}(t)\right)\left(\frac{\sqrt{p_2}}{\sqrt{fm_{00}}}\left(\frac{\gamma_2 m_{00}}{m_0}\right)^{1/2}\right), \tag{5.18}$$

$$\dot{\Omega}_2^{\sec} = -\frac{3}{2}\frac{a_2}{p_2}\frac{\sin\omega_2}{\sin i_2}e_2\left(\tilde{\varPi}_{2n}^{\sec} + F_{2n}(t)\right)\left(\frac{\sqrt{p_2}}{\sqrt{fm_{00}}}\left(\frac{\gamma_2 m_{00}}{m_0}\right)^{1/2}\right), \tag{5.19}$$

$$\begin{aligned}\dot{\pi}_2^{\sec} = \Bigg[&-a_2\frac{3}{2}\left(\frac{\ddot{\gamma}_2}{\gamma_2} + \frac{e_2}{\gamma_2 p_2}\,\mathrm{tg}\frac{i_2}{2}\sin\omega_2\cdot F_{2n}(t)\right) - \tilde{\varPi}_{2r}^{\sec} - \\ &-F_{2r}(t) - \frac{a_2 e_2}{\gamma_2 p_2}\,\mathrm{tg}\frac{i_2}{2}\sin\omega_2\frac{3}{2}\tilde{\varPi}_{2n}^{\sec}\Bigg]\left(\frac{\sqrt{p_2}}{\sqrt{fm_{00}}}\left(\frac{\gamma_2 m_{00}}{m_0}\right)^{1/2}\right),\end{aligned} \tag{5.20}$$

$$\dot{\lambda}_2^{\sec} = n_2 \left(\frac{m_0}{m_{00}} \right)^2 + \left[\frac{2a_2^2 \ddot{\gamma}_2}{p_2 \gamma_2^2} \left(1 + \frac{3}{2} e_2^2 \right) \sqrt{1 - e_2^2} - \right.$$
$$- \left(1 + \frac{e_2^2}{2} \right) \frac{2a_2}{\gamma_2 p_2} \sqrt{1 - e_2^2} \left(1 + F_{2r}(t) \right) - \frac{a_2 e_2}{\gamma_2 p_2} \cdot tg \frac{i_2}{2} \cdot \sin \omega_2 \frac{3}{2} \tilde{\Pi}_{2n}^{\sec} +$$
$$+ \frac{a_2}{\gamma_2 p_2} \left(tg \frac{i_2}{2} \cdot F_{2n}(t) \cos \omega_2 + \frac{1}{1 + \sqrt{1 - e_2^2}} F_{2\tau}(t) \right) - \tag{5.21}$$
$$- \frac{3e_2 a_2}{2\gamma_2} \left(\frac{\sin \omega_2}{p_2} \cdot tg \frac{i_2}{2} \cdot F_{2n}(t) + \frac{\ddot{\gamma}_2}{1 + \sqrt{1 - e_2^2}} \right) +$$
$$\left. + \frac{e_2}{1 + \sqrt{1 - e_2^2}} \left(\tilde{\Pi}_{2r}^{\sec} + F_{2r}(t) \right) \right] \left(\frac{\sqrt{p_2}}{\sqrt{f m_{00}}} \left(\frac{\gamma_2 m_{00}}{m_0} \right)^{1/2} \right),$$

where

$$\frac{1}{2\pi} \int_0^{2\pi} \tilde{\Pi}_{2\tau} d\lambda_2 = \tilde{\Pi}_{2\tau}^{\sec}, \qquad \frac{1}{2\pi} \int_0^{2\pi} \tilde{\Pi}_{2n} d\lambda_2 = \tilde{\Pi}_{2n}^{\sec}, \qquad \frac{1}{2\pi} \int_0^{2\pi} \tilde{\Pi}_{2r} d\lambda_2 = \tilde{\Pi}_{2r}^{\sec}. \tag{5.22}$$

Conclusion. We obtained the evolution equations of motion of a less massive body in osculating elements. Evolution equations of a infinitesimal mass body in osculating elements are obtained in principle, in fact it is necessary, with the necessary exactitude, to give a expansion of the Newtonian force function. Such expansion of the perturbative function of the Newtonian gravitational force will be obtained using the Wolfram Mathematica package. The evolution equations are suitable for the general case, i.e. when the masses of bodies change non-isotropically at different rates (both mass growth and mass decrease), in the presence of reactive forces.

After the expansion of the perturbative function generated by the Newtonian gravitational force, we further investigate it numerically, using Wolfram Mathematica mathematical package. The resulting equations will be suitable not only for planetary systems, but also for binary stars.

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МАССАЛАРЫ АЙНЫМАЛЫ ШЕКТЕЛГЕН ҮШ ДЕНЕ МӘСЕЛЕСІНІҢ ЭВОЛЮЦИЯЛЫҚ ТЕҢДЕУЛЕРІ

Аннотация. Классикалық аспан механикасында кеплер қоғалысы негізінде ұйытқу теориясы жақсы дамыған. Ұйытқыған қозғалыс теңдеулерінің түрлері конондық теңдеулермен, Лагранж және Ньютон теңдеулерімен сипатталған. Сонымен қатар массалары тұрақты денелердің, нүкте ретінде немесе сфералық дене ретінде қарастырылады. Көптеген астрономиялық мәселелерді зерттеуде классикалық ұйытқу теориясы басты құрал болып табылады. Заманауи бақылау астрономиясы нақты ғарыштық денелердің массалары уақыт өте өзгеретінін растауда. Шынайы ғарыштық денелер сфералық емес және қатты дене емес. Аспан денелері бейстационар, эволюция кезеңінде олардың массалары, өлшемдері, пішіндері және құрылымы өзгереді. Қазіргі таңда айнымалы массалы аспан механикасы бірқатар бойынша ауқымды зерттеулер библиографиялық жұмыстары бар. Алайда, бейстационар ғарыштық жүйелер үшін ұйытқу теориясы жеткілікті дамымаған. Осыған орай біз шектелген қойылымдағы массалары айнымалы, тығыздығы сфера бойынша үлестірілген гравитацияланушы үш сфералы аспан денелерінен тұратын жүйені қарастырдық. Сонымен қатар, біз реактивті күші бар денелердің массалары изотропты емес әр түрлі қарқында өзгеретін жалпы жағдайды қарастырдық. Массасы аз дене екі негізгі денелердің қозғалысына әсер етпейді деп есептейміз. Аталған мәселе бейстационар гравитацияланатын жүйелер үшін даярланған квазиконустық қима бойынша апериодты қозғалыс негізіндегі ұйытқыған теория әдістерімен зерттелді. Салыстырмалы координат жүйесіндегі массивті дене мен массасы аз дененің оскуляциялаушы элементтерінің эволюциялық теңдеулері алынды.

Түйін сөздер: шектелген үш дене мәселесі, массаның изотропты емес өзгеруі, реактивті күштер, квазиконустық қима бойынша апериодты қозғалыс, Ньютон теңдеулері түріндегі ұйытқыған қозғалыс теңдеулері.

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ЭВОЛЮЦИОННЫЕ УРАВНЕНИЯ ОГРАНИЧЕННОЙ ЗАДАЧИ ТРЕХ ТЕЛ С ПЕРЕМЕННЫМИ МАССАМИ

Аннотация. В классической небесной механике хорошо развита теория возмущения на базе кеплеровского движения. Различные виды уравнения возмущенного движения описаны каноническими уравнениями, уравнениями Лагранжа и уравнениями Ньютона. При этом массы тел считается постоянными, тела точечными или сферическими. Классическая теория возмущения является основным инструментом в изучении многих астрономических проблем. Современная наблюдательная астрономия свидетельствует о том, что массы реальных космических тел со временем меняются. Реальные небесные тела не сферичные и не твердые. Небесные тела нестационарные, в процессе эволюции меняются их массы, размеры, формы и структуры. В настоящее время существует ряд исследований с обширной библиографией работ по небесной механике тел переменной массы. Однако теория возмущения для нестационарных космических систем не достаточна развита. В связи с этим мы расмотрели гравитирующую систему состоящийся из трех сферических небесных тел, со сферическими распределениями плотностей, с переменными массами в ограниченной постановке. Мы рассмотрели общий случай, когда массы тел изменяются не изотропно в различных темпах, при наличии реактивных сил. Будем считать, что тело с бесконечно малой массой не влияет на движения двух основных тел. Проблема исследована методами теории возмущении на основе апериодического движения по квазиконическому сечению, разработанных для нестационарных гравитирующих систем. Получены эволюционные уравнения движения менее массивного тела и малого тела в оскулирующих элементах, в относительной системе координат.

Ключевые слова: ограниченная задача трех тел, не изотропное изменения масс, реактивные силы, апериодическое движение по квазиконическому сечению, уравнения возмущенного движения в форме уравнения Ньютона.

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