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SOLITON SOLUTIONS FOR THE (2+1)-DIMENSIONAL INTEGRABLE FOKAS-LENELLS EQUATION

Abstact. Studying of solitons led to the discovery of a number of new directions related to it. There is interest in which is also enhanced in connection with the discovery of new examples in which soliton processes are manifested. The number and variety of nonlinear equations containing solitons as the most interesting solutions significantly increase due to generalizations to the two-dimensional and three-dimensional cases. Such popular transformations as Darboux, Backlund and Hirota's bilinear method are often used to find exact different kind of the solutions of nonlinear equations.

In the present paper, we present Lax pair of the (2+1)-dimensional integrable Fokas-Lenells equation. The bilinear form of the (2+1)-dimensional integrable Fokas-Lenells equation was obtained by the Hirota's bilinear method. By using Hirota's bilinear method, we construct exact one-soliton and two-soliton solutions of the (2+1)-dimensional Fokas-Lenells equation. The graphics of the obtained solutions are presented. The obtained new results have important physical applications.

Keywords: Hirota method, Lax representation, soliton solution, Fokas-Lenells equation.

Introduction. In many areas of science, the object of intensive theoretical and experimental research is soliton which is mean the "solitary" wave (solitary wave). Soliton can be used to transmit information, where the main idea is to use in each bit interval to represent units in the stream of binary signals. Mathematically, solitons are localized stationary solutions of nonlinear partial differential equations or their generalizations.

One of the generalization of nonlinear Schrödinger is the integrable (1+1)-dimensional Fokas-Lenells equation (FL) [1, 2] which was proposed by J. Fokas and A.S. Lenells. At present, studying multidimensional integrable systems containing derivatives in more than two variables is very interesting and relevant and namely finding exact soliton solutions. In this regard, by considering the (1+1)-dimensional FL equation, we obtain the (2+1)-dimensional FL equation and present its Lax pair [1, 2].

The paper is organized as follows. In Section 2, we present Lax representation of the (2+1)-dimensional FL equation. Hirota method is presented in Section 3. Namely, we apply Hirota method for FL equation and find one soliton and two soliton solutions by obtained bilinear equation. In Section 4, we give conclusion.

Lax representation. The (2+1)-dimensional FL equation is given by next form

$$iq_{xt} - iq_{xy} + 2q_x - q_x|q|^2 + iq = 0, \tag{1}$$

where q is the complex shell of the field, the indices *x, y* and *t* denote the partial derivatives with respect to the arguments *x, y* and *t*, and i is the complex number.

To construct solutions of differential equations, a number of conditions must be fulfilled, one of which is to satisfy the compatibility condition [1]. The studied equation (1) satisfies the "compatibility condition" and has the following has a Lax representation

$$\Psi_x = U\Psi, \qquad U = -i\lambda^2\sigma_3 + \lambda Q,$$

$$\Psi_t = \Psi_y + W\Psi, \qquad W = W_0 + \frac{1}{\lambda}W_{-1} - \frac{i}{4\lambda^2}\sigma_3,$$

where Ψ = Ψ (x,t) is a 2×2 matrix-valued eigenfunction, λ is an isospectral parameter, and matrices are given in the form:

$$Q = \begin{pmatrix} 0 & q_x \\ r_x & 0 \end{pmatrix}, \quad W_0 = i\sigma_3 - \frac{iqr}{2}\sigma_3, \quad W_{-1} = \frac{i}{2}\begin{pmatrix} 0 & q \\ -r & 0 \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

Hirota's bilinear method. Soliton solutions of the (2+1)-dimensional FL equation will be constructed by the so-called bilinear Hirota's method. The outline for constructing soliton solutions by this method is as follows:

1. The first we apply the dependent variable transformation for FL equation in order to obtain its bilinear form.
2. The second we consider the formal series of perturbation theory.
3. The third we build multi soliton solutions.

One-soliton solution of the (2+1)-dimensional FL equation. In order to construct soliton solutions of equation (1), we use the bilinear form of FL equation has the form [6] $q = \frac{g}{f}$. Then the bilinear form of the FL equation

$$iD_xD_t(g\cdot f) - iD_xD_y(g\cdot f) + 2D_x(g\cdot f) + igf = 0, \tag{2a}$$

$$iD_xD_t(f\cdot f^*) - iD_xD_y(f\cdot f^*) - \frac{1}{2}D_x(g\cdot g^*) = 0, \tag{2б}$$

$$iD_t(f\cdot f^*) - iD_y(f\cdot f^*) - \frac{1}{2}g\cdot g^* = 0, \tag{2в}$$

$$iD_t(f\cdot f_x^*) - iD_y(f\cdot f_x^*) - \frac{1}{2}g\cdot g_x^* = 0, \tag{2г}$$

where g is complex function, f is real one, the "*" sign means complex conjugation and D_x, D_y, D_t, are bilinear differential operators which defined by

$$D_x^m D_t^n f\cdot f^* = \left(\frac{\partial}{\partial x} - \frac{\partial}{\partial x'}\right)^m \left(\frac{\partial}{\partial t} - \frac{\partial}{\partial t'}\right)^n f(x,t)f^*(x',t')|_{x'=x,t'=t},$$

where x', t' as two formal variables, $f(x,t)$ and $f^*(x',t')$ - two functions, m and n - two non-negative integers.

Then, we expand the functions g and f in bilinear equation (2) with respect to small parameter ε as follows [2]:

$$g(x,y,t) = \varepsilon g_1(x,y,t) + \varepsilon^3 g_3(x,y,t) + \ldots, \tag{3a}$$

$$f(x,y,t) = 1 + \varepsilon^2 f_2(x,y,t) + \varepsilon^4 f_4(x,y,t) + \ldots, \tag{3б}$$

where g_j is complex function, f_n is real ones (j = 1,3,5,...; n = 2,4,6,...).

In the case of a one-soliton solution of equation (1), the formal parameter in equation (3) is taken as $\varepsilon = 1$ and $j = 1,\ n = 2$, i.e.

$$g = \varepsilon g_1, \tag{4a}$$

$$f = 1 + \varepsilon^2 f_2 \tag{4б}$$

and we find the solution according to the following statement:

$$q = \frac{g}{f} = \frac{g_1}{1 + f_2}, \tag{5}$$

where $g_1 = e^{\theta_1}$, $\theta_1 = a_1 x + b_1 t + c_1 y + d_1$ and $\theta_1^* = a_1^* x + b_1^* t + c_1^* y + d_1^*$, here a_1, b_1, c_1, d_1 - complex constants.

Substituting expression (4) into equation (2) and collecting in powers of the parameter ε, we get next system

$$\varepsilon^1 \ : \ (iD_x D_t - iD_x D_y + 2D_x)(g_1 \cdot 1) + ig_1 = 0, \tag{6}$$

$$\varepsilon^3 \ : \ (iD_x D_t - iD_x D_y + 2D_x)(g_1 \cdot f_2) + ig_1 f_2 = 0, \tag{7}$$

$$\varepsilon^2 \ : \ i(D_x D_t - D_x D_y)(f_2 + f_2^*) - \frac{1}{2} D_x (g_1 \cdot g_1^*) = 0, \tag{8}$$

$$\varepsilon^4 \ : \ iD_x D_t (f_2 \cdot f_2^*) - iD_x D_y (f_2 \cdot f_2^*) = 0, \tag{9}$$

$$\varepsilon^2 \ : \ i(D_t - D_y)(f_2 - f_2^*) - \frac{1}{2} g_1 \cdot g_1^* = 0, \tag{10}$$

$$\varepsilon^4 \ : \ i(D_t - D_y)(f_2 \cdot f_2^*) = 0, \tag{11}$$

$$\varepsilon^2 \ : \ (-iD_x D_t + iD_x D_y)(f_2^* \cdot 1) - \frac{1}{2} g_1 g_{1x}^* = 0, \tag{12}$$

$$\varepsilon^4 \ : \ i(D_x - D_y)(f_2 \cdot f_{2x}^*) = 0. \tag{13}$$

Now, applying the properties of the Hirota operator to equations (6) - (13), we obtain

$$\varepsilon^1 \ : \ ig_{1tx} - ig_{1yx} + 2g_{1x} + ig_1 = 0, \tag{14}$$

$$\begin{aligned} \varepsilon^3 \ : \ & ig_{1tx} f_2 - ig_{1t} f_{2x} - ig_{1x} f_{2t} + ig_1 f_{2tx} - ig_{1yx} f_2 + +ig_{1y} f_{2x} + \\ & + ig_{1x} f_{2y} - ig_1 f_{2yx} + 2g_{1x} f_2 - 2g_1 f_{2x} + ig_1 f_2 = 0 \end{aligned} \tag{15}$$

$$\varepsilon^2 \ : \ if_{2tx} + if_{2tx}^* - if_{2yx} - if_{2yx}^* - \frac{1}{2} g_{1x} g_1^* + \frac{1}{2} g_1 g_{1x}^* = 0, \tag{16}$$

$$\begin{aligned} \varepsilon^4 \ : \ & if_{2tx} f_2^* - if_{2t} f_{2x}^* - if_{2x} f_{2t}^* + if_2 f_{2tx}^* - if_{2yx} f_2^* + \\ & + if_{2y} f_{2x}^* + if_{2x} f_{2y}^* - if_2 f_{2yx}^* = 0, \end{aligned} \tag{17}$$

$$\varepsilon^2 \quad : \quad if_{2t} - if_{2t}^* - if_{2y} + if_{2y}^* - \frac{1}{2} g_1 g_1^* = 0, \tag{18}$$

$$\varepsilon^4 \quad : \quad if_{2t} f_2^* - if_2 f_{2t}^* - if_{2y} f_2^* + if_2 f_{2y}^* = 0, \tag{19}$$

$$\varepsilon^2 \quad : \quad -if_{2xt}^* + if_{2xy}^* - \frac{1}{2} g_1 g_{1x}^* = 0, \tag{20}$$

$$\varepsilon^4 \quad : \quad if_{2t} f_{2x}^* - if_2 f_{2xt}^* - if_{2y} f_{2x}^* + if_2 f_{2xy}^* = 0. \tag{21}$$

By solving the system of equation (14)-(21) we can get from equation (14)

$$a_1 = \frac{1}{c_1 - b_1 + 2i}, \tag{22}$$

the equation (15) gives conjugation form of a_1

$$a_1^* = \frac{1}{c_1^* - b_1^* - 2i}, \tag{23}$$

and from equations (16) - (21) we obtain

$$f_2 = \frac{ia_1^2 a_1^*}{2(a_1 + a_1^*)^2} e^{\theta_1 + \theta_1^*}. \tag{24}$$

Then by substituting equation (24) into equation (5), we obtain the one-soliton solution of the (2+1)-dimensional FL equation, which is

$$q = \frac{e^{\theta_1}}{1 + \frac{ia_1^2 a_1^*}{2(a_1 + a_1^*)^2} e^{\theta_1 + \theta_1^*}} = \frac{2(a_1 + a_1^*)^2}{2(a_1 + a_1^*)^2 e^{-\theta_1} + ia_1^2 a_1^* e^{\theta_1^*}}, \tag{25}$$

or if rewrite equation (25), with $\theta_1 = \kappa_1 + \chi_1$, then we get

$$q = \frac{4\alpha_1^2 e^{i\chi_1}}{4\alpha_1^2 e^{-\kappa_1} + \frac{1}{2}(\alpha_1^2 + \beta_1^2)(i\alpha_1 - \beta_1) e^{\kappa_1}}, \tag{26}$$

where

$$\chi_1 = \beta_1 x + \nu_1 t + \tau_1 y + n_1, \quad \kappa_1 = \alpha_1 x + \mu_1 t + \sigma_1 y + m_1$$

with

$$a_1 = \alpha_1 + i\beta_1, \;\; b_1 = \mu_1 + i\nu_1, \;\; c_1 = \sigma_1 + i\tau_1, \;\; d_1 = m_1 + in_1.$$

Finally, the equation (26) has the form

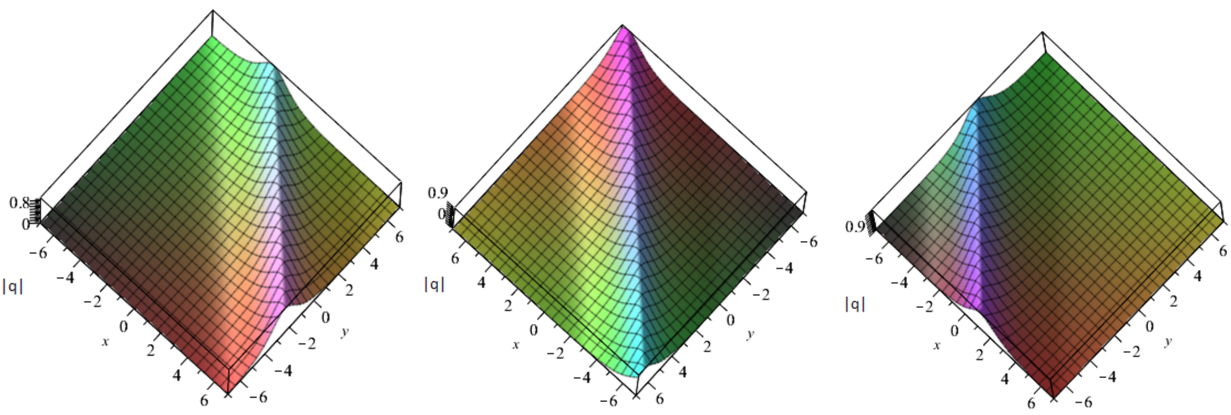
$$q = \frac{p^2 e^{i\chi_1}}{p\sqrt{h}\left(\frac{\sqrt{h}}{p}e^{\kappa_1} + \frac{p}{\sqrt{h}}e^{-\kappa_1}\right)}, \tag{27}$$

where $p = 2\alpha_1$ and $h = \frac{i}{2}(\alpha_1^2 + \beta_1^2)\alpha_1 - \frac{1}{2}(\alpha_1^2 + \beta_1^2)\beta_1$.

In addition, when $\frac{\sqrt{h}}{p} = e_1^{\delta}$ and $\frac{p}{\sqrt{h}} = e^{-\delta_1}$ the equation (27) can be rewritten as

$$q = \frac{pe^{i\chi_1}}{2\sqrt{h}} sech(\kappa_1 + \delta_1) = \frac{pe^{i\chi_1}}{2\sqrt{h}\cosh(\kappa_1 + \delta_1)}. \tag{28}$$

So, the equation (28) is one-soliton solution to the (2+1)-dimensional FL equation. Plot of the one-soliton solution is presented in Figure 1.



(a) (b) (c)

Figure 1 - Dynamics of the one-soliton solution with next parameters:
$a_1 = 1-i$, $c_1 = 1-i$, $b_1 = 1-i$ and $d_1 = 1-i$. $t = -5$ (a); t = 0 (b); t = 5 (c).

Two-soliton solution of the (2+1)-dimensional FL equation. To find the two-soliton solution of equation (1), the formal parameter in equation (3) is taken, as $\varepsilon = 1$ and $j = 1,3,\ n = 2,4$, i.e.

$$\varepsilon^1 \quad : \quad (iD_xD_t - iD_xD_y + 2D_x)(g_1 \cdot 1) + ig_1 = 0, \tag{30}$$

$$\varepsilon^3 \quad : \quad (iD_xD_t - iD_xD_y + 2D_x)(g_1 \cdot f_2 + g_3 \cdot 1) + ig_1 f_2 + ig_3 = 0, \tag{31}$$

$$\varepsilon^5 \quad : \quad (iD_xD_t - iD_xD_y + 2D_x)(g_1 \cdot f_4 + g_3 \cdot f_2) + ig_1 f_4 + ig_3 f_2 = 0, \tag{32}$$

$$\varepsilon^7 \quad : \quad (iD_xD_t - iD_xD_y + 2D_x)(g_3 \cdot f_4) + ig_3 f_4 = 0, \tag{33}$$

$$\varepsilon^2 \quad : \quad i(D_xD_t - D_xD_y)(f_2 + f_2^*) - \frac{1}{2}D_x(g_1 \cdot g_1^*) = 0, \tag{34}$$

$$\varepsilon^4 \quad : \quad iD_xD_t(f_2 \cdot f_2^* + f_4 + f_4^*) - iD_xD_y(f_2 \cdot f_2^* + f_4 + f_4^*) -$$

$$-\frac{1}{2}D_x(g_1g_3^* + g_3g_1^*) = 0, \tag{35}$$

$$\varepsilon^6 \quad : \quad iD_xD_t(f_2 \cdot f_4^* + f_4 \cdot f_2^*) - iD_xD_y(f_2 \cdot f_4^* + f_4 \cdot f_2^*) -$$

$$-\frac{1}{2}D_x(g_3g_3^*) = 0, \tag{36}$$

$$\varepsilon^8 \quad : \quad iD_xD_t(f_4 \cdot f_4^*) - iD_xD_y(f_4 \cdot f_4^*) = 0, \tag{37}$$

$$\varepsilon^2 \quad : \quad i(D_t - D_y)(f_2 - f_2^*) - \frac{1}{2}g_1 \cdot g_1^* = 0, \tag{38}$$

$$\varepsilon^4 \quad : \quad i(D_t - D_y)(f_2 \cdot f_2^*) + i(D_t - D_y)(f_4 - f_4^*) - \frac{1}{2}(g_1 \cdot g_3^* + g_1^* g_3) = 0, \tag{39}$$

$$\varepsilon^6 \quad : \quad i(D_t - D_y)(f_2 \cdot f_4^*) - i(D_t - D_y)(f_2^* \cdot f_4) - \frac{1}{2}(g_3 \cdot g_3^*) = 0, \tag{40}$$

$$\varepsilon^8 \quad : \quad i(D_t - D_y)(f_4 \cdot f_4^*) = 0. \tag{41}$$

$$\varepsilon^2 \quad : \quad (-iD_xD_t + iD_xD_y)(f_2^* \cdot 1) - \frac{1}{2}g_1g_{1x}^* = 0, \tag{42}$$

$$\varepsilon^4 \quad : \quad i(D_x - D_y)(f_2 \cdot f_{2x}^*) - i(f_{4xt}^* - f_{4xy}^*) - \frac{1}{2}(g_1g_{3x}^* + g_3g_{1x}^*) = 0, \tag{43}$$

$$\varepsilon^6 \quad : \quad i(D_t - D_y)(f_2 \cdot f_{4x}^* + f_4 \cdot f_{2x}^*) - \frac{1}{2}g_3g_{3x}^* = 0, \tag{44}$$

$$\varepsilon^8 \quad : \quad i(D_t - D_y)(f_4 \cdot f_{4x}^*) = 0. \tag{45}$$

By applying the properties of the Hirota operator we solve the system of equation (30)-(45), and can get the two-soliton solution of equation (1), which has the next form

$$q = \frac{g_1 + g_3}{1 + f_2 + f_4}, \tag{46}$$

where

$$g_1 = e^{\theta_1} + e^{\theta_2},$$
$$g_3 = k_1 e^{\theta_1 + \theta_2 + \theta_1^*} + k_2 e^{\theta_1 + \theta_2 + \theta_2^*},$$
$$f_2 = l_1 e^{\theta_1 + \theta_1^*} + l_2 e^{\theta_1 + \theta_2^*} + l_3 e^{\theta_1^* + \theta_2} + l_4 e^{\theta_2 + \theta_2^*},$$
$$f_4 = m e^{\theta_1 + \theta_1^* + \theta_2 + \theta_2^*},$$

with

$$\theta_1 = a_1 x + b_1 t + c_1 y + d_1, \qquad \theta_1^* = a_1^* x + b_1^* t + c_1^* y + d_1^*,$$

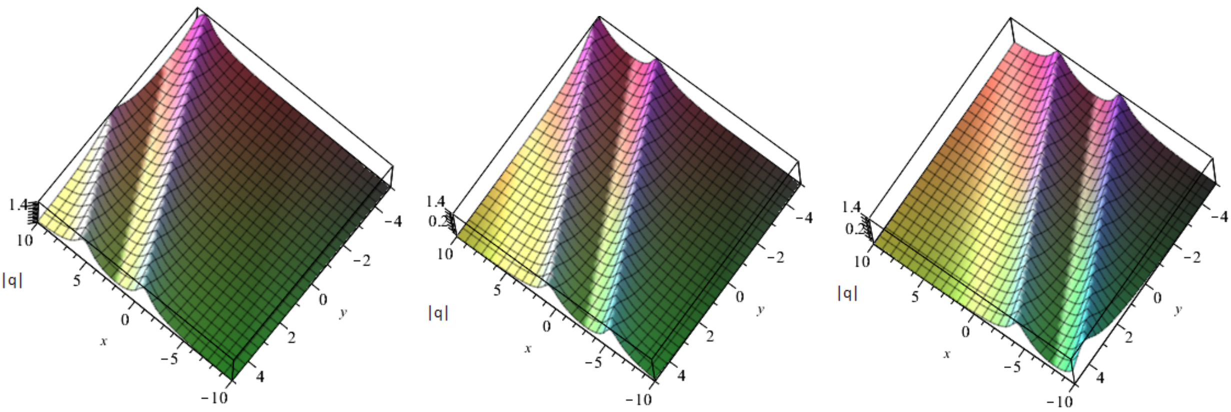
$$\theta_2 = a_2 x + b_2 t + c_2 y + d_2, \quad \theta_2^* = a_2^* x + b_2^* t + c_2^* y + d_2^*,$$

$$l_1 = \frac{i a_1^2 a_1^*}{2(a_1 + a_1^*)^2}, \quad l_2 = \frac{i a_1^2 a_2^*}{2(a_1 + a_2^*)^2}, \quad l_3 = \frac{i a_2^2 a_1^*}{2(a_1^* + a_2)^2}, \quad l_4 = \frac{i a_2^2 a_2^*}{2(a_2 + a_2^*)^2},$$

$$k_1 = \frac{i(a_1^*)^3 (a_2 - a_1)^2}{2(a_1^* + a_2)^2 (a_1 + a_1^*)^2}, \quad k_2 = \frac{i(a_2^*)^3 (a_2 - a_1)^2}{2(a_1 + a_2^*)^2 (a_2 + a_2^*)^2},$$

$$m = -\frac{(a_2 - a_1)^2 (a_2^* - a_1^*)^2 (a_1 + a_2 + a_1^* + a_2^*) a_1^* a_2^* a_1^2 a_2^2}{2(a_1 + a_1^*)^2 (a_2 + a_2^*)^2 (a_1 + a_2^*)^2 (a_2 + a_1^*)^2},$$

and a_n, b_n, c_n, d_n - complex constants, $n = 1,2$.



(a) (b) (c)

Figure 2 - Dynamics two-soliton solution with next parameters:
$a_1 = 0.5 + 0.5i$, $c_1 = 0.5 + 0.5i$, $b_1 = 0.5 + 0.5i$ and $d_1 = 0.5 + 0.5i$. $t = -5$ (a); t = 0 (б); t = 5 (c).

Conclusion. Thus, we studied (2+1)-dimensional Fokas-Lenells equation by the Hirota's bilinear method which is considered one of the effective methods for finding exact solutions of integrable equations. By using this method, exact one-soliton and two-soliton solutions of the (2+1)-dimensional FL equation are constructed. Additionally, we present the graphical representation of the obtained soliton solutions.

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(2+1)-ӨЛШЕМДІ ИНТЕГРАЛДАНАТЫН ФОКАС-ЛЕНЭЛЛС ТЕҢДЕУІНІҢ СОЛИТОНДЫ ШЕШІМДЕРІ

Аннотация. Солитондарды зерттеу солитонмен байланысты бірқатар жаңа бағыттардың ашылуына алып келді. Сонымен қатар, солитонды процестер байқалатын жаңа бағыттардың ашылуымен байланысты қызығушылықтар бар. Солитон құрайтын сызықты емес теңдеулердің саны мен алуандығы екі қлшемді және үш өлшемді жалпылауларға көшу арқылы күннен-күнге артуда. Дарбу, Бэклунд түрлендіруі және Хиротаның бисызықты әдісі сияқты танымал түрлендірулер сызықты емес теңдеулердің әр түрлі шешімдерін табу кезінде жиі қолданылады.

Демек, берілген жұмыс алдыңғы табылған (2+1)-өлшемді интегралданатын Фокас-Ленэлсс теңдеуі және оның бисызықты түрі атты жұмысымыздың жалғасы болып табылады. Енді, зерттеулерімізді жалғастыра отырып, Хирота әдісі арқылы интегралданатын (2+1)-өлшемді Фокас-Ленэлсс теңдеуінің 1-солитонды және 2-солитонды шешімдері табылып, графиктері тұрғызылды. Біздің тапқан нәтижелеріміз маңызды физикалық

қолданысқа ие.

Түйін сөздер: Хирота әдісі, Лакс көрінісі, солитонды шешім, Фокас-Ленэллс теңдеуі.

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СОЛИТОННЫЕ РЕШЕНИЯ (2+1)-МЕРНОГО ИНТЕГРИРУЕМОГО УРАВНЕНИЯ ФОКАСА-ЛЕНЭЛЛСА

Аннотация. Успехи в исследовании солитонов привели к открытию целого ряда новых направлений связанных с ним, тем вдохновили бурной активностью исследователей в данных направлениях. Кроме того, интерес, к которому усиливается также в связи обнаружениями новых примеров, в которых проявляются солитонные процессы. Количество и разнообразие нелинейных уравнений, содержащих солитоны в качестве наиболее интересных решений, существенно увеличиваются благодаря обобщениям на двумерные и трехмерные случаи. Для нахождения солитонных решений нелинейных уравнений часто применяются такие популярные преобразования, как Дарбу, Бэклунда и метод Хироты.

Таким образом, данная работа является продолжением нашей предыдущей работы, в которой было найдено (2+1)-мерное интегрируемое уравнение Фокаса-Ленэллса и построена ее билинейная форма методом Хироты. Теперь, продолжая наши исследования, методом Хироты найдены его точные 1-солитонное и 2-солитонное решения с помощью уже полученной нами билинейной формы (2+1)-мерного уравнения Фокаса-Ленэллса и построены их графики. Найденные нами результаты имеют важные физические приложения.

Ключевые слова: метод Хироты, представление Лакса, солитонное решение, уравнение Фокаса-Ленэллса.

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