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# FORM FACTORS AND DENSITY DISTRIBUTIONS
# OF PROTONS AND NEUTRONS IN $^7$Li AND $^7$Be

**Abstract.** Form factors and density distribution of protons and neutrons in $^7$Li and $^7$Be are investigated within a microscopic two-cluster model. The model correctly treat the Pauli principle and make use of the oscillator basis to expand wave function of two-cluster system. Dynamics of two-cluster system is totally governed by a semirealistic nucleon-nucleon potential. We demonstrate that the model used correctly reproduces form factors for the ground state of $^7$Li and $^7$Be.

**Key words:** two-cluster system, form factors, density distribution, nucleon-nucleon potential etc.

**Introduction**

In this paper we consider the form factor of the elastic scattering of electron and density distribution of protons and neutrons in light nuclei $^7$Li and $^7$Be. In previous paper [1] we have investigated properties of light nuclei $^5$He, $^5$Li, $^6$Li, $^7$Li, $^7$Be and $^8$Be within a microscopic two-cluster model. This model is the algebraic version of the resonating group method. In Ref. [1] main attention was paid to the spectrum and wave functions of bound and resonance states in these nuclei, and also to phase shifts of the elastic two-cluster scattering. It was shown in Ref. [1] that the two-cluster model used correctly reproduces the main properties of light nuclei $^5$He, $^5$Li, $^6$Li, $^7$Li, $^7$Be and $^8$Be. The aim of the present investigations is to obtain complementary information on structure of bounds states in light nuclei $^7$Li and $^7$Be. This

Table 1 - Main input parameters of calculations and energy of the ground state in $^7$Li and $^7$Be.

| Nucleus | Potential | $b$, fm | $m/u$ | $f_{LS}$ | $E_{GS}$, MeV |
|---------|-----------|---------|-------|----------|---------------|
| $^7$Li  | MP        | 1.3451  | 0.969 | 1.00     | -2.468        |
|         | MHNP      | 1.362   | 0.0002| 1.00     | -2.467        |
| $^7$Be  | MP        | 1.3451  | 0.969 | 1.00     | -1.630        |
|         | MHNP      | 1.362   | 0.0002| 1.00     | -1.588        |

will be done within the same two-cluster model. It is well known that the form factor provides information on distribution of charge in a nucleus. Values of the form factor in the region of small transferred momenta allows one to determine the proton root-mean-square radius.

**Details of calculations and results**

As in previous paper [1], we represent $^7$Li and $^7$Be as two-cluster system $\alpha+t$ and $\alpha+^3$He, respectively. We use a common oscillator length $b$ to discribe distribution of nucleons in each cluster.

Two semirealistic nucleon-nucleon interactions: the Minnesota potential (MP) [2], [3] and the modified Hasegawa-Nagata potential (MHNP) [4, 5, 6] are employed in our calculations. As in Ref. [1] we select the oscillator length $b$ to minimize the two-cluster threshold. We also slightly modified the Majorana parameter $m$ of MHNP and the parameter $u$ of MP to reproduce the energy of the ground state accounted from the two-

cluster threshold. In Table 1 we display all input parameters of our calculations. In all our calculations we make of 200 oscillator functions. This number of functions guarantees a good precision for the energy and wave functions of bound states, even for weakly bound states. We have also checked that this number of oscillator functions provides form factors and densities distributions with acceptable precision.

Since we consider light nuclei $^7$Li and $^7$Be within two-cluster microscopic model, wave function of bound and continuos spectrum states is represented in the following form:

$$\Psi_{EJ^\pi} = \hat{A}\left\{\left[\Phi_1(A_1, s_1)\Phi_2(A_2, s_2)\right]_S \psi_{ELS}^{J^\pi}(q)Y_L(\hat{q})\right\}_J \tag{1}$$

Notations which we are using here are identical to those of Ref. [1].

Since wave functions $\Phi_1(A_1, s_1)$ and $\Phi_2(A_2, s_2)$, describing internal motion of nucleons inside each cluster are known and fixed, thus we have to find the inter-cluster function $\psi_{ELS}^{J^\pi}(q)$ by solving the dynamic equations of the resonating group method (RGM). In the standard version of the RGM, they are the integro-differential equation. In the algebraic version of RGM, which we employ, the dynamic equations are transformed into a set of linear algebraic equations. This is achieved by using a full set of the radial part of oscillator functions $\{\Phi_{nL}(q, b)\}$. By expanding the inter-cluster function $\psi_{ELS}^{J}(q)$ over oscillator functions

$$\psi_{ELS}^{J}(q) = \sum_{n=0}^{\infty} C_{nL;SJ}\Phi_{nL}(q, b) \tag{2}$$

or the total two-cluster function $\Psi_{EJ}$ over cluster oscillator functions $\{\mid nL; SJ\rangle\}$

$$\Psi_{EJ} = \sum_{n=0}^{\infty} C_{nL;SJ}\mid nL; SJ\rangle, \tag{3}$$

we arrive to a system of linear algebraic equations

$$\sum_{n=0}^{\infty}\{\langle nL|\hat{H}|\tilde{n}L\rangle - E\langle nL|\tilde{n}L\rangle\}C_{nL;SJ} = 0. \tag{4}$$

In real calculations, the infinite set of equations (4) can be reduced a finite set of equations. And thus to find energies and wave functions of bound state, we need to obtain eigenvalues and corresponding eigenfunctions of the $N \times N$ matrix

$$\left\|\langle nL|\hat{H}|\tilde{n}L\rangle\right\|_N.$$

It worthwhile noticing that, by assuming that the wave function $\Psi_{EJ^\pi}$ of a bound state is obtained, one has to calculate the matrix element

$$F_p(q) = \left\langle \Psi_{EJ^\pi}\left|\hat{F}_p\right|\Psi_{EJ^\pi}\right\rangle \tag{5}$$

$$F_n(q) = \left\langle \Psi_{EJ^\pi}\left|\hat{F}_n\right|\Psi_{EJ^\pi}\right\rangle \tag{6}$$

in order to determine the proton and neutron form factors, and one has also to calculate matrix elements

$$D_p(\boldsymbol{r}) = \left\langle \Psi_{EJ^\pi}\left|\frac{1}{2}\sum_{i=1}^{A}(1 + \hat{\tau}_{iz})\delta(\boldsymbol{r} - \boldsymbol{r}_i)\right|\Psi_{EJ^\pi}\right\rangle, \tag{7}$$

$$D_n(\boldsymbol{r}) = \left\langle \Psi_{EJ^\pi}\left|\frac{1}{2}\sum_{i=1}^{A}(1 - \hat{\tau}_{iz})\delta(\boldsymbol{r} - \boldsymbol{r}_i)\right|\Psi_{EJ^\pi}\right\rangle, \tag{8}$$

to obtain the density distribution of protons $D_p(\boldsymbol{r})$ and neutrons $D_n(\boldsymbol{r})$. Here the operators $\hat{F}_p$ and $\hat{F}_n$ are

$$\hat{F}_p = \frac{1}{2}\sum_{i=1}^{A}(1 + \hat{\tau}_{iz})\exp\{i(\mathbf{qr_i})\} \tag{9}$$

$$\hat{F}_n = \frac{1}{2}\sum_{i=1}^{A}(1 - \hat{\tau}_{iz})\exp\{i(\mathbf{qr_i})\} \tag{10}$$

and the operators $\frac{1}{2}(1 + \hat{\tau}_{iz})$ and $\frac{1}{2}(1 - \hat{\tau}_{iz})$ are projection operators on proton and neutron state, respectively. In equations (5), (6), (7) and (8), integration is carried out over all spatial, spin and isospin coordinates.

It is worth noticing that by calculating both proton and neutron form factor or densities we can easily construct the matter form factors or densities from the relations

$$F_m(q) = F_p(q) + F_n(q)$$

$$D_m(\boldsymbol{r}) = D_p(\boldsymbol{r}) + D_n(\boldsymbol{r})$$

Recall that the nucleon densities are normalized by the conditions

$$\int dr D_p(\boldsymbol{r}) = Z, \quad \int dr D_n(\boldsymbol{r}) = N, \quad \int dr D_m(\boldsymbol{r}) = A,$$

where Z is the total number of protons, N is the total number of neutrons, and the mass number A=Z+N.

**Form factor**

In section we consider form factor of elastic scattering of electrons.

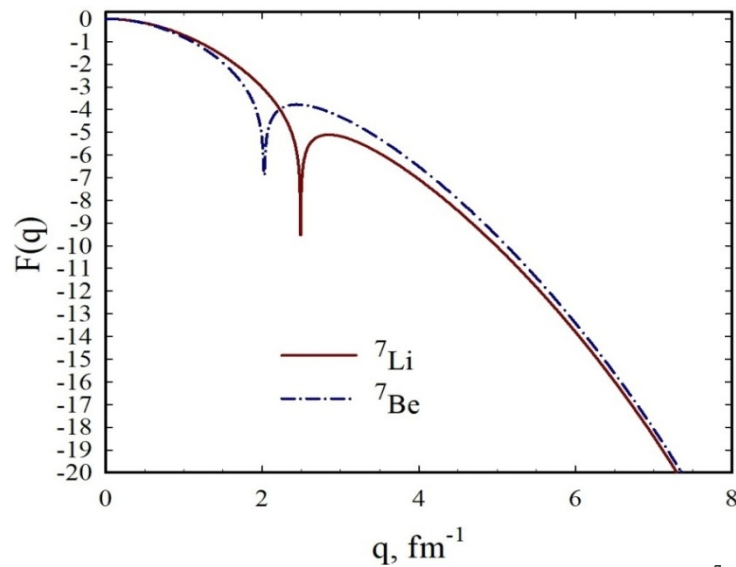The elastic form factors for ground $3/2^-$ states of $^7$Li and $^7$Be are presented in Fig. 1. The form factors are calculated with MHNP.



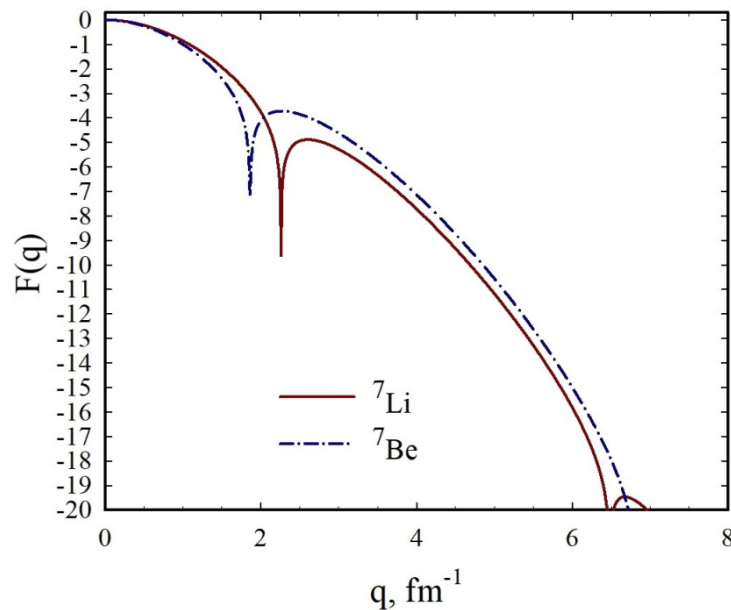Figure 1 - Form factors of the elastic electron scattering from the ground states of $^7$Li and $^7$Be.



Figure 2 - Form factor F(q) of elastic electron scattering on the first excited $1/2^-$ states in $^7$Li and $^7$Be.

In Fig. 2 we display form factor of electron elastic scattering from the excited $1/2^-$ states in [7]Li and [7]Be. These results are also obtained with MHNP.

On both figures 1 and 2 we use the logarithmic scale and indicate only an exponent of $\log(F(q))$. We can see that the form factors for the ground and first excited states rapidly turn to the zero with increasing of the momentum transfer q. The form factor for the ground state of [7]Li has a node at q=2.49 fm$^{-1}$, while for [7]Be the first node appear at q=2.03 fm$^{-1}$. For the first excited $1/2^-$ states, the position of a node slightly shifted to small values of the momentum transfer q and equals to q=2.26 fm$^{-1}$ for [7]Li and equals to q=1.86 fm$^{-1}$ for [7]Be.

The present results are in agreement with other microscopic model. For example, the form factors for the ground state of [7]Li and [7]Be, presented above, are very close to those obtained in Refs. [7], [8], [9] within the other realization of the resonating group method. There is also fairly good agreement with the experimental data Ref. [10].

**Density distributions**

Now we turn our attention to the density distribution of proton and neutrons. The proton and neutron density distributions for the [7]Li ground states are shown in Fig. 3. They are calculated with MHNP. As we see the protons are mainly concentrated at small distances, while the neutron density distribution is more dispersed in the space.

In Figure 4 we display the proton and neutron density distribution for the ground state of [7]Be. This nucleus is mirror to the nucleus [7]Li, and thus we have inverse picture for proton and neutron density distributions with respect to Fig. 3. Despite that the energy of the [7]Be is smaller than the ground state energy of [7]Li, both nuclei have similar density distributions of proton and neutron.

**Conclusion**

We have applied a two-cluster microscopic model to study density distribution of protons and neutrons in light atomic nuclei [7]Li and [7]Be. We also have studied the form factors of elastic electron scattering from these nuclei. The microscopic model which has been employed is the algebraic version of the resonating group method. The later is using the full set of oscillator functions to expand wave functions of bound and scattering states. Nucleon-nucleon interaction being a key ingredient of any microscopic model was represented by two semirealistic potentials which are often used in two- and three-cluster microscopic models.

It was demonstrated that the present two-cluster model reproduces fairly well the elastic form factors of [7]Li and [7]Be as a function of the momentum transfer. It was also demonstrated that our results are compatible with results of other theoretical models.
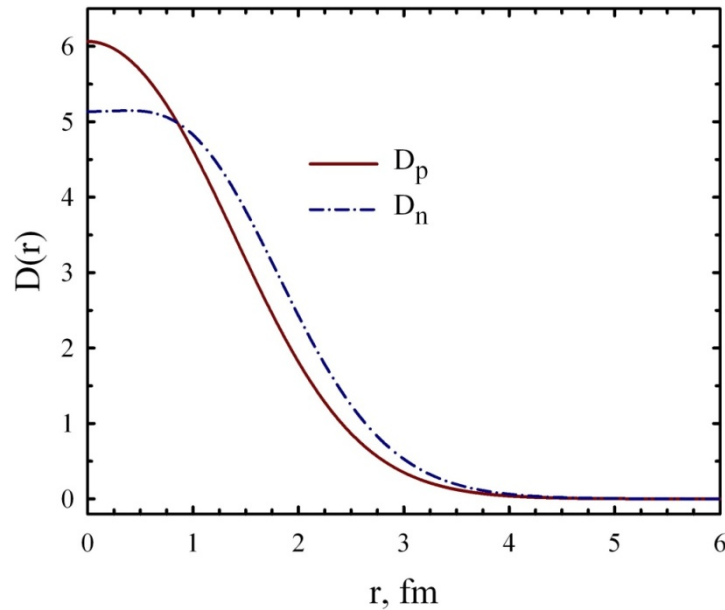


Figure 3 - The proton and neutron density distribution in the ground state of [7]Li.

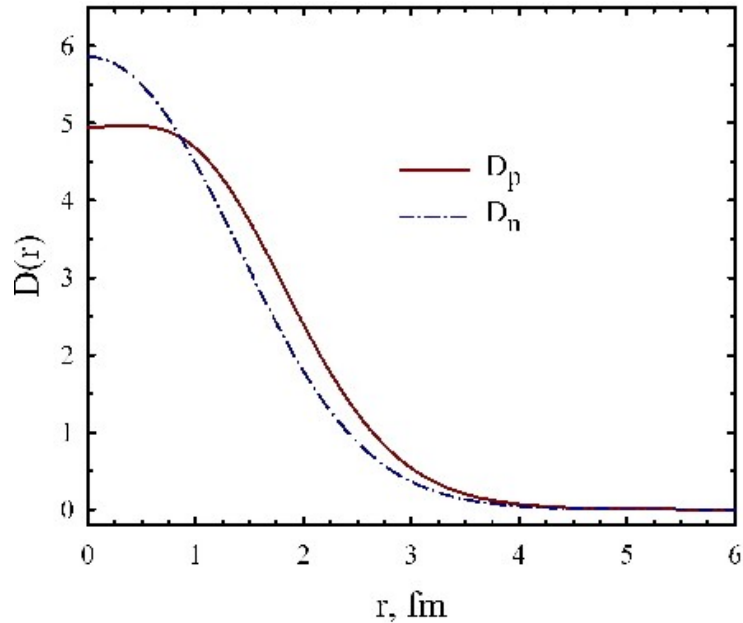Figure 4 - The proton and neutron density distributions in the ground state of $^7$Be

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### $^7$LI ЖӘНЕ $^7$BE ЯДРОЛАРЫНДАҒЫ ПРОТОНДАР МЕН НЕЙТРОНДАР ТЫҒЫЗДЫҒЫНЫҢ ТАРАЛУЫ МЕН ФОРМ ФАКТОРЛАРЫ

**Аннотация.** $^7$Li және $^7$Be ядроларындағы нейтрондар мен протондар тығыздығының таралуы мен форм факторлары микроскопиялық екі кластерлік модель ретінде зерттелінді. Бұл модель Паули принципін ескере отырып, екі кластерлік жүйенің толқындық функциясын жіктеу үшін осцилляторлық негізді қолданады. Екі кластерлік жүйенің динамикасы толығымен жартылай реалистік нуклон-нуклондық потенциалымен анықталады. Қолданылып отырған модель $^7$Li және $^7$Be ядроларының негізгі күйлері үшін форм-факторды дұрыс беретіндігін көрсетеміз.

**Түйін сөздер:** екі кластерлік жүйе, форм факторлар, тығыздықтың таралуы, нуклон-нуклондық потенциал және т.б.

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## ФОРМ ФАКТОРЫ И РАСПРЕДЕЛЕНИЕ ПЛОТНОСТИ ПРОТОНОВ И НЕЙТРОНОВ В ЯДРАХ $^7$LI И $^7$BE

**Аннотация.** Форм факторы и распределение плотности протонов и нейтронов в ядрах $^7$Li и $^7$Be исследованы в рамках микроскопической двухкластерной модели. Модель правильно учитывает принцип Паули и использует осцилляторную основу для разложения волновой функции двухкластерной системы. Динамика двухкластерной системы полностью определяется полуреалистическим нуклон-нуклонным потенциалом. Мы показываем, что используемая модель правильно воспроизводит форм-факторы для основного состояния $^7$Li и $^7$Be.

**Ключевые слова:** двухкластерная система, форм факторы, распределение плотности, нуклон-нуклонный потенциал и т.д.

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