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SPECTRAL DECOMPOSITION OF A FIRST ORDER FUNCTIONAL DIFFERENTIAL OPERATOR

Abstract. In this paper we study spectral properties of a boundary value problem of a first-order differential equation with constant coefficients and a deviating argument. By spectral properties we mean completeness and basis property of the system of eigenfunctions and associated functions of the boundary value problem, as well as the Volterra property.

Keywords: equation with deviating argument, completeness, basis property, Volterra property, Sturm-Liouville operator, Riesz basis.

1. Introduction

In applications the following eigenvalue problem is often occurred in a more general form [1, p. 520]:

$$Au = \lambda Su .$$

If the operator S is determined by the equality $(Sf)(x) = f(-x)$, let us say, for all $f \in L_2$, then the following generalized spectral problem occurs:

$$Au = \lambda u(-x) .$$

In the case when A is a differential operator, we get a differential equation with deviating argument.

Theory of differential equations with deviating argument is the subject of a huge number of works, among which we note only monographs by A.D. Myshkis [2], L.E. Elsgolts and S.B. Norkin [3], which provides an extensive bibliography. Study of the Sturm - Liouville type boundary value problems for an equation with deviating argument is the subject of a monograph by S.B. Norkin [4]. In this and other works, the spectral questions of differential equations with deviations in higher terms, where there is no spectral parameter, are investigated. Only a few works are devoted to the case when the deviating argument is contained in a term with a spectral parameter. In this regard, we note the works of T.Sh. Kalmenov, S.T. Akhmetova and A.Sh. Shaldanbaev [5], A.M. Ibraimkulov [6], S.T. Akhmetova [7].

Apparently, main theorems of the theory of solvability of differential equations with deviating (delayed) arguments were formulated in the monograph by A.D. Myshkis [2].

T.Sh. Kalmenov [5] laid the foundation to a systematic study of spectral questions for a first-order differential equation with a deviating argument of the indicated form. These ideas were developed in [7].

Apparently, generalized spectral problems of the type

$$Au = \lambda u(-x) ,$$

where A is some differential operator of the first order, were first investigated, at the initiative of T.Sh. Kalmenova, in [5]. In [7], various spectral properties of the generalized spectral problem were studied:

$$y'(x) = \lambda y(1-x), \quad 0 < x < 1,$$

$$\alpha y(0) + \beta y(1) = 0,$$

including conditions of self-adjointness, Volterra property and basis property.

Theorem 1.7.7 of the above paper [7] states the Riesz basis property of the system of root functions of the considered generalized spectral problem for

$$(|\alpha| + |\beta|)(\alpha^4 - \beta^4) \neq 0.$$

Results of this work were transferred to the interval [-1 1], by using another method, in [8], where the final solution to basis property questions of root functions of the generalized spectral problem is given

$$u'(-x) = \lambda u(x), \quad -1 < x < 1,$$

$$u(-1) = \alpha u(1).$$

From results of this work it follows that any such correct boundary-value problem is either Volterra or the system of its root functions forms a Riesz basis.

The method of [5] was generalized in [9], in particular, in this paper two abstract theorems on eigenvalues and root vectors of the operators A and A^2, which may be of independent interest, are proved. We give their statement.

Consider a linear operator A in the Hilbert space H. We suppose that domain $D(A)$ of the operator A is dense in H. Then there exists an operator A^*, which is conjugate to the operator A. Let spectrum of the operator A be discrete. The following proposition holds.

Theorem 1.1. Let number λ_0^2 be an eigenvalue of the operator A^2. If the number λ_0 is not eigenvalue of the operator A, then λ_0 is an eigenvalue of the operator A.

In the next theorem we consider the case of root vectors.

Theorem 1.2. Let λ_0^2 be an eigenvalue of the operator A^2. If λ_0 is not eigenvalue of the operator A, then any root vector u_1 of the operator A^2 (if, ofcourse, it exists), corresponding to the eigenvalue λ_0^2, is an associated element of the operator A, corresponding to the eigenvalue λ_0.

In the work of A.M. Ibraimkulov [6] completeness of the root vectors of a second-order equation was studied. The studies of this author were continued in [10] - [14].

Among recent studies we can note works of W.Watkins [15–16], in which questions on solvability of one-dimensional differential equations with involution were considered, A.P. Khromov and his followers [17-18], which considered solvability of integral and differential equations in partial derivatives with involution.

The variable separation method for solving partial differential equations is based on the spectral theory of one-dimensional differential operators. Spectral theory of self-adjoint and non-self-adjoint ordinary differential operators, which originated in the bowels of mathematical physics equations and began with the classical works of Sturm, Liouville, Steklov and others, has received quite complete development over the past century. The spectral theory of self-adjoint ordinary differential operators is almost complete. In the field of the spectral theory of non-self-adjoint ordinary differential operators, significant results on completeness and basis property of eigenfunctions and associated functions were obtained in the works of M.V. Keldysh [19], V.A. Ilyin [20-25], M. Otelbaev [26], A.A. Shkalikov [27], Radzievsky [28] and many other mathematicians.

Theory of basis property of systems of eigenfunctions and associated functions of non-self-adjoint ordinary differential operators, proposed by V.A. Ilyin, got rapid development. A fairly complete idea about development of the theory of basis property by V.A. Ilyin was given in review articles [29-30].

Compared with the spectral theory of ordinary differential operators, the spectral theory of one-dimensional differential operators with involution is in its infancy. Apparently, the first works on the spectral theory of one-dimensional differential operators with involution were carried out by initiative of T.Sh. Kalmenov [31-35] in the last decade of the present century. These studies were continued in the cycle of works by M.A. Sadybekov and A.M. Sarsenby [36-41]. Over the past decade, researchers' interest in differential equations with deviating arguments has grown markedly, as evidenced by publications [42–58]. Theory of bases is described in detail in [60–63].

In this work, we continue the studies begun in [5].

Problem Formulation. Find a spectral decomposition of the operator

$$Au = u'(1-x),\ \ x \in (0,1), \tag{1}$$

$$D(A) = \{u(x) \in C^1(0,1) \cap C[0,1]: \alpha u(0) + \beta u(1) = 0\}, \tag{2}$$

where α, β are arbitrary complex numbers, satisfying the condition:

$$|\alpha| + |\beta| \neq 0. \tag{3}$$

2. Research Method.

The method is based on the following theorem of N.K. Bari [58].

Theorem. If the sequence $\{\psi_j\}$ is complete in a Hilbert space H, it corresponds to the complete biorthogonal sequence $\{\varphi_j\}$ for any $f \in H$

$$\sum_{j=1}^{\infty} \left|(f, \psi_j)\right|^2 < \infty,\ \ \sum_{j=1}^{\infty} \left|(f, \varphi_j)\right|^2 < \infty, \tag{4}$$

then the system $\{\psi_j\}$ forms a Riesz basis of the Hilbert space H.

Therefore, we first show completeness of the system of eigenfunctions of the operator (1) - (2); then we find complete biorthogonal systems of functions and prove inequalities (4).

2.1. On spectrum of the operator.

Consider the following boundary value problem:

$$y'(1-x) = \lambda y(x); x \in (0,1), \tag{5}$$

$$\alpha y(0) + \beta y(1) = 0 \tag{6}$$

where α, β are arbitrary complex numbers, satisfying the condition (3), and λ is a spectral parameter.

It is easy to note that a general solution of the equation (5) has the form

$$y(x, \lambda) = A\left[\cos\lambda\left(\tfrac{1}{2} - x\right) - \sin\lambda\left(\tfrac{1}{2} - x\right)\right], \tag{7}$$

where $A \neq 0$ is an arbitrary nonzero constant.

Putting (7) into the boundary condition (2), we have

$$\alpha u(0) + \beta u(1) = \alpha A\left(\cos\frac{\lambda}{2} - \sin\frac{\lambda}{2}\right) + \beta A\left(\cos\frac{\lambda}{2} + \sin\frac{\lambda}{2}\right) =$$

$$= A\left[(\alpha+\beta)\cos\frac{\lambda}{2} - (\alpha-\beta)\sin\frac{\lambda}{2}\right] = A^\circ\Delta(\lambda) = 0,$$

since $A \neq 0$, therefore

$$\Delta(\lambda) = (\alpha+\beta)\cos\frac{\lambda}{2} - (\alpha-\beta)\sin\frac{\lambda}{2} = 0.$$

Assuming, that $\alpha^2 - \beta^2 \neq 0$, we get

$$\alpha - \beta \neq 0, \qquad \alpha + \beta \neq 0, =>$$

$$tg\frac{\lambda}{2} = \frac{\alpha+\beta}{\alpha-\beta}, \qquad \frac{\lambda_n}{2} = n\pi + arctg\frac{\alpha+\beta}{\alpha-\beta}, =>$$

$$\lambda_n = 2n\pi + 2arctg\frac{\alpha+\beta}{\alpha-\beta}.$$

Lemma 2.1. If

$$\alpha^4 - \beta^4 \neq 0, \tag{8}$$

then the boundary value problem

$$y'(1-x) = \lambda y(x); x \in (0,1), \quad (5)$$

$$\alpha y(0) + \beta y(1) = 0 \quad (6)$$

has infinite set of eigenvalues

$$\lambda_n = 2n\pi + 2arctg\frac{\alpha+\beta}{\alpha-\beta},\ n = 0, \pm 1, \pm 2, \ldots \quad (9)$$

and the eigenfunctions corresponding to them

$$y_n(x) = A_n\left[\cos\lambda_n\left(\frac{1}{2}-x\right) - \sin\lambda_n\left(\frac{1}{2}-x\right)\right],\ n = 0, \pm 1, \pm 2, \ldots \quad (10)$$

where A_n are arbitrary constants.

All eigenvalues are simple, i.e. if λ_n is an eigenvalue, then $\Delta(\lambda_n) = 0$ and $\dot{\Delta}(\lambda_n) \neq 0$, where the icon $(\dot{\ })$ means the derivative with respect to the spectral parameter λ.

There are no associated functions.

Proof. From the condition of Lemma it follows that $\alpha + \beta \neq 0,\ \alpha - \beta \neq 0$, then from the equation

$$\Delta(\lambda) = (\alpha+\beta)\cos\frac{\lambda}{2} - (\alpha-\beta)\sin\frac{\lambda}{2} = 0,$$

we have

$$tg\frac{\lambda}{2} = \frac{\alpha+\beta}{\alpha-\beta}.$$

This equation does not have any roots only in two cases:

$$\frac{\alpha+\beta}{\alpha-\beta} = \pm i.$$

This condition holds only when $\alpha = \pm i\beta$, i.e. $\alpha^2 + \beta^2 = 0$, which is possible due to the condition of Lemma 3.1.

In all other cases our equation has roots, which are given by the formulas

$$\lambda_n = 2n\pi + 2arctg\frac{\alpha+\beta}{\alpha-\beta}, \qquad n = 0, \pm 1, \pm 2, \ldots$$

By (7) we find the corresponding eigenfunctions:

$$y_n(x) = A_n\left[\cos\lambda_n\left(\frac{1}{2}-x\right) - \sin\lambda_n\left(\frac{1}{2}-x\right)\right], \quad (10)$$

where A_n are arbitrary constants.

If λ is a multiple root of the equation $\Delta(\lambda) = 0$, then the system of equations

$$\begin{cases}\Delta(\lambda) = 0,\\ \dot{\Delta}(\lambda) \neq 0\end{cases}$$

implies that $\alpha^2 + \beta^2 = 0$, that is also impossible.

Indeed,

$$\Delta(\lambda) = (\alpha+\beta)\cos\frac{\lambda}{2} - (\alpha-\beta)\sin\frac{\lambda}{2} = 0,$$

$$\dot{\Delta}(\lambda) = \frac{1}{2}\left[-(\alpha+\beta)\sin\frac{\lambda}{2} - (\alpha-\beta)\cos\frac{\lambda}{2}\right] = 0, =>$$

$$\begin{cases}(\alpha+\beta)\cos\frac{\lambda}{2} - (\alpha-\beta)\sin\frac{\lambda}{2} = 0,\\ (\alpha-\beta)\cos\frac{\lambda}{2} + (\alpha+\beta)\sin\frac{\lambda}{2} = 0.\end{cases}$$

Since this system of equations has a nontrivial solution, then its determinant will vanish, i.e.

$$\Delta=\begin{vmatrix}\alpha+\beta & -(\alpha-\beta)\\ \alpha-\beta & \alpha+\beta\end{vmatrix}=(\alpha+\beta)^2+(\alpha-\beta)^2=2(\alpha^2+\beta^2)=0.$$

Definition 2.1. If $u(x)$ is an eigenfunction of the boundary value problem

$$Au=u'(1-x)=\lambda u(x);x\in(0,1),$$

$$\alpha u(0)+\beta u(1)=0,$$

then solution of the boundary value problem

$$Bv=v'(1-x)-\lambda v(x)=u(x),$$

$$\alpha v(0)+\beta v(1)=0$$

is called associated function of this boundary value problem.

Now we show that if the condition $\alpha^4-\beta^4\neq 0$ holds, our boundary value problem (5) - (6) does not have associated functions.

Let $u(x)$ be an eigenfunction of the boundary value problem (5) - (6), and $v(x)$ is its corresponding associated function. Then differentiating (5) - (6) by the spectral parameter λ, we have

$$\dot{u}'(1-x)=\lambda\dot{u}+u(x),\quad \alpha\dot{u}(0)+\beta\dot{u}(1)=0,\qquad =>$$

$$\dot{u}'(1-x)-\lambda\dot{u}=u(x),\quad \alpha\dot{u}(0)+\beta\dot{u}(1)=0.$$

Consequently, the difference $z(x)=\dot{u}(x)-v(x)$ is an eigenfunction of our boundary value problem (5) - (6). Then, obviously, the function

$$\dot{u}(x)=v(x)+z(x)$$

is an associated function of our boundary value problem. We prove that it is not possible. Indeed,

$$\alpha u(0)+\beta u(1)=0=\left|u(x)=A\left[\cos\lambda\left(\frac{1}{2}-x\right)-\sin\lambda\left(\frac{1}{2}-x\right)\right]\right|=$$

$$=\alpha A\left(\cos\frac{\lambda}{2}-\sin\frac{\lambda}{2}\right)+\beta A\left(\cos\frac{\lambda}{2}-\sin\frac{\lambda}{2}\right)=$$

$$=A\left[(\alpha+\beta)\cos\frac{\lambda}{2}-(\alpha-\beta)\sin\frac{\lambda}{2}\right]=A\cdot\Delta(\lambda).$$

Differentiating this formula by the spectral parameter λ, we get

$$\frac{d}{d\lambda}[\alpha u(0)+\beta u(1)]=A\cdot\dot{\Delta}(\lambda),$$

$$\alpha\dot{u}(0)+\beta\dot{u}(1)=A\cdot\dot{\Delta}(\lambda)\neq 0,$$

where λ is an eigenvalue of the boundary value problem (5) - (6).

2.2. On completeness.

Lemma 2.2. If

$$\alpha^4-\beta^4\neq 0, \tag{8}$$

then eigenfunctions of the boundary value problem

$$y'(x)=\lambda y(1-x);x\in(0,1), \tag{5}$$

$$\alpha y(0)+\beta y(1)=0 \tag{6}$$

form a complete system in the space $L^2(0,1)$.

Proof. Let $\{y_n\}, n=0,\pm1,\pm2,\ldots$ be a system of eigenfunctions of the boundary value problem (5) - (6). We assume that

$$\int_0^1\overline{f(x)}y_n(x)dx=0\,,\qquad \int_0^1\overline{f(x)}y_{-n}(x)dx=0\,,n=0,\pm1,\pm2,\ldots$$

then

$$\int_0^1\overline{f(x)}\left[\cos(2n\pi+2\varphi)\left(\frac{1}{2}-x\right)-\sin(2n\pi+2\varphi)\left(\frac{1}{2}-x\right)\right]dx=0,$$

$$\int_0^1 \overline{f(x)}\left[\cos(2n\pi-2\varphi)\left(\frac{1}{2}-x\right)+\sin(2n\pi-2\varphi)\left(\frac{1}{2}-x\right)\right]dx=0,$$

where

$$\varphi=arctg\frac{\alpha+\beta}{\alpha-\beta}.$$

Supposing $t=\frac{1}{2}-x$, from the first formula we have

$$x=\frac{1}{2}-t,\qquad dx=-dt,=>$$

$$\int_{-1/2}^{1/2}\bar{f}\left(\frac{1}{2}-t\right)[\cos(2n\pi+2\varphi)t-\sin(2n\pi+2\varphi)\,t](-dt)=$$

$$=\int_{-1/2}^{1/2}\bar{f}\left(\frac{1}{2}-t\right)[\cos(2n\pi+2\varphi)t-\sin(2n\pi+2\varphi)\,t]dt=0; \tag{12}$$

Similarly, we obtain

$$=\int_{-1/2}^{1/2}\bar{f}\left(\frac{1}{2}-t\right)[\cos(2n\pi-2\varphi)t+\sin(2n\pi-2\varphi)\,t]dt=0. \tag{13}$$

Summing up equalities (12) and (13), we get

$$\int_{-1/2}^{1/2}\bar{f}\left(\frac{1}{2}-t\right)[2\cos 2n\pi t\cos 2\varphi t-2\cos 2n\pi t\sin 2\varphi t]dt=0,$$

$$\int_{-1/2}^{1/2}\bar{f}\left(\frac{1}{2}-t\right)(\cos 2\varphi t-\sin 2\varphi t)\cos 2n\pi t\,dt=0, n=0,1,2,\dots$$

In this formula supposing $2\pi t=x$, we have

$$t=\frac{x}{2\pi},\qquad dt=\frac{dx}{2\pi},$$

$$\int_{-\pi}^{+\pi}\bar{f}\left(\frac{1}{2}-\frac{x}{2\pi}\right)\left(\cos\frac{\varphi x}{\pi}-\sin\frac{\varphi x}{\pi}\right)\cos nx dx=0,\ \ n=0,1,2,\dots \tag{14}$$

Further, subtracting the formula (13) from (12), we get

$$\int_{-1/2}^{1/2}\bar{f}\left(\frac{1}{2}-t\right)[-2\sin 2n\pi\cdot\sin 2\varphi t-2\sin 2n\pi t\cdot\cos 2\varphi t]dt=0,=>$$

$$\int_{-1/2}^{1/2}\bar{f}\left(\frac{1}{2}-t\right)[\cos 2\varphi t+\sin 2\varphi t]\sin 2n\pi t\,dt=0,\qquad n=1,2,\dots \tag{15}$$

Now from (14) we have

$$\int_{-\pi}^{+\pi}\bar{f}\left(\frac{1}{2}-\frac{x}{2\pi}\right)\left(\cos\frac{\varphi x}{\pi}-\sin\frac{\varphi x}{\pi}\right)\cos nx dx=0.=>$$

In this formula assuming , we obtain

$$-x = \xi, dx = -d\xi, =>$$

$$-\int_{-\pi}^{+\pi} \bar{f}\left(\frac{1}{2}+\frac{\xi}{2\pi}\right)\left(\cos\frac{\varphi\xi}{\pi}+\sin\frac{\varphi\xi}{\pi}\right)\cos n\xi d\xi = 0, =>$$

$$\int_{-\pi}^{+\pi} \bar{f}\left(\frac{1}{2}+\frac{\xi}{2\pi}\right)\left(\cos\frac{\varphi\xi}{\pi}+\sin\frac{\varphi\xi}{\pi}\right)\cos n\xi d\xi = 0,$$

or denoting ξ by x, we get

$$\int_{-\pi}^{+\pi} \bar{f}\left(\frac{1}{2}+\frac{\varphi}{2\pi}\right)\left(\cos\frac{\varphi x}{\pi}+\sin\frac{\varphi x}{\pi}\right)\cos nxdx = 0,\ n = 0,1,2,\dots \tag{16}$$

Adding (14) to (16), we receive

$$\int_{-\pi}^{+\pi}\left[\frac{\bar{f}\left(\frac{1}{2}-\frac{x}{2\pi}\right)+\bar{f}\left(\frac{1}{2}+\frac{x}{2\pi}\right)}{2}\cos\frac{\varphi x}{\pi}+\frac{\bar{f}\left(\frac{1}{2}+\frac{x}{2\pi}\right)-\bar{f}\left(\frac{1}{2}-\frac{x}{2\pi}\right)}{2}\sin\frac{\varphi x}{\pi}\right]\cdot$$

$$\cdot\cos nxdx = 0. \tag{17}$$

The function in the integral in (17) is even, therefore

$$\int_{0}^{\pi}\left(P\bar{f}\cos\frac{\varphi x}{\pi}+Q\bar{f}\sin\frac{\varphi x}{\pi}\right)\cos nxdx = 0, \qquad n = 0,1,2,\dots,$$

where

$$P\bar{f}(x) = \frac{\bar{f}\left(\frac{1}{2}-\frac{x}{2\pi}\right)+\bar{f}\left(\frac{1}{2}+\frac{x}{2\pi}\right)}{2}, Q\bar{f}(x) = \frac{\bar{f}\left(\frac{1}{2}+\frac{x}{2\pi}\right)-\bar{f}\left(\frac{1}{2}-\frac{x}{2\pi}\right)}{2}.$$

Due to completeness of the system of functions $\{\cos nx\}, n = 0,1,2,\dots$ in the space $L^2(0,\pi)$, we have

$$P\bar{f}\cos\frac{\varphi x}{\pi}+Q\bar{f}\sin\frac{\varphi x}{\pi} = 0$$

Now we transform the formula (15). Assuming $t = -x$, we get

$$\int_{-1/2}^{1/2}\bar{f}\left(\frac{1}{2}+x\right)(\cos 2\varphi x-\sin 2\varphi x)(-dx) =$$

$$\int_{-1/2}^{1/2}\bar{f}\left(\frac{1}{2}+x\right)(\cos 2\varphi x-\sin 2\varphi x)dx = 0.$$

Consequently, we have a pair of the formulas

$$\int_{-1/2}^{1/2}\bar{f}\left(\frac{1}{2}-x\right)(\cos 2\varphi x+\sin 2\varphi x)\sin 2n\pi xdx = 0, \tag{18}$$

$$\int_{-\frac{1}{2}}^{\frac{1}{2}}\bar{f}\left(\frac{1}{2}+x\right)(\cos 2\varphi x-\sin 2\varphi x)\sin 2n\pi xdx = 0. \tag{19}$$

Subtracting the formula (18) from the formula (19), then dividing the result by 2, we obtain

$$\int_{-1/2}^{1/2}\left[\frac{\bar f\left(\frac{1}{2}+x\right)-\bar f\left(\frac{1}{2}-x\right)}{2}\cos 2\varphi x-\frac{\bar f\left(\frac{1}{2}+x\right)+\bar f\left(\frac{1}{2}-x\right)}{2}\sin 2\varphi x\right]\cdot$$

$$\cdot\sin 2n\pi x dx=0,\ n=1,2,\dots. \qquad (20)$$

The function in the integral in (20) is even, thus

$$\int_0^{1/2}\left(Q\bar f\cos 2\varphi x-P\bar f\sin 2\varphi x\right)\sin 2n\pi x dx=0, \qquad (21)$$

where

$$Q\bar f(x)=\frac{\bar f\left(\frac{1}{2}+x\right)-\bar f\left(\frac{1}{2}-x\right)}{2}, P\bar f(x)=\frac{\bar f\left(\frac{1}{2}+x\right)+\bar f\left(\frac{1}{2}-x\right)}{2}.$$

Assuming $t=2\pi x$, we change the variable in the formula (21), then

$$x=\frac{t}{2\pi},\qquad dx=\frac{dt}{2\pi},$$

$$\int_0^{\pi}\left[\frac{\bar f\left(\frac{1}{2}+\frac{t}{2\pi}\right)-\bar f\left(\frac{1}{2}-\frac{t}{2\pi}\right)}{2}\cos\frac{\varphi t}{\pi}-\frac{\bar f\left(\frac{1}{2}+\frac{t}{2\pi}\right)+\bar f\left(\frac{1}{2}-\frac{t}{2\pi}\right)}{2}\sin\frac{\varphi t}{\pi}\right]\cdot$$

$$\cdot\sin ntdt=0,\qquad n=1,2,\dots.$$

Due to completeness of the system of functions in the space $L^2(0,\pi)$, we have

$$\frac{\bar f\left(\frac{1}{2}+\frac{x}{2\pi}\right)-\bar f\left(\frac{1}{2}-\frac{x}{2\pi}\right)}{2}\cos\frac{\varphi x}{\pi}-\frac{\bar f\left(\frac{1}{2}+\frac{x}{2\pi}\right)+\bar f\left(\frac{1}{2}-\frac{x}{2\pi}\right)}{2}\sin\frac{\varphi x}{\pi}=0.$$

Consequently,

$$\begin{cases}\frac{\bar f\left(\frac{1}{2}-\frac{x}{2\pi}\right)+\bar f\left(\frac{1}{2}+\frac{x}{2\pi}\right)}{2}\cos\frac{\varphi x}{\pi}+\frac{\bar f\left(\frac{1}{2}+\frac{x}{2\pi}\right)-\bar f\left(\frac{1}{2}-\frac{x}{2\pi}\right)}{2}\sin\frac{\varphi x}{\pi}=0,\\ \frac{\bar f\left(\frac{1}{2}+\frac{x}{2\pi}\right)+\bar f\left(\frac{1}{2}-\frac{x}{2\pi}\right)}{2}\sin\frac{\varphi x}{\pi}-\frac{\bar f\left(\frac{1}{2}+\frac{x}{2\pi}\right)-\bar f\left(\frac{1}{2}-\frac{x}{2\pi}\right)}{2}\cos\frac{\varphi x}{\pi}=0.\end{cases}$$

Since the determinant of this system

$$\Delta=\begin{vmatrix}\cos\frac{\varphi x}{\pi} & \sin\frac{\varphi x}{\pi}\\ \sin\frac{\varphi x}{\pi} & -\cos\frac{\varphi x}{\pi}\end{vmatrix}=-\cos^2\frac{\varphi x}{\pi}-\sin^2\frac{\varphi x}{\pi}=-1\neq 0,$$

then

$$\frac{\bar f\left(\frac{1}{2}-\frac{x}{2\pi}\right)+\bar f\left(\frac{1}{2}+\frac{x}{2\pi}\right)}{2}=0,\qquad \frac{\bar f\left(\frac{1}{2}+\frac{x}{2\pi}\right)-\bar f\left(\frac{1}{2}-\frac{x}{2\pi}\right)}{2}=0.$$

Summing up these formulas, we get $\bar f\left(\frac{1}{2}+\frac{x}{2\pi}\right)=0$ almost for all $x\in[0,\pi]$, consequently, almost for all $x\in[0,1]$ we have the equality $\bar f=0$, that is required to prove.

2.3. On the conjugate boundary value problem.

We find conjugate boundary value problem to our boundary value problem

$$Ay=y'(1-x)=\lambda y(x), x\in(0,1); \qquad (5)$$

$$\alpha y(0)+\beta y(1)=0. \qquad (6)$$

Let $z(x)\in D(L^+)$, i.e. belong to the domain of the conjugate problem, then we have the formula

$$(Ay,z)=(y,A^+z),\qquad \forall y\in D(A),\qquad z\in D(A^+).$$

Expanding this formula and integrating by parts, we find $D(A^{+})$.

$$(Ay,z)=\int_0^1 Sy'(x)\overline{z(x)}\,dx=\left(Sy',z\right)=\left(y',Sz\right)=|Sz(x)=z(1-x)|=$$

$$=\int_0^1 \overline{Sz}\,dy=Sz\cdot y|_0^1-\int_0^1(\overline{Sz})'y(x)dx=\overline{z(1-x)}\cdot y(x)\big|_0^1-\int_0^1(\overline{Sz})'y(x)dx=$$

$$=\overline{z(0)}y(1)-\overline{z(1)}y(0)+\int_0^1\overline{z'(1-x)}y(x)dx.$$

Equating to zero, outside the integral term, we compose a system of equations:

$$\begin{cases}\alpha y(0)+\beta y(1)=0\\ \overline{z(1)}y(0)-\overline{z(0)}y(1)=0.\end{cases}$$

Since the system of equations has a non-trivial solution, then the determinant vanishes, i.e.

$$-\alpha\bar{z}(0)-\beta\bar{z}(1)=0,\Rightarrow\bar{z}(0)+\beta\bar{z}(1)=0,\Rightarrow\bar{\alpha}z(0)+\bar{\beta}z(1)=0.$$

From the equality

$$(y,A^{+}z)=\int_0^1 y(x)\,\overline{z'(1-x}dx$$

we have

$$A^{+}z=z'(1-x)$$

consequently, conjugate boundary value problem has the following form:

$$A^{+}z=z'(1-x)=\mu z(x),\ x\in(0,1); \tag{5}^{+}$$

$$\bar{\alpha}z(0)+\bar{\beta}z(1)=0. \tag{6}^{+}$$

It is easy to note that this problem similar to the boundary value problem (5) - (6). Since $(\bar{\alpha})^4-\left(\bar{\beta}\right)^4=0\Leftrightarrow\alpha^4-\beta^4=0$, then the conditions on its solvability are also similar. In particularly, Lemma 2.1 yields the following Lemma 2.3.

Lemma 2.3. If

$$(\bar{\alpha})^4-(\beta)^4\neq 0 \tag{8}^{+}$$

then the boundary value problem $(5)^{+}$ - $(6)^{+}$ has infinite set of eigenvalues:

$$\mu_m=2m\pi+2arctg\frac{\bar{\alpha}+\bar{\beta}}{\bar{\alpha}-\bar{\beta}},\ m=0,\pm1,\pm2,... \tag{9}^{+}$$

and their corresponding eigenfunctions:

$$z_m(x)=B_m\left[\cos\mu_m\left(\frac{1}{2}-x\right)-\sin\mu_m\left(\frac{1}{2}-x\right)\right]\ m=0,\pm1,\pm2,... \tag{10}^{+}$$

where B_m are arbitrary constants.

All eigenvalues μ_m are simple.

There are no associated functions.

Lemma 2.4. If

$$(\bar{\alpha})^4-\left(\bar{\beta}\right)^4\neq 0 \tag{8}^{+}$$

then eigenfunctions $\{z_n\}$ of the boundary value problem

$$A^{+}z=z'(1-x)=\mu z(x),\ x\in(0,1); \tag{5}^{+}$$

$$\bar{\alpha}z(0)+\bar{\beta}z(1)=0 \tag{6}^{+}$$

form a complete system in the space $L^2(0,1)$.

2.4. On the biorthogonal system

Lemma 2.5. If the functions

$$y_n(x) = A_n\left[\cos\lambda_n\left(\frac{1}{2}-x\right) - \sin\lambda_n\left(\frac{1}{2}-x\right)\right], \qquad n = 0, \pm1, \pm2, \ldots$$

are eigenfunctions of the boundary value problem

$$Ay = y'(1-x) = \lambda y(x),\ x \in (0,1); \tag{5}$$

$$\alpha y(0) + \beta y(1) = 0, \tag{6}$$

then the functions

$$z_n(x) = \frac{1}{\bar{A}_n}\left[\cos\bar{\lambda}_n\left(\frac{1}{2}-x\right) - \sin\bar{\lambda}_n\left(\frac{1}{2}-x\right)\right]$$

are eigenfunctions of the conjugate boundary value problem

$$A^{+}z = z'(1-x) = \mu z(x),\ x \in (0,1); \qquad (5)^{+}$$

$$\bar{\alpha}z(0) + \bar{\beta}z(1) = 0; \qquad (6)^{+}$$

moreover, we have the formula

$$(y_n, z_m) = \delta_{nm},$$

where δ_{nm} is the Kronecker symbol.

Proof.

$$(y_n, z_m) =$$

$$= \int_0^1 \left[\cos\lambda_n\left(\frac{1}{2}-x\right) - \sin\lambda_n\left(\frac{1}{2}-x\right)\right] \cdot \overline{\left[\cos\bar{\lambda}_m\left(\frac{1}{2}-x\right) - \sin\bar{\lambda}_m\left(\frac{1}{2}-x\right)\right]}\, dx$$

$$= \int_0^1 \left[\cos\lambda_n\left(\frac{1}{2}-x\right) - \sin\lambda_n\left(\frac{1}{2}-x\right)\right]\left[\cos\lambda_m\left(\frac{1}{2}-x\right) - \sin\lambda_m\left(\frac{1}{2}-x\right)\right] dx =$$

$$= \int_0^1 \left\{\left[\cos\lambda_n\left(\frac{1}{2}-x\right)\cos\lambda_m\left(\frac{1}{2}-x\right) + \sin\lambda_n\left(\frac{1}{2}-x\right)\sin\lambda_m\left(\frac{1}{2}-x\right)\right] -\right.$$

$$-\left[\cos\lambda_n\left(\frac{1}{2}-x\right)\sin\lambda_m\left(\frac{1}{2}-x\right)\right] - \left[\cos\lambda_n\left(\frac{1}{2}-x\right)\sin\lambda_m\left(\frac{1}{2}-x\right) +\right.$$

$$\left.\left. + \sin\lambda_n\left(\frac{1}{2}-x\right)\cos\lambda_m\left(\frac{1}{2}-x\right)\right]\right\} dx =$$

$$= \int_0^1 \left[\cos(\lambda_n - \lambda_m)\left(\frac{1}{2}-x\right) - \sin(\lambda_n + \lambda_m)\left(\frac{1}{2}-x\right)\right] dx =$$

$$= \left[-\frac{\sin(\lambda_n - \lambda_m)\left(\frac{1}{2}-x\right)}{\lambda_n - \lambda_m}\Bigg|_0^1 - \frac{\cos(\lambda_n + \lambda_m)\left(\frac{1}{2}-x\right)}{\lambda_n + \lambda_m}\Bigg|_0^1\right] =$$

$$= \frac{2\sin\frac{\lambda_n - \lambda_m}{2}}{\lambda_n - \lambda_m} = \frac{\sin\frac{\lambda_n - \lambda_m}{2}}{\frac{\lambda_n - \lambda_m}{2}} = \left|\begin{matrix} \frac{\lambda_n}{2} = n\pi + arctg\frac{\alpha+\beta}{\alpha-\beta} \\ \frac{\lambda_n - \lambda_m}{2} = (n-m)\pi \end{matrix}\right| =$$

$$= \frac{\sin(n - m\pi)}{(n-m)\pi} = 0,$$

when $n \neq m$.

If $n = m$, then

$$(y_n, y_m) = \int_0^1 \left[\cos\lambda_n\left(\frac{1}{2}-x\right) - \sin\lambda_n\left(\frac{1}{2}-x\right)\right]^2 dx =$$

$$= \int_0^1 \left[1 - 2\sin\lambda_n\left(\frac{1}{2}-x\right)\cos\lambda_n\left(\frac{1}{2}-x\right)\right] dx =$$

$$= \int_0^1 [1 - \sin\lambda_n(1-2x)]dx = \left[x - \frac{\cos\lambda_n(1-2x)}{2\lambda_n}\right]\Bigg|_0^1 = 1$$

2.5. On basis property

Definition 2.1. Sequence $\{\varphi_j\}$ of vectors of the Banach space B is called basis of this space, if each vector $x \in B$ is expended uniquely in a series

$$x = \sum_{j=1}^{\infty} c_j \times \psi_j$$

converging by the norm of the space B.

Any bounded invertible operator transforms any orthonormal basis into some other basis of the space H. The basis $\{\psi_j\}$ of the Hilbert space H, obtained from the orthonormal basis by using such transformation, is called basis equivalent to orthonormal or the Riesz basis.

Theorem (N.K. Bari). If the sequence $\{\psi_j\}$ is complete in the Hilbert space H, it corresponds to a complete biorthogonal sequence $\{\psi_j\}$ and for all $f \in H$

$$\sum_{j=1}^{\infty}\left|(f, \psi_j)\right|^2 < \infty,\ \sum_{j=1}^{\infty}\left|(f, \varphi_j)\right|^2 < \infty \tag{4}$$

then the system $\{\psi_j\}$ forms Riesz basis in the space H.

Using this theorem, we prove basis property of the system of eigenfunctions of our boundary value problem

$$Ay = y'(1-x) = \lambda y(x),\ x \in (0,1); \tag{5}$$

$$\alpha y(0) + \beta y(1) = 0, \tag{6}$$

where α, β are arbitrary constants, satisfying the condition

$$\alpha^4 - \beta^4 \neq 0. \tag{8}$$

and λ is a spectral parameter.

Our sequence is complete (see Lemma 2.2) in the space $H = L^2(0,1)$, and it corresponds to a biorthogonal sequence $\{z_n\}$ (see Lemma 2.5), which is also complete in H (see Lemma 2.4), therefore, it only remains for us to prove inequalities (4):

Let

$$a_n = (f, y_n) = \int_0^1 f \cdot \bar{y}_n(x)dx,\ b_n = (f, z_n) = \int_0^1 f(x)\bar{z}_n(x)dx. \tag{22}$$

In our case

$$y_n(x) = (-1)^n\left[\cos\lambda_n\left(\frac{1}{2}-x\right) - \sin\lambda_n\left(\frac{1}{2}-x\right)\right],$$

$$z_n(x) = (-1)^n\left[\cos\bar{\lambda}_n\left(\frac{1}{2}-x\right) - \sin\bar{\lambda}_n\left(\frac{1}{2}-x\right)\right].$$

First we transform the integrals (6).

$$a_n = (-1)^n \int_0^1 f(x)\left[\cos\bar{\lambda}_n\left(\frac{1}{2}-x\right) - \sin\bar{\lambda}_n\left(\frac{1}{2}-x\right)\right]dx =$$

$$= (-1)^n \cdot \int_0^1 f(x)\cos\bar{\lambda}_n\left(\frac{1}{2}-x\right)dx - (-1)^n\int_0^1 f(x)\sin\bar{\lambda}_n\left(\frac{1}{2}-x\right)dx;$$

$$\int_0^1 f(x)\cos\bar{\lambda}_n\left(\frac{1}{2}-x\right)dx = \begin{vmatrix} t = \dfrac{1}{2} - x \\ dt = -dx \end{vmatrix} = \int_{-\frac{1}{2}}^{\frac{1}{2}} f\left(\frac{1}{2}-t\right)\cos\bar{\lambda}_n t(-dt) =$$

$$= \int_{-\frac{1}{2}}^{\frac{1}{2}} f\left(\frac{1}{2} - t\right)[\cos 2n\pi t \ \cos 2\varphi t - \sin 2n\pi t \sin 2\varphi t]dt =$$

$$= \left| x = 2\pi t, \qquad t = \frac{2}{2\pi}, \qquad dt = \frac{dx}{2\pi} \right| =$$

$$= \int_{-\pi}^{+\pi} f\left(\frac{1}{2} - \frac{x}{2\pi}\right)\left[\cos nx \cdot \cos\frac{\varphi x}{\pi} - \sin nx \cdot \sin\frac{\varphi x}{\pi}\right]\frac{dx}{2\pi} =$$

$$= \frac{1}{2\pi}\int_{-\pi}^{+\pi} f\left(\frac{1}{2} - \frac{x}{2\pi}\right)\cos\frac{\varphi x}{\pi}\cos nx\, dx -$$

$$- \frac{1}{2\pi}\int_{-\pi}^{\pi} f\left(\frac{1}{2} - \frac{x}{2\pi}\right)\sin\frac{\varphi x}{\pi} \cdot \sin nx\, dx;$$

The system

$$\frac{1}{\sqrt{2\pi}}, \qquad \frac{1}{\sqrt{\pi}}\sin x, \dots, \frac{\cos nx}{\sqrt{\pi}}, \qquad \frac{\sin nx}{\sqrt{\pi}}, \dots$$

forms orthonormal basis of the space $H = L^2(0,1)$. Based on this fact, we estimate the coefficients $a_n$and b_n(see (22)).

$$a_n = \frac{1}{2\sqrt{\pi}}\int_{-\pi}^{+\pi} f\left(\frac{1}{2} - \frac{x}{2\pi}\right)\cos\frac{\varphi x}{\pi} \times \frac{\cos nx}{\sqrt{\pi}} -$$

$$- \frac{1}{2\sqrt{\pi}}\int_{-\pi}^{+\pi} f\left(\frac{1}{2} - \frac{x}{2\pi}\right)\sin\frac{\varphi x}{\pi} \times \frac{\sin nx}{\sqrt{\pi}} dx,$$

$$Rea_n = \frac{1}{2\sqrt{\pi}}\int_{-\pi}^{+\pi} Re\, f\left(\frac{1}{2} - \frac{x}{2\pi}\right)\cos\frac{\varphi x}{\pi} \times \frac{\cos nx}{\sqrt{\pi}} dx -$$

$$- \frac{1}{2\sqrt{\pi}}\int_{-\pi}^{+\pi} Ref\left(\frac{1}{2} - \frac{x}{2\pi}\right)\sin\frac{\varphi x}{\pi} \times \frac{\sin nx}{\sqrt{\pi}} dx, \qquad n \neq 0$$

Let

$$u(x) = \frac{Ref\left(\frac{1}{2} - \frac{x}{2\pi}\right)\cos\frac{\varphi x}{\pi}}{2\sqrt{\pi}}, v(x) = \frac{1}{2\sqrt{\pi}} Ref\left(\frac{1}{2} - \frac{x}{2\pi}\right)\sin\frac{\varphi x}{\pi}.$$

Then

$$Rea_n = \frac{1}{\sqrt{\pi}}\int_{-\pi}^{+\pi} u(x)\cos nx\, dx - \frac{1}{\sqrt{\pi}}\int_{-\pi}^{+\pi} v(x)\sin nx\, dx = \alpha_n - \beta_n, n \neq 0$$

If $n = 0$, we have

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{+\pi} f\left(\frac{1}{2} - \frac{x}{2\pi}\right)\cos\frac{\varphi x}{\pi} dx,$$

$$Rea_0 = \frac{1}{2\pi}\int_{-\pi}^{+\pi} Ref\left(\frac{1}{2} - \frac{x}{2\pi}\right)\cos\frac{\varphi x}{\pi} dx,$$

$$\alpha_0 = \frac{1}{\sqrt{2\pi}}\int_{-\pi}^{+\pi} u(x)dx = \frac{1}{\sqrt{2\pi}}\int_{-\pi}^{+\pi} \frac{Ref\left(\frac{1}{2} - \frac{x}{2\pi}\right)\cos\frac{\varphi x}{\pi}}{2\sqrt{\pi}} dx =$$

$$= \frac{1}{\sqrt{2}} \times \frac{1}{2\pi}\int_{-\pi}^{+\pi} Ref\left(\frac{1}{2} - \frac{x}{2\pi}\right)\cos\frac{\varphi x}{\pi} dx = \frac{Rea_0}{\sqrt{2}}, \Rightarrow Rea_0 = \alpha_0\sqrt{2}.$$

Further,

$$|Rea_n| \leq |\alpha_n| + |\beta_n|, \Rightarrow$$
$$|Rea_n|^2 \leq (|\alpha_n| + |\beta_n|)^2 \leq |\alpha_n|^2 + 2|\alpha_n| \times |\beta_n| + |\beta_n|^2 \leq$$
$$\leq |\alpha_n|^2 + |\beta_n|^2 + |\alpha_n|^2 + |\beta_n|^2 \leq 2(|\alpha_n|^2 + |\beta_n|^2),$$
$$|Rea_0| \leq \sqrt{2}\,|\alpha_0|, \qquad |Rea_0|^2 \leq 2|\alpha_0|^2.$$

Consequently,

$$\sum_{n=0}^{\infty} |Rea_n|^2 \leq 2\left(\sum_{n=0}^{\infty} |\alpha_n|^2 + \sum_{n=1}^{\infty} |\beta_n|^2\right) \leq 2(\|u\|^2 + \|v\|^2) < \infty.$$

Similarly, we have

$$\sum_{n=0}^{\infty} |Ima_n|^2 < \infty.$$

Therefore,

$$\sum_{n=0}^{\infty} |a_n|^2 = \sum_{n=0}^{\infty} |Ima_n|^2 + |Rea_n|^2 < \infty.$$

Estimation of the series $\sum_{n=0}^{\infty} |b_n|^2$ is carried out similarly.

We have proved the main Theorem 3.1., and Theorem 3.2 is its corollary.

3. Research Results.

Theorem 3.1. Suppose that

$$\alpha^4 - \beta^4 \neq 0, \tag{8}$$

then the system of eigenfunctions of the boundary value problem

$$y'(1-x) = \lambda y(x); x \in (0,1), \tag{5}$$

$$\alpha y(0) + \beta y(1) = 0 \tag{6}$$

forms Riesz basis in the space $L^2(0,1)$, i.e. we have

$$f(x) = \sum_{-\infty}^{+\infty} (f, z_n)\, y_n(x),$$

Converging in the space $L^2(0,1)$, where

$$y_n(x) = (-1)^n \left[\cos \lambda_n \left(\frac{1}{2} - x\right) - \sin \lambda_n \left(\frac{1}{2} - x\right)\right],$$

$$z_n(x) = (-1)^n \left[\cos \overline{\lambda_n} \left(\frac{1}{2} - x\right) - \sin \overline{\lambda_n} \left(\frac{1}{2} - x\right)\right],$$

$$\lambda_n = 2n\pi + 2arctg\,\frac{\alpha+\beta}{\alpha-\beta},\ n = 0, \pm 1, \pm 2, \dots \tag{9}$$

and $f(x)$ is an arbitrary element in the space $L^2(0,1)$.

Theorem 3.2. If $\alpha^4 - \beta^4 \neq 0$, then for any element $u(x) \in D(A)$ we have the spectral expansion

$$Au = \sum_{-\infty}^{+\infty} \lambda_n (u, z_n)\, y_n(x) \tag{11}$$

converging in the space $L^2(0,1)$, where

$$\lambda_n = 2n\pi + 2arctg\,\frac{\alpha+\beta}{\alpha-\beta},\ n = 0, \pm 1, \pm 2, \dots \tag{9}$$

$$z_n(x) = (-1)^n \left[\cos \overline{\lambda_n} \left(\tfrac{1}{2} - x\right) - \sin \overline{\lambda_n} \left(\tfrac{1}{2} - x\right)\right], \tag{10$^+$}$$

$$y_n(x) = (-1)^n \left[\cos \lambda_n \left(\tfrac{1}{2} - x\right) - \sin \lambda_n \left(\tfrac{1}{2} - x\right)\right]. \tag{10}$$

Theorem 3.2. is the main result of this work.

4. Discussion.

Formula (11) is not possible if there are associated vectors, the well-known Kesselman - Mikhailov theorem [61- [62] states that, not the system of eigenvectors, the system of root vectors is basic, and this is significance of the results of this work. Formula (11) can find application in electrical engineering, information theory, crystallography, and signal transmission theory. It can be useful in study various boundary value problems by the method of variables separation.

5. Conclusion.

1) Operator (1) - (2) is not semi-bounded;

2) There is an alternative: either the boundary value problem (5) - (6) is Volterra i.e. has no eigenvalues, or the system of its eigenvectors forms a Riesz basis of the space $L^2(0,1)$.

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БІРІНШІ РЕТТІ ФУНКЦИОНАЛ-ДИФФЕРЕНЦИАЛ ОПЕРАТОРДЫҢ СПЕКТРӘЛДІ ТАРАЛЫМЫ

Аннотация. Бұл еңдекте аргументі ауытқыған бірінші ретті дифференциалдық теңдеудің спектрәлдік қасиеттері зерттелді.Шекаралық есептің спектрәлдік қасиеттері ретінде біз оның меншікті және олармен еншілес функцияларының системасының толымдылығы мен базистігін,сондайақ, оның вөлтерлігін таныймыз.

Түйін сөздер: аргументі ауытқыған теңдеу,толымдылық,базистік, вөлтерлік, Штурм -Лиувиллдің операторы, Рисстің базисі.

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СПЕКТРАЛЬНОЕ РАЗЛОЖЕНИЕ ФУНКЦИОНАЛЬНО-ДИФФЕРЕНЦИАЛЬНОГО ОПЕРАТОРА ПЕРВОГО ПОРЯДКА

Аннотация. В настоящей работе получено спектральное разложение функционаьно-дифференциального оператора первого порядка,с помощью прямого доказательства полноты системы системы собственных функций и теоремы Н.К.Бари.

Ключевые слова:уравнение с отклоняющимся аргументом, полнота, базисность, вольтерровость, операторы Штурма-Лиувилля, базис Рисса.

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